MULTICENTRIC CALCULUS: STILL
ANOTHER LOOK AT $\varphi(A)$

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Abstract

We outline a path to multicentric calculus for evaluating $\varphi(A)$. Consider

$$V_p(A) = \{ z \in \mathbb{C} : |p(z)| \leq \|p(A)\| \}$$

where $p$ is a polynomial and $A$ a bounded linear operator (or matrix). Intersecting these sets over polynomials of degree 1 gives the closure of the numerical range, while intersecting over all polynomials gives the spectrum of $A$, with possible holes filled in [1].

Outside any set $V_p(A)$ one can write the resolvent down explicitly and this leads to multicentric holomorphic functional calculus [3], [4].

In this talk we shall mention how the multicentric calculus leads to a nontrivial extension of von Neumann theorem

$$\|f(A)\| \leq \sup_{|z| \leq 1} \|f(z)\|$$

where $A$ is a contraction in a Hilbert space, [5], and then conclude with some new unpublished results on nonholomorphic functional calculus for operators for which $p(A)$ is (similar to ) normal at a nontrivial polynomial $p$. The results are new even for matrices. So, if the matrix has nontrivial Jordan blocks, one does not need to assume that $\varphi$ is differentiable at those eigenvalues. We give a natural Banach algebra of functions - which are easy to write down and for which $\varphi(A)$ are well defined.

References