Environmental Superstatistics

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6 Environmental superstatistics: rainfall fluctuations, flooding, heat-waves, global warming, and more...
1 What is superstatistics?

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Consider Brownian particle moving through spatio-temporal inhomogeneous environment with temperature fluctuations on a large scale.
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Mixture of two statistics:

- locally ordinary Brownian motion at inverse temperature $\beta_i$
- superimposed to this is the stochastic process of inverse temperatures $\beta_i$
Can construct dynamical realization in terms of Langevin equation

\[ \dot{v} = -\gamma v + \sigma L(t) \]


Example: assume probability distribution of \( \beta = \frac{\gamma}{2\sigma^2} \) in the various cells is \( \chi^2 \)-distribution of degree \( n \)

\[ f(\beta) \sim \beta^{n/2-1} e^{-\frac{n\beta}{2\beta_0}} \]

(e.g. \( \beta = \sum_{i=1}^{n} X_i^2 \)) and that \( \beta \) varies on a much larger time scale than local relaxation time.

Conditional prob. \( p(v|\beta)) \sim e^{-\frac{1}{2}\beta v^2} \)

Joint prob. \( p(v, \beta) = f(\beta)p(v|\beta) \)

Marginal prob. \( p(v) = \int_{0}^{\infty} f(\beta)p(v|\beta)d\beta \)

Integration yields

\[ p(v) \sim \frac{1}{(1 + \frac{1}{2}\tilde{\beta}(q - 1)v^2)^{1/(q-1)}} \]

(power-law generalized Boltzmann factors with \( q = 1 + \frac{2}{n+1}, \tilde{\beta} = 2\beta_0/(3 - q), \) and \( E = \frac{1}{2}v^2 \))

\[ \beta_0 = \int f(\beta)\beta d\beta = \text{average of } \beta \]

Very broad interpretation—\( \beta \) need not be inverse temperature
Can generalize the above example to general probability densities $f(\beta)$ and general Hamiltonians.

Superposition of two different statistics: that of $\beta$ and that of ordinary stat. mech.

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Can further generalize to systems outside physics: Consider a relatively simple dynamics locally, depending on parameters $\beta, \lambda, ...$ (biology, finance, ...). Consider a complex environment (networks, nonequilibrium situations, ...) responsible for $\beta, \lambda, ...$ fluctuations on a much larger scale.
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Take this as a simple model of the complex system under consideration. Often some partial analytic treatment is possible and essential features of the complex system can be understood in this way.

Effective Boltzmann factors $B(E)$ of this approach are given by

$$B(E) = \int_{0}^{\infty} f(\beta) e^{-\beta E} d\beta$$

$f(\beta)$: probability distribution of $\beta$.
Many results can be proved for general $f(\beta)$. 
Some recent theoretical developments of the superstatistics concept:

- Can formally define generalized entropies for general superstatistics (Tsallis and Souza, Phys. Rev. E (2003))
- Can study various theoretical extensions of the superstatistics concept (Chavanis (2005), Vignat, Plastino (2005), Grigolini et al. (2005), Crooks (2006), Naudts (2007), Abe (2007))
- Study general symmetry group properties of superstatistics (Gell-Mann et al, PNAS 2012)
- Can consider superstatistical random matrix theory (Abul-Magd 2006-2012)
- Can apply superstatistical techniques to networks (Abe & Thurner 2005)
- Superstatistical path integrals (Jizba & Kleinert 2008-2012)
...and some more practical applications:


- Can apply it to atmospheric turbulence (wind velocity fluctuations at Florence airport, Rizzo & Rapisarda (2004))

- Can apply superstatistical methods to finance (Bouchard 2003, Ausloos 2003, Duarte Queiros 2005, Anteneodo 2008, ...2012)

- Can apply it to cosmic ray and high energy scattering statistics (C.B. 2004, 2009, Wilk 2012)

- Can apply it to hydroclimatic fluctuations (Porporato et al. 2006)

- Can apply it to train delay statistics (Briggs et al. 2007)

- Medical applications (Chen et al. 2008)
2 Typical classes of superstatistics

In experiments, one often observes 3 physically relevant universality classes (C.B., E.G.D. Cohen, H.L.Swinney, PRE 2005):

• (a) $\chi^2$-superstatistics (＝ Tsallis statistics)
• (b) inverse $\chi^2$-superstatistics
• (c) lognormal superstatistics
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Why? Consider, e.g., case (a). Assume there are many microscopic RV $\xi_j$, $j = 1, \ldots, J$, contributing to $\beta$ in an additive way. For large $J$, sum $\frac{1}{\sqrt{J}} \sum_{j=1}^{J} \xi_j$ will approach a Gaussian random variable $X_1$ due to the Central Limit Theorem.

