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Dynamic Macroeconomic Approaches to
the Balance of Payments and the Exchange Rate

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Abstract

Having explored various implications of the comparative statics analysis in the preceding chapter, we now turn to deterministic (i.e., not stochastic) dynamic macroeconomic models of balance of payments adjustment and exchange rate determination. These models are based on postulated aggregate relationships in open-economy environments and are dynamic insofar they are solved under perfect foresight, which is the strongest form of rational expectations. After briefly introducing the concept of rational expectations in section 1, we first review in section 2 the stock, or asset (market), model of the BoP and the NER under flexible prices. This is the monetary approach developed in the late 1960s and the 1970s. Section 3 then switches to sticky prices and presents in detail a very influential article, the Dornbusch (1976) model of exchange rate overshooting, in fact, a dynamic version of the static Mundell-Fleming framework studied in chapter 2. It is, certainly, due to the significant contribution made by each of these three authors that this general set-up and its extensions in the subsequent literature are sometimes called the Mundell-Fleming-Dornbusch tradition (or paradigm) in international macroeconomics. We conclude the chapter by reviewing, in section 4, some empirical implications of the exchange rate models discussed, considering in particular the random walk hypothesis of the exchange rate in Meese and Rogoff (1983 a, b) and later work. As a technical complement underlying the chapter, backward and forward solutions to stochastic difference equations as well as the general solution to deterministic homogeneous differential equations are illustrated with some detail in appropriate contexts; also, the difference among and the rationale for perfect foresight, static, adaptive and rational expectations as well as mean error (ME), mean absolute error (MAE) and root mean square error (RMSE) of forecasts are analytically clarified.

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1 The Rational Expectations Revolution

This first section has the purpose to introduce the concept of rational expectations (RE) and to compare it analytically with a few alternative ways of modelling expectations that have been common in macroeconomics.

1.1 Expectations and Economic Behaviour

The behaviour of economic agents is based on certain expectations concerning the future level or evolution of relevant variables or processes. This fact differentiates social from natural sciences as acknowledged by Evans and Honkapohja (2001): “Modern economic theory recognises that the central difference between economics and natural sciences lies in the forward-looking decisions made by economic agents.” (p. 5). They further ascertain that expectations influence the time path of the macroeconomy, and vice versa, so that there is a profound two-sided relationship, a challenge to modellers.

Evans and Honkapohja (2001) also indicate that systematic economic theories attributing a major role to expectations can be first seen in Thornton (1802) and Cheysson (1887). Classical economists implicitly applied perfect foresight, and Marshall is credited with the concept of static expectations. Long-run expectations of prospective yields and asset prices are main themes in Keynes (1936), while in the 1950s and 1960s adaptive expectations or related lagged adjustment mechanisms had become widespread in macroeconomic models. Muth (1961) is the seminal paper which explicitly defines rational expectations in an economic context. His impact on how to write down and solve economic models under assumptions for rationality of forward-looking agents was somewhat delayed, but crucial indeed for the profession. Since Lucas (1972) and Sargent (1973) introduced and extended his RE methodology to macroeconomics, it has been the dominant feature of dynamic theoretical analysis. Rogoff (2002) attributes to Black (1973) the first consideration of rational expectations in an international finance context.1

Before moving to the perfect foresight models in the next sections, we first ‘catalogue’ the key types of expectations having been employed in economics in a taxonomy allowing clear comparison by means of analytical expressions.

1.2 A Basic Analytical Taxonomy of Modelling Expectations in Economics

Following Evans and Honkapohja (2001), the major types of expectations that have been important in economic modelling are characterised here by their defining formulas.

1.2.1 Perfect Foresight

Perfect foresight is a common assumption in earlier RE models endowing economic agents with superhuman powers. Perfect foresight, in fact, means that these agents make no mistakes in predicting the future value of a variable of interest. I.e., the expected value, $p_t^e$, of a variable $p$ (say, some price) at time $t$ is simply the actual (or realised) value of the same variable:

1Evans (1985) and Evans and Honkapohja (1995, 2001) developed this line of research further to bounded rationality that incorporates statistical learning.
\[ p_t^e = p_t. \]

This is, naturally, not at all a realistic assumption. Yet it disciplines the researcher to restrict attention to the study of behaviour that is model-consistent, as we make clearer below. Moreover, before better techniques for handling analytically expectations in macroeconomics were developed by the late 1970s, perfect foresight had been the prevalent RE hypothesis assumed.

1.2.2 Naive or Static Expectations

Static (or naive) expectations imply that the expected value of a variable \( p \) at time \( t \) is the actual (or realised) value of the same variable at the previous date (i.e., at time \( t - 1 \)):

\[ p_t^e = p_{t-1}. \]

Static expectations thus predict no change.

1.2.3 Adaptive Expectations

This type of modelling expectations appears to have been explicitly introduced in Cagan (1956). It implies that the expected value of a variable \( p \) at time \( t \) is the expected value of the same variable \( p \) at time \( t - 1 \) plus a term accounting for the forecast error \( p_{t-1} - p_t^e \) at time \( t - 1 \) (an error correction term):

\[ p_t^e = p_{t-1} + \omega (p_{t-1} - p_t^e). \]

The weight on the forecast error is given above by the constant \( 0 < \omega < 1 \) (yet, more generally, weights can change too, so that \( \omega_t \) then would be the right notation). The above equation can be solved backwards for \( p_t^e \). Write it as\(^2\)

\[ p_t^e = (1 - \omega) p_{t-1}^e + \omega p_{t-1}. \]

and start by lagging it one period (compare the subscripts):

\[ p_{t-1}^e = (1 - \omega) p_{t-2}^e + \omega p_{t-2}. \]  \hspace{1cm} (1)

Then substitute for \( p_{t-1}^e \) from the latter into the former expression:

\[ p_t^e = (1 - \omega) [(1 - \omega) p_{t-2}^e + \omega p_{t-2}] + \omega p_{t-1}, \]

\[ p_t^e = (1 - \omega)^2 p_{t-2}^e + \omega [(1 - \omega) p_{t-2} + p_{t-1}]. \]  \hspace{1cm} (2)

Now lag equation (1) by one period,

\[ p_{t-2}^e = (1 - \omega) p_{t-3}^e + \omega p_{t-3} \]

and substitute for \( p_{t-2}^e \) from the latter expression into (2) to obtain:

\(^2\)An alternative notation to implement the same solution in a more compact writing makes use of the lag operator.
\[ p^e_t = (1 - \omega)^3 p^e_{t-3} + \omega \left( (1 - \omega)^2 p_{t-3} + (1 - \omega) p_{t-2} + p_{t-1} \right). \]

Continuing in the same fashion, one can see that for the general case of \( n \) lags, the above formula transforms into

\[ p^e_t = (1 - \omega)^n p^e_{t-n} + \omega \left( (1 - \omega)^{n-1} p_{t-n} + (1 - \omega)^{n-2} p_{t-n-1} + \ldots + p_{t-1} \right), \]

so that one could compactly write

\[ p^e_t = (1 - \omega)^n p^e_{t-n} + \omega \sum_{i=0}^{n-1} (1 - \omega)^i p_{t-1-i}. \]

Finally, if \( n \rightarrow \infty \), the first term above would become negligibly small, since \( (1 - \omega)^n \) with \( 0 < \omega < 1 \) and \( n \rightarrow \infty \) is a miniscule fraction. We can, therefore, ignore it in the limit, writing the equation as

\[ p^e_t = \omega \sum_{i=0}^{\infty} (1 - \omega)^i p_{t-1-i}. \]

This latter expression is also known as a distributed lag polynomial with exponentially decaying weights, and constitutes an alternative writing of adaptive expectations. Other distributed lag formulations have been employed to model expectations formation in the macroeconomic literature, mostly in the course of the 1970s and the 1980s.

1.2.4 Rational Expectations

"The rational expectations revolution begins with the observation that adaptive expectations, or any other fixed-weight distributed lag formula, may provide poor forecasts in certain contexts and that better forecast rules may be readily available. The optimal forecast method will in fact depend on the stochastic process which is followed by the variable being forecast, and [...] this implies an interdependency between the forecasting method and the economic model which must be solved explicitly." Evans and Honkapohja (2001), p. 11.

Rational expectations (RE) is an equilibrium concept. The actual stochastic process that a variable (such as the price \( p \) in our examples above) follows depends on the forecast rules used by economic agents. The optimal choice of the forecast rule by each agent is, in turn, conditional on the choices of others. In effect, RE equilibrium imposes the consistency condition that each agent’s choice is a best response to the choices by others. In the simplest models with a representative agent these choices are, of course, identical.

Rational expectations can be written as

\[ p^e_{t+1} = E [p_{t+1} | \Omega_t] \text{ or } p^e_{t+1} = E_t [p_{t+1}], \]

where \( E [p_{t+1} | \Omega_t] \) or its shorthand \( E_t [p_{t+1}] \) denotes the mathematical (or statistical) expectation of \( p_{t+1} \) conditional on all available relevant information observable at time \( t, \Omega_t \), which also includes past data.
1.3 Muth (1961) and the Rational Expectations Revolution in Macroeconomics

Muth (1961) was the first to formulate the *rational expectations hypothesis* as a theory of expectations formation and to show that its implications are roughly consistent with the relevant data. In essence, he defined rational expectations, or “informed predictions of future events”, as “the same as the predictions of the relevant economic theory”, that is, in the sense of *model-consistency*. More precisely, the hypothesis states: “expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the ‘objective’ probability distributions of outcomes).” Muth (1961), p. 316.

As clarified in the original article, the RE hypothesis asserts three propositions:

1. The economic system does not waste scarce information.
2. Expectations formation depends specifically on the structure of the relevant system describing the economy.
3. Public predictions will have no substantial effect on the operation of the economic system, unless based on inside information.

After formulating the RE hypothesis, Muth (1961) applied it to a simple economic environment, introducing the basic mathematics to handle such analysis.

The overwhelming influence of Muth’s (1961) RE methodology on the mainstream of contemporary macroeconomics has commonly been termed the *rational expectations revolution*. Starting with Lucas (1972) and Sargent (1973) in closed-economy set-ups, and with Black (1973) in an open-economy framework, the profession has been accumulating better techniques to address expectations formation and to analyse its impact in various economic environments, up until the current proliferation of the literature on *bounded rationality* with statistical learning, to which we shall briefly return by the end of this book.3

2 Flexible-Price Stock Approach under Perfect Foresight: The Monetary Model

With the notions of RE and perfect foresight now defined, this section goes on to outline in more detail the so-called *stock* approach to BoP adjustment and NER determination in subsections 2.1 and 2.2.4 It is important to understand a major difference in perspective with respect to chapter 2, which focused on *flow* approaches. The models that belong to the group of the asset (market) or stock approaches to the BoP and/or the NER emphasise the view that flow adjustments of financial variables are ultimately driven

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3 Recent macroeconomics literature within the maximizing agents framework has also criticized the widespread use of the basic notion of RE in Dynamic Stochastic General Equilibrium (DSGE) models. This is the case, for instance, in Sims’s (2003) idea of “rational inattention”.

4 We focus in this chapter on the monetary model only because of its relevance for modern research in international macroeconomics and the lasting influence it has exerted. There is, however, a (now somewhat obsolete) literature on “portfolio balance models” (PBMs) of the exchange rate. These include stock-flow approaches such as Branson and Buijer (1983) for general equilibrium and Branson and Henderson (1985) for partial equilibrium. A concise summary of PBMs can be found in Sarno and Taylor (2002).
by the adjustment of the underlying desired stocks of assets held in various currency
denominations as part of wealth portfolios of economic agents. The stock approaches
do not, however, totally ignore the flow of funds arising from trade, highlighted in the
models of chapter 2. Rather, they point to the fact that there is much more to flow
adjustment, i.e., in addition to flow adjustment, and that the magnitude of stocks is
generally much higher than the magnitude of flows, so that even a small percentage change
in stock-supply or stock-demand may generate a large percentage change of corresponding
flows. Moreover, the stock and flow adjustments are, by definition, influencing each
other, so another term – and approach – commonly employed in analysis of the BoP
and/or the NER has been termed stock-flow models.

