Compressed fast solution of the acoustic wave equation

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Motivation:

Seismic Oil Exploration

- Seismic oil exploration requires the processing of very large scale data sets. The data is collected in the time domain, but maps of the underground are required in the space domain.

- The most sophisticated processing is called Depth Migration and until recently only approximate solutions to the wave equation have been used, such as the one way or parabolic approximation. This has some limitations in complex areas.

- One of the currently more expensive processes (Reverse Time Migration) uses full wave simulation from the sources and backward propagation of the seismic data from the receivers in order to create these depth migrated images.

- In 3D it is not unusual to manipulate many terabytes of data in large computer clusters. These problems are still a challenge even with large dedicated clusters.
Feasibility

- Thus, it would be of interest to be able to run these repeated simulations faster and using less data.

- Model order reduction is a technique that has been used in many application domains to do just that.

- Since there is a paucity of work for applications to the wave equation, we are performing some feasibility studies in order to see if the resulting approximate solutions are cost effective for some of these problems.

First efforts were published in: **FASTWAVE PROPAGATION BY MODEL ORDER REDUCTION. V. PEREYRA AND B. KAELIN. ETNA 30, 2008.**
Consider the wave equation in inhomogeneous media, discretized in space:

$$\frac{\partial^2 w}{\partial t^2} = Aw + Bu(t),$$

$$v = Cw,$$

Where $x \in \mathbb{R}^l$, $w(t) \in \mathbb{R}^n$ and $A$ (discrete Laplacian), $B$ (source term), $C$ (output) are appropriate matrices, $u(t)$ is an input forcing function and $v$ is the desired output. $n$ will usually be very large.
Basic Idea

- The dynamics is obtained by integrating this very large system of Ordinary Differential Equations (method of lines).

- The procedure we propose attempts to obtain an approximate solution by reducing drastically the size of this system of ODE’s.

- This is specially important and useful when rapid response or repeated simulation are required.

- It is assumed that we have the ability to perform a few full size (high fidelity) solves.
Proper Orthogonal Decomposition

- From one or a few full fidelity simulations we extract a certain number of snapshots: i.e., spatial states of the field at different times, \( f_i, i=1,\ldots,m \).

- We form a matrix \( F \) (\( n \times m \)), where \( n \) is the number of degrees of freedom of the original discretization (veeery large) and \( m \) is the number of snapshots (relatively small).

- Then we proceed to calculate its Singular Value Decomposition:

\[
F = U_m S_m V_m^T.
\]

- This calculation serves two purposes. On one hand, the matrix \( U_m \) is an orthogonal basis for the space of columns of \( F \), the snapshots. On the other hand, the Singular Values, \( \text{diag} (S_m) \), give us information on which columns of \( F \) are most important.
Reduced System

• Now we use Galerkin collocation on the original equations, by proposing the Ansatz:

\[ w = U a(t). \]

• The key here is to introduce the time dependence through the weights \( a(t) \).

• Replacing in the wave equation we obtain the reduced system (of size \( m \) or smaller):

\[ a_{tt} = (U^T A U) a(t) + (U^T B) u(t) \]

Observe that we still can input a source (it does not have to be the same source wavelet or even be applied at the same location as in the original problem); we can also change the material properties or change the output in this reduced equation. i.e., we can change \( A, B \) (not drastically, obviously).
High Fidelity and MOR Codes

- As a full fidelity code we use an 8th order finite difference approximation in space and a Runge-Kutta-Fehlberg fourth order integrator in time. Absorbing boundary conditions are included (R. Kosloff and D. Kosloff, Absorbing boundaries for wave propagation problems. J. Comp. Physics 63:363-376 (1986).)

- The second order acoustic wave equation is written in first order form (as required by RKF45), so that doubles the number of ODE’s for the time integration.

- The MOR code implements the procedure indicated above, using snapshots of previously run HF shots. We are currently using two HF simulations to generate the basis of snapshots, corresponding to the end points of a sequence of evenly spaced shots.
In this example we consider an inhomogeneous model with the following specs:

- \( dx = dz = 6.25 \text{m} \); \( dt = 0.05 \text{sec} \) (snapshot spacing)
- \( nx = 2001 \), \( nz = 2001 \) (total number of space mesh \( n = 4004001 \)).
- Nine shots are placed on the free surface, starting at \( x = 5931.25 \text{ m} \), spaced by 25 m for a total distance of 200 m.
- We take 200 snapshots from the first and last shot simulations, for a total of 400 snapshots. With a threshold of the SV’s of 0.01 we select \( m = 363 \) basis snapshots.
- We use RKF45, a standard Runge-Kutta 4th order solver to integrate the 8008002 ODE’s in time. RKF45 adapts its integration step to preserve stability and achieve a prescribed accuracy (in this case 0.001).
A Complex, Large Scale 2D Model. Vel4, a Salt Body Courtesy of BP

Velocity model. 2001 x 2001 mesh. Salt velocity is very high 13,000 ft/sec
Observe that if we consider the preprocessing time and prorate it over the seven internal shots, then the gain in time is much smaller, but still significant: 2.53.

This factor would be much larger in 3D or if we consider more internal shots.
Crossplot of a Trace: HF vs MOR
Midway Shot

Crossplot for model vel4. Trace at x=5931.25.
Shot at x=6312.5

[Graph showing HF and MOR]
Progressive QR instead of SVD

- As a faster alternative we can process incoming snapshots as they are produced and do a tentative QR decomposition.

- If the angle between the new snapshot and the existing basis is sufficiently large, then we accept it into the basis, otherwise we reject it.

- In this way we have an adaptive snapshot selection, and at the end we have a well conditioned basis. The Q in the QR decomposition is our orthogonal basis.

- This has been tested and gives very good results.
The key next step

- In this evaluation stage one can monitor the true error by comparing the MOR and the HF solutions as one walks away from the problems that generated the basis snapshots. We have done that and as we showed in the example above, there are increasing errors as we solve problems farther away, but for instance, the errors seem to be concentrated in the amplitudes, while the phases of the events are very good. This is good, since the phases are more important than the amplitudes in migration algorithms.

- Thus, it is important not to lose sight of the final objective of the simulations (whatever that might be). We are planning to test this technique (with Chevron Research Technology Co.) for different applications in seismic data processing, to see how the approximation error affects the final result. The more MOR we can use, the more efficient the overall approach will be. (Somebody should write a Limerick with this material)