There can be $n$ Gaussian random variables $X_1, \ldots, X_n$ due to various relevant degrees of freedom in the system.

$\beta$ positive $\Rightarrow \beta = \sum_{i=1}^{n} X_i^2$ is $\chi^2$-distributed with degree $n$,

$$f(\beta) = \frac{1}{\Gamma\left(\frac{n}{2}\right)} \left( \frac{n}{2\beta_0} \right)^{n/2} \beta^{n/2-1} e^{-\frac{n\beta}{2\beta_0}} \quad (1)$$
(b) Same considerations can be applied if the 'temperature' $\beta^{-1}$ rather than $\beta$ itself is the sum of several squared Gaussian random variables arising out of many microscopic degrees of freedom $\xi_j$. Resulting $f(\beta)$ is the inverse $\chi^2$-distribution:

$$f(\beta) = \frac{\beta_0}{\Gamma(n/2)} \left( \frac{n\beta_0}{2} \right)^{n/2} \beta^{-n/2-2} e^{-n\beta_0/2\beta}.$$  

(2)

It generates superstatistical distributions $p(E) \sim \int f(\beta)e^{-\beta E}$ that that decay as $e^{-\tilde{\beta}\sqrt{E}}$ for large $E$. 

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It generates superstatistical distributions \( p(E) \sim \int f(\beta) e^{-\beta E} \) that decay as \( e^{-\tilde{\beta} \sqrt{E}} \) for large \( E \).

(c) \( \beta \) may be generated by multiplicative random processes. Local cascade random variable \( X_1 = \prod_{j=1}^{J} \xi_j \), where \( J \) is the number of cascade steps and the \( \xi_j \) are positive microscopic random variables. By the Central Limit Theorem, for large \( J \) the RV \( \frac{1}{\sqrt{J}} \log X_1 = \frac{1}{\sqrt{J}} \sum_{j=1}^{J} \log \xi_j \) becomes Gaussian for large \( J \). Hence \( X_1 \) is log-normally distributed. In general there may be \( n \) such product contributions to \( \beta \), i.e., \( \beta = \prod_{i=1}^{n} X_i \). Then \( \log \beta = \sum_{i=1}^{n} \log X_i \) is a sum of Gaussian random variables; hence it is Gaussian as well. Thus \( \beta \) is log-normally distributed, i.e.,

\[
f(\beta) = \frac{1}{\sqrt{2\pi s\beta}} \exp \left\{ -\frac{(\ln \beta - \mu)^2}{2s^2} \right\}, \tag{3}
\]
3 Application to traffic delays

Departure delay statistics on the British rail network

0th-order model: Poisson process $P(t|\beta) = \beta e^{-\beta t}$. 
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Departure delay statistics on the British rail network

0th-order model: Poisson process \( P(t|\beta) = \beta e^{-\beta t} \).

Does not agree with data (red). Much better fit by power law (green).

![Graph showing departure delay statistics with green line representing the prediction of the model (K. Briggs and C. Beck, Modelling train delays with q-exponential functions, Physica A 378, 498 (2007))](image)
What may cause this power law? There are fluctuations in the parameter $\beta$ as well. These fluctuations describe large-scale temporal or spatial variations of the British rail network environment. Examples

- begin of holiday season with lots of passengers
- problem with the track
- bad weather conditions
- extreme events such as derailments, industrial action, terror alerts, etc.

Long-term distribution of train delays is then a mixture of exponential distributions where the parameter $\beta$ fluctuates.

$$p(t) = \int_0^\infty f(\beta)p(t|\beta)\,d\beta = \int_0^\infty f(\beta)\beta e^{-\beta t}.$$  

(4)

$\chi^2$-distributed $\beta$ with $n$ degrees of freedom yields

$$p(t) \sim (1 + b(q - 1)t)^{1/1-q}$$  

(5)

where $q = 1 + 2/(n + 2)$ and $b = 2\beta_0/(2 - q)$. Our model generates $q$-exponential distributions of train delays by a simple mechanism, namely a $\chi^2$-distributed parameter $\beta$ of the local Poisson process.
4 Application to turbulence

Bodenschatz et al., Nature (2001)

Measurements of acceleration $\vec{a}$ of single tracer particle in turbulent flow.
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Measurements of acceleration $\vec{a}$ of single tracer particle in turbulent flow.

colour code:

blue.......green...yellow

$|\vec{a}| = 0..................16000 \ m/s^2$
Experimental data one wants to understand:

- **Histogram of accelerations (strongly non-Gaussian):**
- **Correlations of acceleration components (cross-like structure):**
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2 possible theoretical approaches:
- give up! (system too complex!)
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2 possible theoretical approaches:
— give up! (system too complex!)
— construct simple superstatistical solvable model
Superstatistical Lagrangian model for 3-dim velocity difference of tracer particle $\vec{u}(t) := \vec{v}(t + \tau) - \vec{v}(t)$. (C.B., PRL 98, 064502 (2007))

(note that $\vec{a} = \vec{u}/\tau$ for small $\tau$)

$$\dot{\vec{u}} = -\gamma \vec{u} + B \vec{n} \times \vec{u} + \sigma \vec{L}(t).$$

(6)

$\gamma$ and $B$: constants
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Noise strength $\sigma$ and the unit vector $\vec{n}$ evolve stochastically on a large time scale $T_{\sigma}$ and $T_{\vec{n}}$, respectively. ($\implies$ superstatistics) $T_{\sigma}\gamma \sim T_L/\tau_\eta \sim R_\lambda \gg 1$

Time scale $T_{\vec{n}} \gg \tau_\eta$ describes the average life time of a region of given vorticity surrounding the test particle.
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Time scale $T_\vec{n} >> \tau_\eta$ describes the average life time of a region of given vorticity surrounding the test particle.

Define $\beta := 2\gamma / \sigma^2$

$$\beta^{-1} \sim \nu^{1/2} \langle \epsilon \rangle^{-1/2} \epsilon,$$ where $\nu$ is the kinematic viscosity and $\langle \epsilon \rangle$ the average energy dissipation.

Probability density of the stochastic process $\beta(t)$ assumed to be a lognormal distribution

$$f(\beta) = \frac{1}{\beta s \sqrt{2\pi}} \exp \left\{ -\frac{(\log \frac{\beta}{m})^2}{2s^2} \right\}.$$  \hspace{1cm} (7)
For very small $\tau$ the acceleration of the particle is given by $a_x = u_x/\tau$ and one gets the 1-point distribution

$$p(a_x) = \frac{\tau}{2\pi s} \int_{0}^{\infty} d\beta \beta^{-1/2} \exp \left\{ -\frac{(\log \frac{\beta}{m})^2}{2s^2} \right\} e^{-\frac{1}{2} \beta \tau^2 a_x^2}. \quad (8)$$
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Experimentally measured probability distribution of acceleration:

strongly non-Gaussian
3-dim superstatistics induces correlations between components: Study $R := p(a_x, a_y)/(p(a_x)p(a_y))$.
For independent acceleration components this ratio would always be given by $R = 1$. However, our 3-dim superstatistical model yields prediction

$$R = \frac{\int_0^\infty \beta f(\beta) e^{-\frac{1}{2}\beta \tau^2 (a_x^2 + a_y^2)} d\beta}{\int_0^\infty \beta^{1/2} f(\beta) e^{-\frac{1}{2}\beta \tau^2 a_x^2} d\beta \int_0^\infty \beta^{1/2} f(\beta) e^{-\frac{1}{2}\beta \tau^2 a_y^2} d\beta}$$

(9) General formula.
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General formula.

Note that $R = 1$ for $f(\beta) = \delta(\beta - \beta_0)$
$R := \frac{p(a_x, a_y)}{p(a_x)p(a_y)}$ as predicted by lognormal superstatistics:
\( R := p(a_x, a_y)/(p(a_x)p(a_y)) \) as predicted by lognormal superstatistics:

\( R \) as measured by Bodenschatz (New Journal Physics 2005)
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$R$ as measured by Bodenschatz (New Journal Physics 2005)

Big mystery: Why do highly nonlinear complex systems effectively generate such a simple superstatistical process?
5 Application in medicine

Complex cell migration processes in cancerous systems

Gives the following prediction for probability density function of survival time $t$ once diagnosed with cancer

$$p(t) = \int_0^\infty t^{n-1} \frac{\lambda^n e^{-\lambda t}}{\Gamma(n)} \frac{\lambda_0 (n\lambda_0/2)^{n/2}}{\Gamma(n/2)} \lambda^{-n/2-2} e^{-\frac{n\lambda_0}{2\lambda}} d\lambda,$$

(10)

or

$$p(t) = \frac{(n\lambda_0)^{3n/4}}{\Gamma(n)\Gamma(n/2)} \left(\frac{t}{2}\right)^{3n/4-1} \left[ \sqrt{2n\lambda_0 t} K_{n/2+1} \left(\sqrt{2n\lambda_0 t}\right) - K_{n/2} \left(\sqrt{2n\lambda_0 t}\right) \right],$$