The rationale for the shift of emphasis from flow to asset approaches in academic
research can also partly be seen as a consequence of institutional changes in the global
monetary system, with an increasing capital mobility and deregulation of international
financial transactions after the demise of Bretton Woods. As Lindbeck (1976), p. 139,
duly remarks, such changes have led to different conceptual interpretations of the rela-
tions between the current (CA) and financial (KA) accounts of the BoP. In analyses
before World War II, long-term capital movements were usually assumed to be deter-
mined by differentials among real rates of return on physical assets; consequently, the
CA was assumed to adjust via changes in relative prices, stock of assets, exchange rates
or income. During the Bretton Woods system of fixed parities under heavily regulated
capital flows, largely inconvertible currencies and small stocks of foreign claims relative
to the flows of trade, the emphasis in analyses of the BoP and the NER moved to the
CA; the KA, then, was assumed to adjust to the state of the CA, via capital flows in-
duced by interest rate differentials or political control. Lindbeck (1976) concludes that
the increasing size of the stock of foreign financial assets and (related) capital flows since
the early 1970s has once again modified the focus through which researchers analyse the
interaction between the CA and the KA. In line with the asset approach, establishing
itself more firmly since about that same time, the most important short-run effects on
the BoP and the NER were asserted to come from shifts in stock-demand and stock-
supply unrelated to the current account. However, these effects were accumulating on
the current account as time passed by, and with subsequent feedbacks on the markets
for assets and foreign exchange.\textsuperscript{5}

Models along the above lines that view the exchange rate as an asset price under per-
fect foresight were developed employing assumptions of both flexible prices (this section)
and sticky prices (section 3). Among the flexible-price perfect foresight models, our focus
is on the most prominent one, the monetary model of the exchange rate. It is often the
case, and we shall note that in this book on several occasions, that open-economy mod-
els are in essence extensions of corresponding, methodologically similar, closed-economy
models. The monetary model under flexible prices presented further down is, thus, an
extension of the empirical set-up of Cagan (1956) analysing hyperinflations in a closed
economy.\textsuperscript{6}

\textsuperscript{5}See Johnson (1977) for an overview of the different approaches to the BoP up to the monetary
model.

\textsuperscript{6}For more details on the Cagan (1956) model in its initial closed-economy version and later open-
economy extension, see the original contribution and chapter 8 in Obstfeld and Rogoff (1996).
2.1 The Monetary Approach to the Balance of Payments (Peg)

The monetary approach to the balance of payments was developed in the 1960s, in part within the research department of the IMF under the intellectual leadership of Jacques Polak and in the context of the Bretton Woods system of fixed exchange rates. With its collapse, the approach was extended to cover the case of flexible exchange rates, thus becoming one of the major theories of NER determination. The monetary approach to the balance of payments is described in a comprehensive manner in the volume edited by Frenkel and Johnson (1976). In this and the next subsections, we present its two versions, for a regime of peg or float, respectively.7

No matter that the monetary approach to the BoP found its accomplished form in the 1970s, its proponents, e.g., Frenkel (1976), have claimed that the intellectual origins of this theory go back to Hume (1752), Wheatley (1803), Ricardo (1821) and to Cassel’s (1918, 1921) revival of ideas of the Salamanca School in the 16th century related to the proposition of purchasing power parity (PPP). The Casselian approach to PPP has later been articulated by Samuelson (1964) in the context of a generalisation of the law of one price (LOP) ensured by commodity arbitrage for all internationally traded goods. PPP is definitely violated in the short run, but tends to be a long-run equilibrium concept to which nominal exchange rates gravitate.8 That is why it is often an ingredient in many economic models, including the monetary approach, especially as a long-term condition. For this reason, and also because of the assumption of flexible prices, the monetary model should be understood as a benchmark long-run equilibrium model.

2.1.1 Origins: The Classical (Humean) Price-Specie-Flow Mechanism

In summary, the classical theory of balance of payments adjustment under the gold standard, building on ideas of Hume (1752), is as follows. A surplus in the BoP (or, more precisely, the current account) causes an inflow of gold. The gold inflow increases the price level. This is because of two mechanisms: (i) the link between gold reserves and the amount of money in circulation under the gold standard due to (full) convertibility of (paper) money into metal equivalent; (ii) the link between money and prices originating in the quantity theory of money (QTM) with the economy operating at full employment which states: \( M_t V = P_t Y \). That is, if \( M_t ↑ \), then \( P_t ↑ \) too, since the income velocity of money \( V \) and real output \( Y \) are assumed constant by the QTM. Hence, as the goods of the surplus country become relatively more expensive in the international market, the increased demand for imports by residents and the reduced demand for exports by nonresidents lead to a gradual reduction of the initial trade surplus. Analogous logic applies to the case of a CA deficit. Thus, the so-called price-specie(=gold)-flow mechanism ensures automatically BoP equilibrium under the gold standard.

A different perspective on the same process is based on the notion of the optimum or desired distribution of the stock of specie (gold) among all countries in the world. Any BoP disequilibrium, which is a flow disequilibrium, is therefore determined by the underlying stock disequilibrium, i.e., a distribution of the stock of gold available in the world which does not coincide with the desired one, ensuring BoP equilibrium.

The important point of Hume’s price-specie-flow mechanism is that the ultimate cause of balance of payments disequilibria is to be found in money stock disequilibria:

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7 Our exposition is based on original articles, e.g., Lindbeck (1976), Frenkel (1976) and Mussa (1976), as well as on Mark (2001), chapter 3, and Gandolfo (2001), parts of chapters 12, 13 and 15.

8 Chapter 6 will discuss the evidence on PPP.
a divergence of the quantity of money (gold) supplied and the quantity demanded by each country’s residents. It is from such considerations that the monetary approach to the BoP took its source.

2.1.2 Main Assumptions

1. **PPP** is a key assumption in the monetary approach, justified as an aggregation of the law of one price (LOP) in the markets for goods and services.

2. **UIP** is assumed to hold as well, so we have perfect substitutability between home and foreign assets.

3. In contrast to the flow approaches to BoP adjustment we studied thus far, all prices are assumed *flexible* under the monetary approach.

4. Again, opposite to the flow models explored earlier, the focus of the monetary approach is on conditions for *stock* equilibrium in the *money* market: the BoP is essentially a monetary phenomenon, and should therefore be analysed in terms of adjustment of money stocks.

5. Production is assumed at the level of full employment, so real income is *fixed*. This assumption, together with flexible prices ensure no short-run output-inflation trade-off (i.e. a vertical Phillips Curve).

6. A *stable money demand* function is taken to exist.

7. We will consider below the *small* open-economy case, although *two-country* versions of the model are straightforward to write down.

The monetary approach to BoP adjustment (under *fixed* exchange rates) or to NER determination (under *flexible* exchange rates) has been widely used for policy analysis. We shall see in later chapters that many of its predictions are confirmed by more complicated *optimising* models, both in flexible-price and sticky-price environments.

As is customary in the literature, the model will be presented in a notation distinguishing between *levels* of the variables, denoted by upper-case letters, and corresponding (natural) *logarithms* of the variables, denoted by lower-case letters (except for interest rates, which are themselves in (net) per-unit terms and are in levels).

2.1.3 Set-Up and Derivation of Key Result

Using the notation we introduced so far, we start by writing the definition of the *money supply*, in *levels*:

\[ MS_t \equiv \mu MB_t \equiv \mu (DC_t + IR_t), \]  

where \( \mu \equiv \frac{E[MS_t]}{E[MB_t]} \) is the *money multiplier*, assumed *constant*.

We next proceed to a *logarithmic expansion* of the money supply and its components about their *mean* values. Such first-order expansion of (3) could be done as follows. Taking expectations from both sides of (3),

\[ E[MS_t] \equiv \mu E[DC_t] + \mu E[IR_t], \]
subtracting the above equation from (3) and rearranging, yields:

\[ MS_t - E[MS_t] \equiv \mu (DC_t - E[DC_t]) + \mu (IR_t - E[IR_t]). \]

Dividing through by \( E[MS_t] \),

\[ \frac{MS_t - E[MS_t]}{E[MS_t]} = \frac{\mu (DC_t - E[DC_t])}{E[MS_t]} + \frac{\mu (IR_t - E[IR_t])}{E[MS_t]}, \]

substituting for the definition of \( \mu \) and rewriting gives:

\[ \frac{MS_t - E[MS_t]}{E[MS_t]} = \frac{(DC_t - E[DC_t])}{E[MB_t]} + \frac{(IR_t - E[IR_t])}{E[MB_t]}, \]

\[ \frac{MS_t - E[MS_t]}{E[MS_t]} = \frac{(DC_t - E[DC_t])}{E[MB_t]} + \frac{(IR_t - E[IR_t])}{E[IR_t]}, \]

Defining \( \theta \equiv \frac{E[IR_t]}{E[MB_t]} \) and, hence, \( (1 - \theta) \equiv \frac{E[DC_t]}{E[MB_t]} \), we get, by substitution:

\[ \frac{MS_t - E[MS_t]}{E[MS_t]} = \frac{(DC_t - E[DC_t])}{E[MB_t]} + \frac{(IR_t - E[IR_t])}{E[IR_t]}, \]

\[ \frac{MS_t - E[MS_t]}{E[MS_t]} = \frac{(1 - \theta)(DC_t - E[DC_t])}{E[DC_t]} + \theta (IR_t - E[IR_t]). \]

Now, since for a random variable \( Z_t, \frac{Z_t - E[Z_t]}{E[Z_t]} \approx \ln(Z_t) - \ln(E[Z_t]) \), ignoring an arbitrary constant depending on the mean values, we can write the money supply in the corresponding logarithms:

\[ m^∗_t \approx (1 - \theta) d_t + \theta i r_t. \]  

(4)

A standard transactions and speculation motive gives rise to the following demand for money function in levels:

\[ \frac{M^d_t}{P_t} = \gamma_t e^{-\lambda_t} e^{\epsilon_t}. \]

Written in logarithms, i.e., taking logs from both sides, this yields

\[ m^d_t - p_t = \phi y_t - \lambda t + \epsilon_t, \]  

(5)

with \( 0 < \phi < 1 \) being the income elasticity of money demand, \( \lambda > 0 \) the interest semi-elasticity (since the interest rate is not in natural logs, as noted earlier) of money demand and the error term assumed independently and identically distributed (iid) with zero mean and constant variance, \( \epsilon_t \overset{iid}{\sim} (0, \sigma^2) \).

We next use PPP and UIP, as assumed within the monetary approach framework.

Since the exchange rate is fixed, say at \( \overline{\pi} \), PPP implies (in logs as below, from \( P_t = \overline{\pi} P^*_t \) in levels) that the domestic price level in the SOE is determined by the exogenous foreign (RoW) price level:

\[ p_t = \overline{\pi} + p^*_t. \]  

(6)
Moreover, assuming a credible peg, market participants do not expect a change in the NER, i.e., \( E_t[S_{t+1}] = S_t = S = \text{const} \) for any \( t \) (hence \( \ln \frac{E_t[S_{t+1}]}{S_t} = \ln \frac{S}{S} = \ln 1 = 0 \)), so that UIP reduces (taking log approximations) to

\[
\iota_t = \iota^*_t,
\]

which implies that the interest rate in the SOE is equal to the exogenous foreign (RoW) interest rate.

