PhD Project Proposal 6

Large-scale least squares solvers with applications

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This project, which is a joint project between the University of Reading and the Science and Technology Facilities Rutherford Appleton Laboratory (RAL), focuses on developing a new generation of algorithms for solving sparse numerical linear algebra problems that arise in practical applications, with a strong emphasis on large-scale least squares and related problems. The algorithms need to be able to tackle extremely large problems and to be accompanied by rigorous error analysis to guarantee their reliability. Because of the problem size, advantage must be taken of the underlying structure.

Solution methods for least squares problems based on sparse matrix factorizations can handle modest-size linear systems but when the problem size is so large that storing and factoring the system matrix is not feasible, or if a low accuracy solution is required cheaply, an iterative solver is employed. In general, to achieve acceptable performance, a preconditioner that transforms the system to a 'nicer' system is needed. Currently, limited options are available for the ill-conditioned systems arising from least squares problems and those preconditioners that have been proposed are not fully robust and/or are not suitable for executing in parallel.

Domain decomposition (DD) preconditioners are constructed by decomposing the global problem into local subproblems that can be solved independently, making them suitable for use on emerging architectures. Although initially proposed for solving partial differential equations, DD methods have been used for solving the linear systems arising in other contexts. However, there have been limited attempts to use them in the solution of least squares problems.

This project will design and implement novel DD preconditioners. It will build a general DD framework for least squares problems and will consider sparse problems, sparse problems that include some dense rows, weighted least squares, and sequences of least squares problems. The use of variable precision arithmetic will be explored to speed up the computation and further expand the sizes of systems that can be solved.

Solving practical applications will be a key aspect of the project. It is envisaged that these will include image reconstruction, in particular, bundle adjustment (BA), cosmic microwave background (CMB) data analysis in astrophysics, and data fitting in weather forecasting. BA refines a visual reconstruction to produce jointly optimal structure and viewing parameter estimates. BA is a key component in the algorithms behind the street view in Google Maps. It is often formulated as a nonlinear least squares problem, which is solved by solving a series of linear systems. This is normally the computational bottleneck. Similarly, CMB data analysis involves solving many thousands of very large linear systems while for weather forecasting not only are the systems very large but they must be solved rapidly.

Requirements: A strong background in mathematics, including numerical linear algebra, is essential and programming skills in MATLAB/Python/Julia or a high level language are desirable.

The supervisory team will meet with the student each week. The student will spend time at RAL on a regular basis. Thus the project offers a unique opportunity not only to belong to the Department of Mathematics and Statistics at the University of Reading but also to be a part of the Computational Mathematics Group at the internationally renowned RAL.