(11)

$K_\nu(z)$: modified Bessel function

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(a) linear

(b) log

(c) semi-log

(d) log-log
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Idea: Define ‘super-Hamiltonian’ with artificially constructed energy levels

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Small circles: local system, energy levels \( E_i \).
entire system (super-Hamiltonian): \( E_i^{(j)} \), \( \beta_0 \): average inverse temperature
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Do ordinary statistical mechanics with the super-Hamiltonian, translate results back to original setting. Leads effectively to more general Boltzmann factors, entropies, free energies by averaging over \( \beta \).


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Conjecture (to be tested)
Many dynamical phenomena observed on Planet Earth are well-described by spatio-temporal superstatistical models.
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New interdisciplinary Collaboration funded by EPSRC with 2 Mio (start date 15th January 2013): Flood MEMORY (Newcastle, Queen Mary University of London, Edinburgh, Aberdeen, Swansea, Univ. West England, Cranfield, Nottingham, Southampton, Univ West England, NOC)

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Theoretical emphasis on clustered and superstatistical processes. Look, for example, at Poisson processes for rainfall events where the rate constant is fluctuating in a spatio-temporal way, include memory kernels in the superstatistical Langevin equation, ...
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Test predictions from extreme value theory, compound Poisson process models, hitting time statistics, ... (Vaienti, Holland, Pollicott, Freitas\(^2\), Lucarini) on real data, compare with various applied approaches (Deidda, Speranza, Kantz, Kurths)
Besides precipitation statistics one can look at temperature statistics:

Interesting to just look at distributions of inverse temperature $f(\beta)$ as observed at various locations on Planet Earth—superstatistics with these $f(\beta)$ relevant for thermodynamic devices that are supposed to work in open air.

Did that for Darwin, Santa Fe, Dubai, Sydney, London, Vancouver, Hong Kong, Ottawa, Eureka, ...

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Again asymptotic decay of $B(E)$ for large $E$ can be understood using techniques from large deviation theory (H. Touchette, C.B., PRE 2005) Relevant for the behavior of $B(E)$ for $E \to \infty$ is the behaviour of $f(\beta)$ for $\beta \to 0$, i.e. the statistics of heatwaves in very hot summers.
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The original superstatistics concept was for sharply peaked distributions $f(\beta)$ with a single maximum (C.B., E.G.D. Cohen, 2003). But in an environmental context this concept needs to broadened: Typically observed distributions at various locations on Planet Earth are double-peaked (due to seasonal variations), not single-peaked.
FIG. 23: Distribution of the daily measured inverse mean temperature in Santa Fe for 1998-2011

FIG. 24: Distribution of the daily measured inverse mean temperature in Dubai for 1974-2011
FIG. 26: Distribution of the daily measured inverse mean temperature in Central England (Lancashire, London and Bristol) for 1910-2011

FIG. 27: Distribution of the daily measured inverse mean temperature in Vancouver for 1937-2011
FIG. 32: Average of daily measured mean temperature of every single year in Eureka 1951-2011

FIG. 33: Average of daily measured mean temperature of every single year in Eureka 1973-2011
FIG. 35: Average of daily measured mean temperature of every single year in Sydney 1910-2011

FIG. 36: Average of daily measured mean temperature of every single year in Vancouver 1937-2011
Summary:

- **Superstatistical** techniques can generally be applied if there is sufficient time scale separation.

- So far three major physically relevant **universality classes** have been studied: $\chi^2$-superstatistics, inverse $\chi^2$-superstatistics, and lognormal superstatistics. These arise as **universal limit statistics** for many different systems.

- Train delays on British railway networks: Example of $\chi^2$ superstatistics = Tsallis statistics

- A superstatistical model of **Lagrangian turbulence** (C.B., PRL 2007) is in excellent agreement with the experimental data for probability densities, correlations between components, decay of correlations, Lagrangian scaling exponents, ... (lognormal superstatistics). Recently generalization to **quantum turbulence** (C.B., S. Miah, arXiv:1207.4062)

- Cancer survival described by inverse $\chi^2$ superstatistics

- Flood MEMORY project on spatio-temporally clustered processes: Aim is to develop superstatistical models to better understand rainfall and flooding data (2013)

- Environmentally relevant superstatistical distributions $f(\beta)$ for Planet Earth are more complicated: Double-peak structure, nonstationarity due to global warming, etc. (C. Yalcin, C.B., arXiv:1212.5783)