We finally assume that the money market is continuously in equilibrium, so that we can equate the supply of money (4) to the demand for money (5):

\[
(1 - \theta) d_t + \theta \iota_t = \underbrace{\pi_t + \phi y_t}_\text{=} -\underbrace{\lambda \iota_t}_\text{=} + \epsilon_t.
\]

Rearranging, one obtains:

\[
\theta \iota_t = \pi_t + \phi y_t - \lambda \iota^*_t + \epsilon_t - (1 - \theta) d_t.
\]

### 2.1.4 Analysis and Interpretation

Equation (8) summarises the main insights of the monetary approach to the balance of payments, or, which is the same, of the monetary model (of the balance of payments) under peg. It is evident from the money demand function (5) that if the SOE in question experiences any of the following:

- positive income growth, \( y_t \uparrow \);
- declining interest rates, \( \iota_t \downarrow \);
- rising prices, \( p_t \uparrow \);

the demand for nominal money balances will grow, \( m^d_t \uparrow \).

Equation (8) further shows that if this increased demand for money is not satisfied by an accommodating increase in domestic credit, \( d_t \), so that \( m^d_t > (1 - \theta) d_t \) in (8), the public will obtain the additional money it desires to hold by running an overall BoP surplus, i.e., an increase in international reserves, \( \iota_t \); if, on the other hand, the central bank engages in a domestic credit expansion that exceeds the growth of money demand, so that \( m^d_t < (1 - \theta) d_t \) in (8), the public will eliminate the excess supply of money (it does not wish to hold) by spending or investing it abroad and thus running a(n overall) BoP deficit, i.e., a decrease in international reserves. Consequently, the money supply, \( m^s_t \equiv (1 - \theta) d_t + \theta \iota_t \), in the monetary model under peg is endogenous, that is, determined by expression (8).

Note that this endogeneity of money supply is also a common feature of the Mundell-Fleming model analysed in chapter 2. In essence, the conclusion that under fixed exchange rates the monetary authority loses control of domestic credit (the policy instrument in this model) remains unchanged. Note also that, with fixed exchange rates, the correction of any disequilibrium in the BoP induced by, say, a shock to money demand \( \epsilon_t \) is brought about by an adjustment of the level of reserves (the specie-flow). In other words, the stock disequilibrium in the money market is corrected by a flow adjustment of reserves.
2.2 The Monetary Approach to the Exchange Rate (Float): The Monetary Model

Changing the assumption of a peg with the alternative of a flexible exchange rate regime transforms the above model into what has become known as the monetary approach to the (nominal) exchange rate.9

The similarities between the monetary model under peg and under float are many indeed. However, there are also differences, originating in the alternative assumption about the NER regime: these differences are evident below in the expressions for PPP and UIP, which still hold but in a modified analytical form. We write the model assuming two countries, Home and Foreign (RoW). We also assume that the pair of money demand equations has identical parameters for the two countries.

2.2.1 Set-Up and Derivation of Key Results

Under float, the money supply is exogenous, i.e., it is determined by the monetary authority in each of the countries, since the central bank does not need to target a certain value of the NER. Equilibrium in the money market in Home and Foreign is then given by the pair of equations:

\[ m_t - p_t = \phi y_t - \lambda t, \quad (9) \]

\[ \text{supply of real balances in } H \quad \text{demand for real balances in } H \]

\[ m_t^* - p_t^* = \phi y_t^* - \lambda t^*, \quad (10) \]

\[ \text{supply of real balances in } F \quad \text{demand for real balances in } F \]

International capital market equilibrium is determined by UIP, now allowing for expected depreciation under the float regime (i.e., with \( \ln E_t \left[ S_{t+1} \right] = \ln E_t \left[ S_{t+1} \right] - \ln S_t \equiv E_t \left[ S_{t+1} \right] - S_t \)):

\[ \iota_t - \iota_t^* = E_t \left[ S_{t+1} \right] - S_t. \quad (11) \]

PPP relates the price levels in the two economies through the exchange rate, given that LOP holds for the individual products ensuring equilibrium in the market for goods via arbitrage:

\[ s_t = p_t - p_t^*. \quad (12) \]

A usual simplification of the notation at this point, which allows certain interpretation as to what constitutes the (economic) fundamentals of the exchange rate, is:

\[ f_t \equiv (m_t - m_t^*) - \phi (y_t - y_t^*), \quad (13) \]

where \( f_t \) is the vector of fundamentals that determine the exchange rate multiplied by their respective coefficients (vector \([1, -\phi]^t\)).

Substituting (9), (10) and then (11) and (13) into (12) and rearranging, we get:

\[ s_t = m_t - \phi y_t + \lambda t - (m_t^* - \phi y_t^* + \lambda t^*), \]

\[ =p_t, \text{ from (9)} \]

\[ =p_t^*, \text{ from (10)} \]

---

9 Obstfeld and Rogoff (1996), pp. 526-530, also call the monetary model “the Cagan model”, in honour of the mentioned early contribution by Cagan (1956) with application to hyperinflation.
\[ s_t = (m_t - m_t^*) - \phi (y_t - y_t^*) + \lambda \left( t_t - t_t^* \right), \]

\[ \equiv f_t, \text{ from (13)} \]

\[ = E_t[s_{t+1}] - s_t, \text{ from (11)} \]

\[ s_t = f_t + \lambda \left( E_t \left[ s_{t+1} \right] - s_t \right). \]  

(14)

If we now solve for \( s_t \) we obtain:

\[ s_t = f_t + \lambda E_t \left[ s_{t+1} \right] - \lambda s_t, \]

\[ (1 + \lambda) s_t = f_t + \lambda E_t \left[ s_{t+1} \right], \]

\[ s_t = \frac{1}{1 + \lambda} f_t + \frac{\lambda}{1 + \lambda} E_t \left[ s_{t+1} \right], \]

\[ \equiv \gamma \]

\[ = \left( 1 - \gamma \right) E_t \left[ s_{t+1} \right]. \]  

(15)

(15) is the central equation in the monetary model. It is a first-order dynamic-stochastic difference equation that states that the current value of the exchange rate is a function of future expected values of the exchange rate. From the composition of the fundamentals term \( f_t \) it is clear that high relative money growth in Home, \( m_t - m_t^* > 0 \), leads to a depreciation of the Home currency, \( s_t \uparrow \), while high relative income growth, \( y_t - y_t^* > 0 \), tends to appreciate it, \( s_t \downarrow \). In the long-run equilibrium or steady state, when \( s_t = E_t \left[ s_{t+1} \right] \), we then have that \( s_t = f_t \).

2.2.2 General Forward-Looking Solution

Equation (15) can be solved forward for the exchange rate, under an (implicit or explicit) assumption of rational expectations.\(^{10}\) This is done by first writing (15) one period forward,

\[ s_{t+1} = \gamma f_{t+1} + (1 - \gamma) E_{t+1} \left[ s_{t+2} \right], \]

then taking expectations conditional on time \( t \) information,

\[ E_t \left[ s_{t+1} \right] = \gamma E_t \left[ f_{t+1} \right] + (1 - \gamma) E_t \left[ E_{t+1} \left[ s_{t+2} \right] \right], \]

and using the law of iterated expectations,\(^{11}\) to obtain

\[ E_t \left[ s_{t+1} \right] = \gamma E_t \left[ f_{t+1} \right] + (1 - \gamma) E_t \left[ s_{t+2} \right], \]

by which we substitute \( E_t \left[ s_{t+1} \right] \) back into (15) to get

\[ s_t = \gamma f_t + (1 - \gamma) \left( \gamma E_t \left[ f_{t+1} \right] + (1 - \gamma) E_t \left[ s_{t+2} \right] \right), \]

\[ = E_t \left[ s_{t+1} \right]. \]

\(^{10}\)Note the parallels in the algorithm with the backward solution for the expected price in the subsection introducing adaptive expectations earlier in the chapter.

\(^{11}\)Here meaning, in particular, that \( E_t \left[ E_{t+1} \left[ s_{t+2} \right] \right] = E_t \left[ s_{t+2} \right] \). That is, with the information set available at time \( t \), agents “expect to expect” in period \( t + 1 \) exactly what they expect in period \( t \).
Further rearranging, one could write

\[ s_t = \gamma \{ E_t [f_t] + (1 - \gamma)E_t [f_{t+1}] \} + (1 - \gamma)^2 E_t [s_{t+2}] , \]

\[ s_t = \gamma \sum_{j=0}^{1} (1 - \gamma)^j E_t [f_{t+j}] + (1 - \gamma)^2 E_t [s_{t+2}] . \]

We can now repeat the same procedure by advancing time in (15) by two, three, four periods and so forth, say up to \( k \) periods ahead. The result will be the general equivalent of the equation above:

\[ s_t = \gamma \sum_{j=0}^{k} (1 - \gamma)^j E_t [f_{t+j}] + (1 - \gamma)^{k+1} E_t [s_{t+k+1}] . \]  (16)

### 2.2.3 No-Bubbles Solution

If now we let \( k \to \infty \), the second term in (16) will vanish, becoming negligible asymptotically:\(^{12}\)

\[ \lim_{k \to \infty} (1 - \gamma)^k E_t [s_{t+k}] = 0. \]  (17)

Equation (17) is called in similar models a *transversality condition* (TVC). By imposing it, we restrict the rate at which the exchange rate can grow asymptotically to obtain the unique *fundamentals* (or *no-bubbles*) solution:\(^{13}\)

\[ s_t = \gamma \sum_{j=0}^{k} (1 - \gamma)^j E_t [f_{t+j}] . \]  (18)

(18) expresses the current period \((t)\) exchange rate as the *present discounted value* (PDV) of expected future values of the fundamentals. That is, the current NER reflects not only the current value of the fundamentals, but all the future expected path of fundamentals. Hence, the NER is a reflection of market participants' expectations, just like in finance theory of asset pricing. For the latter theory a stock price, for instance, is the present discounted value of the firm's expected dividends. Because of this similarity, the monetary model is sometimes referred to as the *asset approach* to the exchange rate.

From the perspective of this approach, it is natural to expect the exchange rate to be more volatile than fundamentals, just like the prices of assets such as stocks are more volatile than the corresponding dividends, an issue directly addressed by the Dornbusch overshooting model to be analysed in section 3.

---

\(^{12}\) Recall that \( 0 < (1 - \gamma) \equiv \frac{\lambda}{1 + \lambda} < 1 \) because we have, by definition, restricted \( \lambda \) to be positive \((\lambda > 0)\), so that the interest semi-elasticity of money demand, \(-\lambda\), is negative (i.e., a higher interest rate would, ceteris paribus, reduce the demand for money).

\(^{13}\) Note that the TVC will hold only if, as \( k \to \infty \), \((1 - \gamma)^k\) goes to zero faster than \( s_{t+k} \) goes to infinity in expected value.
2.2.4 Rational Bubbles

If the TVC does not hold, the behaviour of the exchange rate will be governed in part by an (asymptotically) explosive bubble, \( b_t \), which will eventually dominate the fundamentals.\(^\text{14}\) We can show why this is the case by assuming that the bubble evolves according to a first-order autoregressive process (AR1), where the autoregressive coefficient (measuring persistence) is defined to be \( \frac{1}{1-\gamma} \) and thus exceeds 1 (\( \frac{1}{1-\gamma} > 1 \) since \( 0 < 1 - \gamma < 1 \)), which means that the bubble process is explosive:

\[
b_t = \frac{1}{1-\gamma} b_{t-1} + \xi_t, \quad (19)
\]

where \( \xi_t \sim \text{iid } N \left( 0, \sigma^2 \right) \). We can now add the bubble process (19) to the fundamentals solution (18) to obtain back the general solution of the first-order stochastic difference equation for the exchange rate we started with, (15), but now assuming a bubble:

\[
\hat{s}_t = s_t + b_t, \quad (20)
\]

One can check that \( \hat{s}_t \) violates the TVC, by simply substituting (20) into (17):

\[
\lim_{k \to \infty} (1-\gamma)^k \mathbb{E}_t [\hat{s}_{t+k}] = \lim_{k \to \infty} (1-\gamma)^k \mathbb{E}_t [s_{t+k}] + \lim_{k \to \infty} (1-\gamma)^k \mathbb{E}_t [b_{t+k}] = b_t \neq 0. \quad (21)
\]

To understand why \( b_t = \lim_{k \to \infty} (1-\gamma)^k \mathbb{E}_t [b_{t+k}] \) in the expression just above, solve forward (19):

\[
(1-\gamma)b_t = b_{t-1} + (1-\gamma)\xi_t,
\]

\[
b_{t-1} = (1-\gamma)b_t - (1-\gamma)\xi_t.
\]

Taking expectation at \( t-1 \):

\[
\mathbb{E}_{t-1} [b_{t-1}] = (1-\gamma)\mathbb{E}_{t-1} [b_t] - (1-\gamma)\mathbb{E}_{t-1} [\xi_t],
\]

\[
b_{t-1} = (1-\gamma)\mathbb{E}_{t-1} [b_t].
\]

Or, which is the same,

\[
b_t = (1-\gamma)\mathbb{E}_t [b_{t+1}].
\]

Hence,

\(^{14}\)The solution path for any variable \( y_t \) defined by a first-order linear difference equation has a particular solution (or integral) and a complementary function. The sum of these two constitutes the general solution. In the case of (16) the TVC simply eliminates the possibility of an explosive complementary function. However, if the TVC does not hold, then the solution will include the particular solution (18) plus an explosive component or bubble.
\[ b_t = (1 - \gamma)E_t \left[ \frac{b_{t+1}}{(1 - \gamma)E_{t+1} \left[ b_{t+2} \right]} \right], \]

and so on, up to lead \( k \):

\[ b_t = (1 - \gamma)^k E_t \left[ b_{t+k} \right]. \]

Now it is clear that \( \lim_{k \to \infty} (1 - \gamma)^k E_t \left[ b_{t+k} \right] = \lim_{k \to \infty} b_t = b_t. \)

No matter that it violates the TVC, \( \tilde{s}_t = s_t + b_t \) is still a solution to the model because it solves (15). To check, substitute (20) into (15):

\[ s_t + b_t = \gamma f_t + (1 - \gamma)E_t [s_{t+1} + b_{t+1}], \]

\[ s_t + b_t = \gamma \underbrace{s_t - (1 - \gamma)E_t \left[ s_{t+1} \right]}_{=f_t, \text{ from (15)}} + (1 - \gamma)E_t \left[ s_{t+1} \right] + (1 - \gamma)E_t \left[ \frac{1}{1 - \gamma}b_t + \xi_{t+1} \right], \]

\[ = s_{t+1}, \text{ from (19)} \]

\[ s_t + b_t = s_t - (1 - \gamma)E_t \left[ s_{t+1} \right] + (1 - \gamma)E_t \left[ s_{t+1} \right] + (1 - \gamma) \frac{1}{1 - \gamma}b_t, \]

\[ s_t + b_t = s_t + b_t. \]

However, even being a solution to (15), \( \tilde{s}_t = s_t + b_t \) is an unstable one, since the bubble will eventually dominate the fundamentals, as we saw in (21), and will thus drive the exchange rate arbitrarily far away from them. Because the bubble arises in a model where people are assumed (by working out the forward-looking solutions to the stochastic difference equation for the behaviour of the exchange rate) to hold rational expectations, it is known as a rational bubble.

The intuition behind this result is simple. Rational agents may still want to hold on to a currency that is overvalued relative to its fundamental value if they can be compensated by an expected capital gain. This, in itself, reinforces the bubble, which becomes self-fulfilling. Such was the case of the large overvaluation of the US dollar in the mid-1980s. At that time, as the value of the dollar continued to raise, it was certainly not rational to sell dollars. It may be the case that forex markets are occasionally driven by bubbles, but real-world experience suggests that such bubbles eventually pop, and that the probability of the bubble bursting increases as the NER departs further from fundamentals. This is a reason not to focus too much on the solution with a (rational) bubble to explain the equilibrium behaviour of the exchange rate.

There is, however, evidence that the presence of bubbles is not a rare event in financial markets and, specifically, the forex market. The presence of bubbles was originally tested using the variance bounds test proposed by Shiller (1981) to study the volatility of long-term bonds. Shiller’s method was employed by Huang (1981) to examine the presence of bubbles in the Deutsche mark exchange rate. The volatility test is based on the idea that if prices have a bubble component, causing them to deviate from the
fundamentals solution, then they will vary even if fundamentals are unchanged. In this case, too much price variation suggests the presence of a bubble. This class of test is essentially a test of excess volatility of the actual exchange rate relative to the volatility of the exchange rate based on the fundamentals solution. A problem with this approach is that it relies heavily on the choice of fundamentals. More recent approaches, since Evans (1991), have tested the idea of periodically collapsing bubbles. These include Markov-switching models such as Hall et al. (1999), and random-coefficient autoregressive models such as Psaradakis et al. (2001).

3 Sticky-Price Models of Exchange Rate Dynamics under Perfect Foresight: The Dornbusch (1976) Overshooting Model

We now switch from flexible to sticky prices, but still consider the dynamics of exchange rates under perfect foresight. In this section, we concentrate in detail on the most prominent set-up representative of this line of research, the Dornbusch (1976) model of exchange rate overshooting.

Dornbusch (1976) develops a formal but not optimising framework, in the sense of assuming explicit functional forms not derived from microfoundations. This formal approach enables him to solve analytically for the time path of the endogenous variables under perfect foresight. Dornbusch’s model was very influential both among academic circles and in policy making. Furthermore, as a dynamic, RE reformulation of the static (Keynesian) Mundell-Fleming framework, it has been for long the ‘workhorse’ of open-economy macro and a major tool for policy analysis across the globe. For these reasons at least, it is worth devoting some more attention to the Dornbusch (1976) model: drawing largely on the original article, we analyse in the present section its structure, solution and interpretation.

3.1 Stylised Facts the Model was Designed to Explain

1. The observed large exchange rate volatility – much higher than that of underlying fundamentals such as relative money supplies and demands, relative output or relative interest rates – in the years just after the demise of the Bretton Woods system; this high NER volatility needed to be consistent, at the same time, with rational expectations formation, i.e., the model had to also incorporate rationality (as summarised in the beginning of the chapter).

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15 Wage rigidity is a prominent feature of Keynes (1936). The Keynesian tradition in macroeconomics later on incorporated sticky prices too. Recall that such stickiness of prices (including wages) leads to short-run real effects of monetary policy; yet in the long run, when prices have sufficient time to adjust completely, money growth causes just a proportional growth in the price level, i.e., money homogeneity with respect to the price level \( dp = dm \), which is at the same time money neutrality with respect to output (and other real variables). More recently, nominal rigidity was rationalised by the ‘menu costs’ literature of the New Keynesian school, started by Mankiw (1985) and Akerlof and Yelen (1985). The essential point in it is that even small costs of changing prices, e.g., of a restaurant’s menus, can lead – via some magnification mechanism – to large macroeconomic fluctuations. The assumption of nominal stickiness in (New) Keynesian models is often supported empirically by real-world evidence of wage and price rigidity (of various duration across sectors).

16 See Rogoff (2002).
2. The effects of a monetary expansion, which is observed to generate:

(a) in the short run, an immediate domestic currency depreciation: monetary expansion thus seems to account for fluctuations in the exchange rate and the terms of trade (ToT);

(b) during the adjustment process, rising prices may be accompanied by an appreciating exchange rate so that the trend behaviour of exchange rates and the cyclical behaviour of exchange rates and prices stand potentially in contrast;

(c) during the adjustment process, there is also a direct effect of the exchange rate on domestic inflation: in this context, the exchange rate is identified as a critical channel for the transmission of monetary policy to the aggregate demand for domestic output.

3. The differential speed of adjustment of asset versus goods markets, which was becoming at that time an interesting novel topic for research.

3.2 Key Assumptions of the Model

As before, all lowercase variables except interest rates denote logarithms. The model is set up in continuous time (but explicit time indexing is not used, in principle, except on several occasions, as in the original article). Another ‘value added’ of analysing Dornbusch’s paper is therefore that it will provide some revision of differential equations as a technique to solve continuous-time dynamic models, in addition to the analogous methods of solving difference equations in discrete time we applied in section 2.

3.2.1 General Assumptions

1. We consider the case of a small-open economy (SOE). Hence: (i) the world (nominal) interest rate \( i^* \) is given in the world asset market (i.e., is exogenous to the SOE); (ii) the world price of SOE’s imports \( p^* \) is also given (exogenously) in the world goods market.

2. Domestic output is an imperfect substitute for imports, i.e., for foreign output. From this and the previous assumption it follows that aggregate demand (AD) for domestic goods will determine both the absolute and the relative value of SOE’s imports.

3. Differential adjustment speed of markets: goods markets adjust slowly relative to asset (including exchange rate) markets, the latter adjusting instantaneously.

4. Consistent expectations: expectations are model-consistent, in the sense we explained in section 1, and perfect foresight, i.e. the extreme form of rational expectations, is assumed.

\(^{17}\)For easier reference, the notation is kept on purpose to correspond exactly with that of the original Dornbusch (1976) model. Two exceptions are that – for consistency with the conventions adopted earlier in this book – we use \( i \) and \( s \) (instead of Dornbusch’s \( i \) and \( e \)) to designate the interest rate and (the log of) the exchange rate, respectively.
3.2.2 Capital Mobility and Exchange-Rate Expectations Formation

1. **Perfect capital mobility**: assets denominated in domestic and foreign currency are perfect substitutes given a proper *premium* (or *discount*) to offset anticipated NER changes; equivalently, this is (a form of) the uncovered interest parity (UIP) condition which states that net asset yields, corrected for expected exchange rate movements, should be equalised by arbitrage:

   \[ \iota = \iota^* + x, \]  

   where \( x > 0 \) is interpreted as expected NER *depreciation* and \( \iota \) is the domestic (nominal) interest rate determined by imposing equilibrium in the SOE’s money market (i.e., \( \iota \) is *endogenous*); incipient capital flows ensure that (22), a representation of perfect capital mobility, holds at all times.

2. Exchange-rate expectations formation: agents distinguish between the long-run exchange rate, \( \pi \), to which the economy will eventually converge, and the current-date, or spot, exchange rate, \( s \), which may deviate from its long-run value. More precisely, the expected rate of depreciation, \( x \), of the spot NER is assumed proportional (via some coefficient \( \theta \)) to the *discrepancy* between the long-run exchange rate \( \pi \) and the current spot rate \( s \):

   \[ x = \theta (\pi - s). \]  

Dornbusch (1976) makes the point that while expectations formation according to (23) may appear ad-hoc, it will actually be consistent with perfect foresight. This will be more clearly shown further down.

3.2.3 Money Market Structure and Equilibrium

1. A conventional money demand function: it states that the demand for real money balances is assumed to depend negatively on the domestic interest rate (as a proxy for the opportunity cost of holding money) and positively on real income (as a proxy for the total number of transactions in an economy); it is, in fact, this function that ‘introduces’ money into the model\(^{18}\) and helps determine the *domestic* (nominal) interest rate \( \iota \), *given* also the three assumptions stated next.

2. The nominal quantity of the money stock (or supply), \( m^s \) in logs, is *exogenously determined* by the SOE’s central bank.

3. Real income – or output, or aggregate supply (AS) – is also (initially) *given* at its *full-employment* level, \( y \).\(^{19}\)

\(^{18}\)Money thus enters as a medium of exchange, not through a cash-in-advance (CiA) constraint or a money-in-the-utility (MiU) assumption, popular shortcuts – as we shall see in later chapters – in microfounded monetary models.

\(^{19}\)Dornbusch (1976) relaxes this assumption in Part V of his article, where output is allowed to vary in response to aggregate demand. This modification, however, changes accordingly two key results of the model: (i) the *liquidity effect* of a monetary expansion (namely, the *lower* interest rate); and (ii) the *exchange rate overshooting*. Both these effects, which arise always under *fixed* output (i.e., output unaffected by aggregate demand) *may not occur* under the alternative assumption of *variable* output, as Dornbusch (1976) points out. See question 1 in the exercise set for this chapter.
4. Money market equilibrium: the domestic money market is in equilibrium, i.e. \( m^s = m^d = m \); thus money demand can be written as

\[
m - p = -\lambda \iota + \phi y \Rightarrow \lambda \left( m - p \right) + \phi y, \tag{24}
\]

where the parameters \( \lambda \) and \( \phi \) have the same interpretation as in (5) above. Substituting for \( x \) from (23) in (22), then for \( \iota \) from (24) in (22), and finally solving (22) for \( p - m \), we obtain a relationship linking the spot and long-run exchange rates with the price level:

\[
p - m = -\phi y + \lambda \iota^* + \lambda \theta \left( \pi - s \right). \tag{25}
\]

5. A stationary money supply process: with stationary money supply assumed, long-run equilibrium will imply \( s = \pi \) and, hence, through (23) and (22), \( \iota = \iota^* \); thus, from (25) with \( s = \pi \), an expression for the long-run equilibrium price level is derived:

\[
\pi = m - \phi y + \lambda \iota^*, \tag{26}
\]

where we can observe the standard long-run property of proportionality between money and prices.

Substituting for \( m \) from (26) in (25) – to eliminate it – and solving (25) for \( s \), a key equation showing the relationship between the exchange rate and the price level is obtained:

\[
s = \pi - \frac{1}{\lambda \theta} \left( p - \pi \right). \tag{27}
\]

Solving (27) for \( p \) will determine what Dornbusch (1976) termed the asset market equilibrium (or \( QQ \)) schedule, which we shall need for later use in the graphs illustrating the model:

\[
p = \pi - \lambda \theta \left( s - \pi \right) = \pi - \lambda \theta s + \lambda \theta \pi. \tag{28}
\]

Dornbusch (1976), p. 1164, suggests the following interpretation of key equation (27). For given long-run values \( \pi \) and \( \pi \), the spot exchange rate \( s \) can be determined from (27) as a function of the current price level \( p \). Given the price level \( p \), we have a domestic interest rate \( \iota \) from (24) and an interest rate differential \( \iota - \iota^* \) from (22) determined endogenously. Given also the long-run exchange rate \( \pi \), from (23) there is a unique level of the spot exchange rate \( s \) such that expected depreciation \( x \) matches the interest differential \( \iota - \iota^* \). So an increase in the price level, because it raises the domestic interest rate from (24), causes next an incipient capital inflow that will appreciate from (22) and (23) the spot exchange rate to the point where the anticipated depreciation \( x \) offsets exactly the increase in \( \iota \). That is, an increase in \( p \) needs to generate an expected depreciation for the UIP condition to hold. Hence, the spot rate has to appreciate by more than the equilibrium NER, which happens because of of the capital inflow induced by the increase in the domestic interest rate.
3.2.4 Goods Market Structure and Equilibrium

1. The foreign price level is normalised to unity, and hence $p^* = 0$, and the relative price of domestic goods (in logarithms), $s + p^* - p$, becomes simply $s - p$.

2. A special, ad-hoc form of the aggregate demand function for domestic output is assumed:

$$\ln D = u + \delta (s - p) + \gamma y - \sigma \iota,$$

with $u$ introduced as a shift parameter; (29) says that the demand for domestic output depends on the relative price of domestic goods (in terms of foreign goods), $s - p$, on the interest rate, $\iota$, and on real income, $y$. Note that a decrease in the relative price of domestic output ($s \uparrow$ or $p \downarrow$) raises $\ln D$, as does an increase in real income ($y \uparrow$) or a fall in the interest rate ($\iota \downarrow$).

3. A special, ad-hoc form of the rate of increase of the price of domestic goods, is assumed too: $\dot{p}$ is proportional (via some coefficient $\pi$) to an excess demand measure, $\ln D - \ln Y$:

$$\dot{p} = \pi \left( \ln D - \ln Y \right)_{\equiv u} = \pi \left[ u + \delta (s - p) + (\gamma - 1) y - \sigma \iota \right].$$

4. The long run equilibrium implies that

$$\dot{p} = 0,$$

because $D = Y$, and, since $s = \bar{s}$ we have

$$\iota = \iota^*.$$

Further substituting for $\iota = \iota^*$ from the money demand function (24) in (30) to finally solve (30) for $s$ gives the long run equilibrium exchange rate, $\bar{s}$:

$$\bar{s} = p + \frac{1}{\delta} \left[ \sigma \iota^* + (1 - \gamma) y - u \right],$$

where $\bar{p}$ is defined in (26). It is evident in equation (31) that the long-run exchange rate depends on both monetary and real variables. This is a similar expression to the one obtained from the steady state of the monetary model with flexible prices, which is evident from the fact that, in the long run we assume that prices are flexible.

Now the price adjustment equation (30) can be simplified using (31) to substitute in (30) for $s$ and (22) and (23) to express $x = \iota - \iota^* = \theta (\bar{s} - s)$. Subtract first $s$ from both sides of (31) and rearrange, solving (implicitly) for $s$:

---

20 E.g., the GDP deflator or the producer price index (PPI) in real-world economies.
\[ s - s = \overline{p} - s + \frac{1}{\delta} \left[ \sigma \nu^* + (1 - \gamma) y - u \right], \]

\[ s = (s - \overline{p}) + \overline{p} + \frac{1}{\delta} \left[ \sigma \nu^* + (1 - \gamma) y - u \right]; \]

Plug the above expression for \( s \) in (30):

\[ \dot{p} = \pi \left[ u + \delta \left\{ (s - \overline{p}) + \overline{p} + \frac{1}{\delta} \left[ \sigma \nu^* + (1 - \gamma) y - u \right] - p \right\} + (\gamma - 1) y - \sigma \nu \right], \]

and rearrange, to progressively obtain

\[ \dot{p} = \pi \left[ u + \delta (s - \overline{p}) + \delta \overline{p} + \sigma \nu^* + (1 - \gamma) y - u - \delta p + (\gamma - 1) y - \sigma \nu \right], \]

\[ \dot{p} = \pi \left[ \delta (s - \overline{p}) + \delta (\overline{p} - p) + \sigma (\nu^* - \nu) \right], \]

\[ \dot{p} = \pi \left[ (\delta + \sigma \theta) (s - \overline{p}) + \delta (\overline{p} - p) \right]. \]

Finally, using (27) to substitute out \( s - \overline{p} \) above,

\[ \dot{p} = \pi \left[ (\delta + \sigma \theta) \left\{ \frac{1}{\chi \theta} \right\} (p - \overline{p}) + \delta (\overline{p} - p) \right], \]

\[ \dot{p} = \pi \left[ \left( \frac{\delta + \sigma \theta}{\chi \theta} + \delta \right) (\overline{p} - p) \right], \]

one can get Dornbusch’s (1976) price adjustment equation,

\[ \dot{p} = -\pi \left( \frac{\delta + \sigma \theta}{\theta \lambda} + \delta \right) (p - \overline{p}) = -\nu (p - \overline{p}), \quad (32) \]

where the definition of the speed of price adjustment, parameter \( \nu \), of the above differential equation is:

\[ \nu \equiv \pi \left( \frac{\delta + \sigma \theta}{\theta \lambda} + \delta \right). \quad (33) \]

Recall that a first-order linear homogeneous differential equation of some function of time \( z(t) \),

\[ \frac{dz}{dt} + cz = 0, \quad \Rightarrow z \]
where $c$ is a constant – like $\nu$ in (32) above – has a (definite) solution of the form

$$z(t) = z(0) e^{-ct}.$$ 

Now solving (32) by analogy (also taking into account the long-run equilibrium constant $\overline{p}$ subtracted from both sides in our case), given some initial $p_0 \equiv p(0)$, yields the dynamics (i.e., the time path) of the price of domestic output in the SOE considered:

$$p(t) - \overline{p} = (p_0 - \overline{p}) e^{-\nu t},$$

$$p(t) = \overline{p} + (p_0 - \overline{p}) e^{-\nu t}.$$ 

That is, the price of domestic output will converge to its long-run equilibrium level at the speed $\nu$, itself determined by (33). Substitution of (34) in (27), finally results into Dornbusch’s (1976) key model equation describing NER dynamics, i.e., the time path of the nominal exchange rate:

$$s(t) = \overline{s} - \frac{1}{\lambda \theta} (p_0 - \overline{p}) e^{-\nu t} = \overline{s} + (s_0 - \overline{s}) e^{-\nu t}.$$ 

We can write the second equality because of (27), which states that

$$s(t) = \frac{1}{\lambda \theta} [p(t) - \overline{p}].$$

From (36) the exchange rate will likewise converge to its long-run level. The domestic currency will appreciate, $s(t) \downarrow$, if prices are initially below their long-run level, $p_0 < \overline{p}$, so that $p_0 - \overline{p} < 0$, hence the sign of the term $-\frac{1}{\lambda \theta} (p_0 - \overline{p}) e^{-\nu t}$ in (36) becomes positive, meaning that the spot rate, $s(t)$, is temporarily higher than its long-run equilibrium value, $\overline{s}$, and will therefore revert back to it, i.e., appreciate; and conversely in the opposite initial situation.

### 3.3 Model Equilibrium and Transition Paths

#### 3.3.1 Graphical Analysis

Equilibrium in Dornbusch’s (1976) model can be illustrated graphically. Figure 1 shows the long-run equilibrium level of the NER and the price level in the SOE considered (point A), as well as the transition paths from an initial position from a higher NER and lower price level (point B, path B to A) and from a lower NER and higher price level (point C, path C to A).

Based on the preceding detailed derivation of the model, we now briefly interpret the elements of the graph and the transitions to long-run equilibrium.

As mentioned earlier, equation (28) defines the so-called QQ schedule describing asset (here also money) market equilibrium. The same equation shows that the slope of QQ is negative, and that it, therefore, should intersect the 45-degree line.

The $\dot{p} = 0$ schedule shows, in turn, all combinations of price levels and exchange rates for which the goods market and the money market clear. This schedule can be derived from equation (30), using $\dot{p} = 0$ and the money demand function (24) to express the domestic interest rate in (30):
Figure 1: The Dornbusch Model: Long-Run Equilibrium and Transition Paths. Source: Authors, based on Dornbusch’s (1976) original Figure 1, p. 1166.

\[
\frac{\dot{p}}{p} = \pi \begin{bmatrix} u + \delta (s - p) + (\gamma - 1) y - \sigma m - p - \phi y \\end{bmatrix},
\]

and then solve for \( p \):

\[
p = \frac{\lambda \delta + \sigma}{\lambda \delta + \sigma} u + \frac{\lambda \delta}{\lambda \delta + \sigma} s + \frac{\sigma}{\lambda \delta + \sigma} m + \frac{\lambda (\gamma - 1) - \phi \sigma}{\lambda \delta + \sigma} y.
\]

The above expression shows that the slope of the \( \dot{p} = 0 \) schedule in Figure 1, \( \frac{\lambda \delta}{\lambda \delta + \sigma} \), is positive and between zero and unity in value. Hence, the schedule must intersect the 45-degree line too.

Finally, the simultaneous interpretation of the two schedules in the figure is the following. For any given price level, the NER adjusts instantaneously to clear the asset (money) market. Accordingly, we are continuously on the QQ schedule, since money market equilibrium and the UIP conditions hold continuously. Goods market equilibrium, to the contrary, is only achieved in the long run. Conditions in the goods market are however critical in moving the SOE from initial disequilibrium (excess demand of, or supply for, domestic output) by inducing rising or falling prices (and hence NER appreciation or depreciation respectively) to the long-run equilibrium at point \( A \). Two such initial conditions (at points \( B \) and \( C \) in the graph) and the corresponding transition paths (the arrows from \( B \) to \( A \) in the former case and, conversely, from \( C \) to \( A \) in the latter) are illustrated. Once the long-run equilibrium (point \( A \)) is attained, interest rates are equal internationally, the goods market clears, prices are constant and expected exchange rate changes are zero.
3.3.2 Alternative Ways of Goods Market Clearing

It is interesting to note that Dornbusch (1976) highlights in his exchange rate dynamics model two alternative (or, rather, complementary) ways of clearing the goods market in the presence of sticky prices. These are evident from equation (30) above, bearing also in mind that equation (24) determines the (nominal) domestic interest rate \( \iota \). With (30), (24) and their relationship via \( \iota \) in front of us, we can easily verify these two channels of goods market adjustment the Dornbusch (1976) model identifies:

1. domestic currency depreciation, \( s \uparrow \), lowers the relative price of domestic output (that is, the relative price of imports goes up), \((s - p) \uparrow\), and thus creates excess demand for domestic output, \( \ln \frac{D}{P} \uparrow \);

2. to restore equilibrium, the price of domestic output increases, \( p \uparrow \) (but proportionally less than \( s \uparrow \) since an increase in \( p \) affects AD both via the relative price effect (the first channel above) and via higher domestic interest rates, \( \iota \uparrow \)), that is, via the last term in equation (30) (the second channel here of goods market adjustment).

3.3.3 Consistent Expectations

Until now, no restrictions have been placed on the formation of expectations other than the assumption concerning anticipated exchange rate changes embodied in (23). But from (35) and (36) we have that the rate at which prices and the exchange rate converge to their long-run equilibrium values is determined in the Dornbusch (1976) model by the parameter \( \nu \). Furthermore, it is clear from (33) that the rate of convergence \( \nu \) is a function of the NER expectations coefficient \( \theta \). Therefore, if the expectations formation process in (23) must correctly predict – in compliance with perfect foresight – the actual path of exchange rates, it must also be true that the two parameters in question coincide, i.e., that \( \theta = \nu \). Accordingly, Dornbusch (1976) argues, the expectations coefficient, \( \theta \), that corresponds to the perfect foresight path, or, equivalently, that is consistent with the model he developed, is given by the solution to the equation

\[
\theta = \nu \equiv \pi \left( \delta + \sigma \theta \frac{\pi}{\theta \lambda} + \delta \right).
\]

The consistent expectations coefficient, \( \tilde{\theta} \), is obtained as a solution\(^{21}\) to (37). \( \tilde{\theta} \) is thus a function of the structural parameters of the model:

\[
\tilde{\theta}(\delta, \lambda, \pi, \sigma) = \pi \frac{a + \delta}{2} + \sqrt{\left( \pi \frac{a + \delta}{2} \right)^2 + \pi \delta \lambda}.
\]

Equation (38) gives the rate at which the Dornbusch (1976) economy will converge to long run equilibrium along the perfect foresight path. If expectations are formed according to (23) and (38), exchange rate predictions will actually be borne out. Dornbusch (1976) defends his interest in this particular path, the perfect foresight path, by emphasising that it is the only expectational assumption (i) that is not arbitrary (given the model) and (ii) that does not involve persistent prediction errors. The perfect foresight path is, obviously, the deterministic equivalent of rational expectations, he concludes.\(^{22}\)

---

\(^{21}\)Dornbusch (1976), footnote 9, p. 1167, clarifies that he has taken the positive and therefore stable root of the quadratic equation implied by (37).

\(^{22}\)Footnote 10, p. 1167.
The characteristics of the perfect foresight path one could infer from (38) are that – for any given price adjustment parameter \(\pi\) – the Dornbusch (1976) economy will converge faster the lower the interest-rate response of money demand, \(\lambda\); and the higher the interest-rate response of goods demand, \(\sigma\), and the price elasticity of demand for domestic output, \(\delta\). Dornbusch (1976) eloquently provides the intuition (which becomes even clearer when checking the respective algebraic expressions):

"... The reason is simply that with a low interest response a given change in real balances will give rise to a large change in interest rates which, in combination with a high interest response of goods demand, will give rise to a large excess demand and therefore inflationary impact. Similarly, a large price elasticity serves to translate an exchange rate change into a large excess demand and, therefore, serves to speed up the adjustment process.”


### 3.4 Model Main Result: Exchange Rate Overshooting

A second graph in Dornbusch (1976) illustrates the effects of a monetary expansion and, notably, the famous exchange rate overshooting result. Figure 2 shows the SOE at its initial equilibrium (point \(A\)), for given long run values of the NER and the price level. It also traces down the transition path from this initial position to the new long run equilibrium (at \(B'\)) following a positive monetary shock (that is, monetary expansion).

![Figure 2: The Dornbusch Model: Exchange Rate Overshooting. Source: Authors, based on Dornbusch’s (1976) original Figure 2, p. 1169.](image)

Initially, at \(A\), \(\bar{p}\) is determined by equation (26), \(\pi\) by equation (31), and the asset-market equilibrium schedule \(QQ\) (that combines money-market equilibrium with perfect capital mobility, or UIP) is given by equation (28). Then \(m\uparrow\), and this shock is anticipated to be permanent. If we substitute for \(\bar{p}\) into the \(QQ\) schedule using (26), it is easily seen that the positive monetary shock shifts \(QQ\) out to \(Q'Q'\). It is also clear from the expression for \(QQ\) that, for given \(y\) and \(\iota^*\), the increased quantity of money has to be matched by higher prices and/or domestic currency depreciation (an increase in the NER), if asset-market equilibrium is to be maintained.
We now describe in greater detail the economic forces that lead to the exchange rate overshooting phenomenon.

1. At the initial level of prices $\bar{p}$, the positive monetary shock, from (24), reduces domestic (nominal) interest rates, that is, $\iota \downarrow$. This is because $y$ is assumed fixed and $p$ is predetermined in the short run. This leads to an anticipation of a domestic currency depreciation in the long run and, therefore, at the current NER, to an expectation of a depreciating exchange rate. Both the falling interest rate and the expected depreciation reduce the attractiveness of domestic-currency denominated assets, thus leading to incipient capital outflow and causing the spot rate to depreciate, $s \uparrow$. “... the extent of that depreciation has to be sufficient to give rise to an anticipation of appreciation at just sufficient a rate to offset the reduced domestic interest rate”, Dornbusch (1976), p. 1168. Hence, at point B, we have that $s > \bar{s} \iff s - \bar{s} > 0 \iff$ expected appreciation. This first stage of the adjustment process of the SOE, only through the NER, corresponds to the constant-price path from A to B, where the economy finds itself in a short-run equilibrium.

2. At point B, however, the lower interest rates and the lower relative price of domestic goods will cause domestic prices to start rising (there is excess demand for domestic output). This gradually reduces real money balances, rising interest rates and, thus, the currency depreciation reverses into an appreciation which continues until the point where the spot NER reaches its new long-run equilibrium. That is, from the $(s - \pi)$ term of the QQ schedule (28), we get a process of a gradual upward adjustment of prices until the point when $s - \pi = 0 \iff s = \bar{s}$. This second stage of the SOE’s adjustment to a permanent monetary shock corresponds to a simultaneous adjustment of the NER and the price level until the new long-run equilibrium is reached at $B'$.

In effect, such a two-stage adjustment implies exchange rate overshooting, a phenomenon whereby the spot exchange rate temporarily exceeds its new long-run value, illustrated by the short-run equilibrium at point B in Figure 2.

To better understand overshooting, let us focus on some important features of the Dornbusch (1976) model. Equation (30) says that the rate of inflation, $\pi$, is proportional (via $\pi$) to excess demand for goods, $\ln D - \ln Y$. As Mark (2001), p. 186, points out, because excess demand is always finite, the rate of change in goods prices is always finite too, so there are no jumps in the price level. If the price level cannot jump, then at any point in time it is instantaneously fixed. The adjustment of the price level toward its long-run value must occur over time, and it is in this sense that goods prices are sticky in the Dornbusch (1976) model.

Now bearing in mind that (i) $p$ is instantaneously fixed because of the sticky-price assumption, (ii) $y$ is fixed too and (iii) $\iota^*$ is constant and taken as given by the SOE, first totally differentiate equation (24). The result from the differentiation shows that an unanticipated monetary expansion, $dm > 0$, will produce the liquidity effect, i.e., $d\iota < 0$, of which there was mention before:

---

23 As stressed by Mark (2001) in footnote 6 on p. 187, this often used thought experiment in economic analysis brings up an uncomfortable question under the common assumption, as in Dornbusch (1976), of perfect foresight: how then can a shock be unanticipated?
\[
\frac{du}{dm} = -\frac{1}{\lambda} < 0. \tag{39}
\]

Also, (iv) an increase in money causes an \emph{equiproportionate} increase in prices and the exchange rate in the long run (i.e., the long run homogeneity property that parallels money neutrality we also mentioned earlier):

\[d\pi = dp = dm.\]

Then totally differentiate (22) using \(d\pi = dm\) (and holding \(t^*\) constant) in (23) to obtain:

\[\frac{dt}{\lambda} = \left(\frac{dm}{ds}\right).\]

Use the above expression to eliminate \(dt\) in (39):

\[\frac{\theta(dm - ds)}{dm} = -\frac{1}{\lambda}.\]

Finally solving for the instantaneous depreciation \(ds\) yields:

\[ds = dm + \frac{1}{\lambda\theta} dm,\]

\[ds = \left(1 + \frac{1}{\lambda\theta}\right) dm, \tag{40}\]

and since

\[1 + \frac{1}{\lambda\theta} > 1,\]

we obtain

\[ds > d\pi,\]

which is the \emph{exchange-rate overshooting} result derived \textit{analytically}.\footnote{An alternative way to obtain the same result, which however misses the point about the liquidity effect, is to totally differentiate directly equation (25):}

\[
\frac{dp}{\lambda} - dm = -\phi \frac{dy}{\lambda} + \lambda \frac{d\pi^*}{\lambda\theta} (d\pi - ds),
\]

or

\[-dm = \lambda\theta (d\pi - ds)\]

and then use again the long run homogeneity assumption allowing us to replace \(d\pi = dm\) and finally solve for \(\frac{ds}{dm} = 1 + \frac{1}{\lambda\theta}.\)
The latter, from (22), requires only a small expectation of appreciation to offset it and therefore, given $\theta$ and $\pi$, only a small depreciation of the spot rate $s$ (in excess of the long run NER) to generate that expectation. A similar interpretation applies to the coefficient of expectations in equation (40), $\theta$.

As Dornbusch (1976) points out, the short run effects of monetary expansion are thus entirely dominated by asset markets and, more specifically, by capital mobility and expectations formation. This feature of his model corresponds to the crucial assumption that asset market prices such as the exchange rate adjust fast relative to the price of domestic output (the price level) in the goods market.

We can also observe that, from (40) the volatility of exchange rate changes is higher than that of changes in fundamentals since $\text{Var}[ds] = (1 + \frac{1}{\lambda\theta})^2 \text{Var}[dm]$. This is one of the prominent facts the model originally aimed to explain. A careful examination of the adjustment process to a monetary shock also shows that the model is compatible with the other stylised facts spelled out in Section 3.1.

4 Empirics of the Nominal Exchange Rate and the Random Walk Hypothesis

So far we have presented in detail two key theoretical models of exchange rate dynamics. A central issue in macroeconomics is how well these models fare when tested against the data. Important changes in the way researchers develop theory are due to empirical findings that lend support or stand against theory conclusions. Possibly nowhere else in international macroeconomics is this interaction so important as in exchange rate determination theory. In this section, hence, we review how successful dynamic macroeconomic models are in explaining the behavior of exchange rates. We focus here mainly on the monetary model, but the results easily apply to other fundamentals-based models. The evidence reviewed pertains mainly to the ability of structural macroeconomic models to forecast exchange rates, although this is not the only possible way of assessing theory.25

4.1 Meese and Rogoff (1983): The Exchange Rate Disconnect Puzzle

There is a very large applied literature studying the power of key exchange rate models to forecast out-of-sample over horizons of up to four years. The literature started with Meese and Rogoff’s (1983 a, b) conclusion that a set of widely used structural (i.e., theory-based) models, including the monetary model and the Dornbusch model, cannot predict future exchange rates better than the simplest, but not very informative, time series model, the random walk. In a nutshell, the random walk without drift is a univariate time series model which implies that the best prediction for the future level of a variable $z_t$ is its level today,26 i.e., $E_t[z_{t+1}] = z_t$.27 In fact, what Meese and Rogoff

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25 Other possible criteria can be performance in-sample, the economic value of predictability for investors (Abhyankar et al., 2005), ability to predict the direction of exchange rate changes and, more generally, economic value for policy makers. It also has to be noted that, following Clements and Hendry (2001), incorrect but simple models can outperform correct models in terms of forecast accuracy.

26 This also represents static expectations, as no change is anticipated. Recall also the discussion in section 3.4. of Chapter 1 about unit-roots and mean-reversion.

27 Kilian (1999) favours comparison with a random walk model with drift, $E_t[z_{t+1}] = v + z_t$, so that the model predicts a constant rate of change of $z_t$ equal to $v$. 
León-Ledesma and Mihailov (January 24, 2011)

(1983) found was often termed the random walk hypothesis of the exchange rate. It basically means that the exchange rate appears to be (close to) a random walk process, hence difficult to predict by structural exchange rate models of the kind we studied in this chapter. The hypothesis also forms part of the wider exchange rate disconnect puzzle in Obstfeld and Rogoff (2001): the very weak connection between the NER and virtually any macroeconomic fundamentals. In a way, such a conclusion made senseless earlier and further attempts to try to explain exchange rate determination and dynamics by modelling macroeconomic details of relevance to the exchange rate. Some of the research on exchange rates that followed, therefore, directed its interest to purely statistical modelling of exchange rates, e.g., the literature on the fundamental equilibrium exchange rate (FEER) or the behavioural equilibrium exchange rate (BEER). Other researchers began focusing on a deeper study of what became known as the microstructure approach to the exchange rate, to which we shall return later in the book. Much of the profession continued to be interested in improving the theoretical foundations of exchange rate modelling, by incorporating additional and more realistic features into traditional exchange rate macro-models, and notably by deriving equilibrium conditions from microeconomic principles of agents’ optimisation, as we shall see in the next parts of the book.

Let us now provide more detail here on the persistent and widely debated empirical failure of structural exchange rate models. Meese and Rogoff (1983a) compared the forecasting performance of two types of exchange rate models: structural, such as the monetary or Dornbusch models, and time series models, such as the random walk model or the unconstrained vector autoregression (VAR) we shall sketch further down. They summarised, in fact, three models, the third one an extension of the Dornbusch model to incorporate consideration of current account effects, into a general (semi-reduced form) specification:

\[ s = a_0 + a_1 (m - m^*) + a_2 (y - y^*) + a_3 (\iota - \iota^*) + a_4 (\pi^e - \pi^e*) + a_5 tb + a_6 tb^* + u. \] (41)

The notation in (41) is as introduced thus far, with the interest rates being short-term, expected inflation rates \((\pi^e, \pi^e*)\) long-run, and \(tb\) and \(tb^*\) being cumulated trade balances. The disturbance term \(u\) may be serially correlated. The Home variables are proxied by data for the US, while the Foreign variables (denoted by asterisk) correspond to data for Germany, the UK, Japan and a trade-weighted index, respectively. As one can see, (41) expresses the exchange rate as a function of Home-Foreign differentials in money supplies, real incomes, interest rates, expected inflation rates and of the levels of cumulative trade balances. The above general specification implies some constraints, nesting the three structural models employed in the Meese-Rogoff tests. For example, one assumption is that \(a_1 = 1\), which in a more formal language states that the exchange rate exhibits homogeneity of degree 1 in relative money supplies. Furthermore, Meese and Rogoff (1983a) impose the constraints that the coefficients to the Home and Foreign

\[^{28}\text{The other manifestation being that transitions to floating-exchange-rate regimes lead to sharp increases in nominal and real exchange rate volatility with no corresponding changes in the volatility of fundamentals.}\]

\[^{29}\text{See, e.g., Clark and MacDonald (1998) and Isard (2007).}\]

\[^{30}\text{We follow in our exposition the original papers, in particular Meese and Rogoff (1983 a, b), Mark (1995), Rogoff (2001), Engel and West (2005), Engel, Mark and West (2007) and the comments to the latter paper.}\]
money demand functions and price adjustment equations are the same (which is not the case, however, for the trade balances). Then, if \( a_4 = a_5 = a_6 = 0 \), the general specification (41) reduces to the monetary model. The Dornbusch model, which implies slow domestic price adjustment, can be obtained instead by setting \( a_5 = a_6 = 0 \), since now interest rate differentials do not cancel out immediately due to price stickiness. The third structural model Meese and Rogoff (1983a) test corresponds to the general specification itself, with no zero coefficients at all. The authors note that all these three regressions perform reasonably well in-sample. Yet out-of-sample they do not forecast better than a simple univariate random walk or an unconstrained multivariate VAR. The random walk is a first logical benchmark against which one would judge about any improvement of fit or forecasts arising from ‘enlightenment’ from economic theories because according to the random walk model no information could help predict future change. The random walk is, thus, agnostic, always forecasting no change. Furthermore, the simple random walk model requires no estimation, while the VAR Meese and Rogoff (1983a) estimate is of the form:

\[
s_t = a_1 s_{t-1} + a_2 s_{t-2} + \ldots + a_n s_{t-n} + B_1' X_{t-1} + B_2' X_{t-2} + \ldots + B_n' X_{t-n} + u_t,
\]

where \( X_{t-j} \) is a vector of the explanatory variables in the single-equation regression (41), lagged \( j \) periods, the \( a_j \)'s are autoregressive coefficients, and \( B_j \) is a vector of coefficients. The other six univariate time series models, in addition to the random walk, that Meese and Rogoff (1983a) employed were simple autoregressive models with different lag selection criteria available in the econometric literature at the time. The models were estimated using Ordinary Least Squares (OLS), Generalised Least Squares (GLS) to correct for residual autocorrelation of orders 1 and 5, and an Instrumental Variables (IV) estimator that also corrects for residual autocorrelation. The forecasting performance of all these models, structural and statistical, was compared along their out-of-sample accuracy, the latter measured by three alternative but complementary statistics, mean error (ME), mean absolute error (MAE) and root mean square error (RMSE), defined as follows:

\[
ME \equiv \frac{\sum_{q=0}^{N_k-1} F(t + q + k) - A(t + q + k)}{N_k},
\]

\[
MAE \equiv \frac{\sum_{q=0}^{N_k-1} |F(t + q + k) - A(t + q + k)|}{N_k},
\]

\[
RMSE \equiv \sqrt{\frac{\sum_{q=0}^{N_k-1} [F(t + q + k) - A(t + q + k)]^2}{N_k}}.
\]

31 Meese and Rogoff (1983a), p. 6, report that no gain in out-of-sample fit results from estimating separate coefficients for the Home and Foreign money supplies and real incomes.

32 Or a constant rate of change in the case of the random walk with drift.

33 In fact, with monthly data Meese and Rogoff (1983a), p. 9 and footnote 9, have estimated the uniform lag length across their seven equations in the VAR system to be 2 for all specifications of the Foreign country, except for Japan, when this lag was 4.

34 They also use an estimator that minimises absolute deviations (MAD). This estimator is believed to be more robust for series that display “fat tails” (and hence non-normality) as is the case with many financial variables.
where \( k = 1, 3, 6, 12 \) is the forecast step, \( N_k \) the total number of forecasts in the projection period for which \( A(t) \) is the actual value and \( F(t) \) the corresponding forecast value. The principal criterion applied for comparing the forecasts is RMSE. Yet RMSE is not appropriate if exchange rates are governed by a non-normal stable Paretian process with infinite variance, and here comes one rationale for adding MAE, the other being that MAE is also useful when the distribution has fat tails (known too as leptokurtic distributions and empirically common among asset prices). Meese and Rogoff’s (1983a) justification for adding ME as well is that by comparing MAE and ME they can ascertain whether a model systematically over- or underpredicts. The main results are summarised in Table 1, which is slightly adapted from the original study (p. 13) and reports only the RMSE.\(^{35}\)

What is evident in the table is that none of the models achieves systematically and significantly lower RMSE than the random walk.\(^{36}\) Moreover, actual values of the explanatory variables in the structural models were used to obtain the exchange rate forecasts instead of forecasts of these explanatory variables. Meese and Rogoff (1983a) claim that such a procedure actually favours structural models, an issue to which we will return later. Thus, the random walk emerges as the model that predicts the exchange rate no worse than either time series models or structural models. Hence, exchange rate dynamics appear to be better approximated by a random walk.

### 4.2 Explaining the Meese and Rogoff (1983) Results

The reaction to the Meese-Rogoff results has become a subdiscipline within international macroeconomics in itself. Exchange rate forecasting is not only a potentially useful tool for investors seeking to gain returns from currency trading, but a way of testing the validity of fundamentals-based models of exchange rate determination. It is not surprising, hence, that the profession has greatly built up and developed on these initial results. Much of the debate has been enriched by advances in the econometric theory of time series, which has formed the core of many responses to the random walk hypothesis. Recently, however, alternative explanations of the “puzzle” also focus on the interpretation of well-established theory results. Here we review several of them in a succinct way.

#### 4.2.1 Econometric Issues

As noted above, the way Meese and Rogoff obtain forecasts of exchange rates using fundamentals is not a proper out-of-sample forecast, as they used the actual values of fundamentals instead of their forecasts. This led to the appearance of true out-of-sample forecast exercises, of which Mark (1995) is perhaps the best exponent. He reports evidence that, at long-horizons, changes in the log of the exchange rate contain a component that is significantly predictable by fundamentals.

This evidence of long-horizon predictability is based on the ideas coming from the theory of integration and cointegration. As discussed in section 3.4 of Chapter 1, a random walk is not mean reverting, meaning that its autoregressive root is equal to 1. A time-series process like \( x_t = x_{t-1} + \epsilon_t \) with \( \epsilon_t \sim iid \ N(0, \sigma_t) \) is a non-stationary process. Its variance grows with time and it can be shown that \( Var(x_t) = \sigma^2 t \). That is,
Table 1: The Meese-Rogo Results

<table>
<thead>
<tr>
<th>Ex. rate</th>
<th>Horizon</th>
<th>Model: Random walk</th>
<th>Forward rate</th>
<th>Univariate VAR</th>
<th>Monetary model</th>
<th>Dornbusch model</th>
<th>Complete eq. (41)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$/DM</td>
<td>1</td>
<td>3.72</td>
<td>3.20</td>
<td>3.51</td>
<td>5.40</td>
<td>3.17</td>
<td>3.65</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8.17</td>
<td>9.03</td>
<td>12.40</td>
<td>11.83</td>
<td>9.64</td>
<td>12.03</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>12.98</td>
<td>12.60</td>
<td>22.53</td>
<td>15.06</td>
<td>16.12</td>
<td>18.87</td>
</tr>
<tr>
<td>$/yen</td>
<td>1</td>
<td>3.68</td>
<td>3.72</td>
<td>4.46</td>
<td>7.76</td>
<td>4.11</td>
<td>4.40</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>11.58</td>
<td>11.93</td>
<td>22.04</td>
<td>18.90</td>
<td>13.38</td>
<td>13.94</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>18.31</td>
<td>18.95</td>
<td>52.18</td>
<td>22.98</td>
<td>18.55</td>
<td>20.41</td>
</tr>
<tr>
<td>$/£</td>
<td>1</td>
<td>2.56</td>
<td>2.67</td>
<td>2.79</td>
<td>5.56</td>
<td>2.82</td>
<td>2.90</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6.45</td>
<td>7.23</td>
<td>7.27</td>
<td>12.97</td>
<td>8.90</td>
<td>8.88</td>
</tr>
<tr>
<td>trade-weighted</td>
<td>1</td>
<td>1.99</td>
<td>n.a.</td>
<td>2.72</td>
<td>4.10</td>
<td>2.40</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6.09</td>
<td>n.a.</td>
<td>6.82</td>
<td>8.91</td>
<td>7.07</td>
<td>6.49</td>
</tr>
</tbody>
</table>

Source: Meese and Rogoff (1983), Table 1, p. 13, slightly shortened by the authors. The table reports the RMSE for the full forecasting period, November 1976 through June 1981, approximately in percentage terms. The three structural models are estimated using and IV estimator to correct for first-order serial correlation. The measure of money is narrow money (M1-B) and the long-term interest rate differential serves as the proxy for the long-run expected inflation differential in the two sticky-price models.

It has infinite memory as it permanently accumulates shocks and hence diverges from its initial value $x_0$. A variable like $x_t$ can be transformed into a stationary variable by applying the first difference operator, so that $\Delta x_t$ is a zero mean constant variance process. This is usually termed an integrated process of order 1 and denoted $I(1)$, or a difference stationary variable, since first differencing it will result in a stationary (or $I(0)$) process.

There is little doubt that many macroeconomic variables can be represented as $I(1)$ processes. However, for any pair of $I(1)$ variables, if there is any linear combination of the two that yields a stationary process, then they are said to be cointegrated. The idea of cointegration was first introduced by Granger (1981). Suppose that we observe two $I(1)$ variables $x_t$ and $y_t$. Then $x_t$ and $y_t$ are cointegrated if there exists a $\beta$ such that $y_t - \beta x_t$ is $I(0)$. This means that in the regression equation

$$y_t = \beta x_t + u_t,$$

the error term $u_t$ is an $I(0)$ process so that $x_t$ and $y_t$ do not drift away from each other but are tied together by a long-run relationship given by the cointegration vector (-1 $\beta$).

See Aksoy and León-Ledesma (2008) for an analysis on a set of 249 macroeconomic variables for the G7 countries.
That is, even if the two variables are non-stationary, they share an equilibrium relation. Hence, by the Engle and Granger (1987) error-correction representation theorem either \( y_t \) or \( x_t \) has to adjust towards this long-run relationship. That is, in the long-run, current (time \( t \)) deviations from equilibrium must be corrected so that changes in the variables from \( t \) to any \( t + k \) must be in the direction of this equilibrium relationship. This means, then, that \( y_t \) should be a predictor for \( \Delta x_{t,t+k} \) or \( \Delta y_{t,t+k} \).

If we go back to equation (14) in the monetary model, we can rearrange it so that

\[
s_t - f_t = \lambda (E_t [s_{t+1}] - s_t).
\]

The right-hand side of this equation can be solved by using the present-value (no-bubbles) solution for \( s_t \) (18), so that

\[
\lambda(E_t [s_{t+1}] - s_t) = \lambda \left( \gamma \sum_{j=0}^{k} (1 - \gamma)^j E_t [f_{t+1+j}] - \gamma \sum_{j=0}^{k} (1 - \gamma)^j E_t [f_{t+j}] \right) = \\
= \lambda \gamma \sum_{j=0}^{k} (1 - \gamma)^j E_t [\Delta f_{t+j+1}].
\]

Hence, given that \( \lambda \gamma = 1 - \gamma \), we obtain

\[
s_t - f_t = \sum_{j=1}^{k} (1 - \gamma)^j E_t [\Delta f_{t+j}]. \tag{43}
\]

Both \( s_t \) and \( f_t \) are commonly found to be non-stationary variables. But looking at equation (43), since \( \Delta f_t \) is stationary, the discounted sum of expected future changes in the fundamentals must also be stationary.\(^{38}\) Hence, the theory suggests that \( s_t - f_t \) should be a predictor for \( \Delta s_{t,t+k} \), and hence fundamentals could help forecast exchange rate returns, at least for long-horizons. This is tested by Mark (1995) using the following regression model:

\[
s_{t+k} - s_t = \beta(s_t - f_t) + \varepsilon_{t+k}, \tag{44}
\]

where \( \varepsilon_{t+k} \) is an error term, \( \beta \) an error-correction coefficient, \( k \) the forecast horizon and \( t = 1, 2, \ldots, T \). If fundamentals help explaining exchange rates, we would expect \( \beta \) to be positive, statistically significant, and an increasing function of \( k \). This equation is used by Mark (1995) to compare the forecasts of a fundamentals-based model with those of a naïve random walk. The value for parameter \( \phi \) for the construction of fundamentals can be either estimated or pre-fixed to any value between 1 and 0.\(^{39}\) Then equation (44) can be estimated for a sub-sample \([1, 2, \ldots, \tau]\), with \( \tau < T \), and used to obtain out-of-sample \( k \)-step ahead forecasts. That is, forecasts from (44) for observations \( \tau + 1, \tau + 2, \ldots, \tau + k \). Then the equation is estimated up to observation \( \tau + 1 \) and forecasts obtained for \( \tau + 2, \tau + 3, \ldots, \tau + k + 1 \). This process is recursively repeated until we obtain the last estimate up to observation \( T - k \). This yields a series of forecasts for

\(^{38}\)If the fundamentals follow a driftless random walk \( f_t = f_{t-1} + \varepsilon_t \), then \( \Delta f_t = \varepsilon_t \). Since \( \varepsilon_t \sim iid N(0, \sigma_t) \), then \( \sum_{j=1}^{k} (1 - \gamma)^j E_t [\Delta f_{t+j}] \) must also be a zero-mean stationary process.

\(^{39}\)The results reported by Mark (1995) use a pre-fixed value of \( \phi = 1 \).
each different horizon $k$. We can then compute the RMSE and compare it to that of a random-walk. A simple, yet statistically not devoid of problems, metric for comparison is to calculate Theil’s statistic $U = \frac{RMSE_f}{RMSE_{RW}}$, where $RMSE_f$ and $RMSE_{RW}$ are the RMSE of the fundamentals and the random walk models respectively. If this ratio is below unity, then fundamentals-based models outperform the random-walk at that particular horizon. Mark (1995) performs forecasts using quarterly data for forecast horizons of $k = 1, 4, 8, 12, 16$ and finds a significant forecastable exchange rate component for the Deutsche Mark, Japanese Yen and Swiss Franc against the US dollar, especially for forecast horizons of 12 and 16 quarters. Mark and Sul (2001) use a panel version of (44) by pooling data for 19 industrialised countries. They find that the monetary model outperforms the random-walk for 16 quarter horizons, but also for 1 quarter in a substantial number of countries.

We reproduced Mark’s (1995) test for the quarterly bilateral exchange rates of Canada, Japan and the UK against the US dollar for the period 1974:1 to 2001:3. We set $t_0$ in 1987:3 so as to capture about half of the sample and forecast on the other half. The results of Theil’s $U$ index are reported in Table 2 for $k = 1, 6, 12, 16$, where we pre-fixed the value of $\phi$ to 1. We can observe that, for 1-quarter ahead forecasts, the model only marginally outperforms the random walk (RW) for Japan, being practically indistinguishable from the RW for the other two. The forecast performance does not improve for Canada. However, for Japan and the UK, it substantially outperforms the RW for the 16 quarters forecast. These results support the evidence in Mark (1995) that fundamentals can improve on a RW when forecasting exchange rates at long-horizons.

### Table 2: Long-Horizon Forecasts

<table>
<thead>
<tr>
<th>Country</th>
<th>Horizon</th>
<th>Canada</th>
<th>Japan</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.981</td>
<td>0.981</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.014</td>
<td>0.955</td>
<td>0.976</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.033</td>
<td>0.939</td>
<td>0.938</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1.009</td>
<td>0.847</td>
<td>0.889</td>
<td></td>
</tr>
</tbody>
</table>

Source: Authors’ own calculations of Theil’s $U = \frac{RMSE_f}{RMSE_{RW}}$. Money is M1 and output is GDP. Data from IMF’s IFS database for 1974:1–2001:3.

The assumption of the literature is that Meese and Rogoff (1983) give an “unfair” advantage to the fundamentals model since they use true values of the fundamentals, rather than their forecasts. Engel, Mark and West (2007) challenge this assumption by arguing that the in-sample fit of the model can be arbitrarily changed by the way we write the model. One can re-scale the variables in such a way that they yield a smaller variance, a problem that is not present in (44). They also claim that, if exchange rates are also driven by “unobserved” fundamentals, the use of actual observed values may worsen forecasts if these are correlated with the “unobserved” variables.

The recent literature has focused on improving the forecast performance of models like (44) by focusing on a variety of econometric problems. These range from the use of historical data (Rapach and Wohar, 2002), panel data (Mark and Sul, 2001 and Groen, 2005), high-persistence (Rossi, 2005), parameter instability (Rossi, 2006), small-sample inference (Kilian, 1999), nonlinear mean-reversion in deviations from fundamentals (Kilian and Taylor, 2003), etc. The findings of these studies all point towards statistically
and economically significant gains from using fundamentals to forecast exchange rates. However, this appears to happen only at long-horizons, and this positive performance is not robust.  

### 4.2.2 Engel and West (2005): The Exchange Rate as a Near-Random Walk

In an important paper, Engel and West (2005) take a different route. They claim that exchange rates and fundamentals are linked in a way that is broadly consistent with asset-pricing models of the exchange rate. Looking back at equation (18), one can easily see that the exchange rate should be a good predictor of the future evolution of fundamentals, as they reflect market participants’ expectations about the future stream of fundamentals. In order to test this hypothesis, they present Granger-causality tests between fundamentals and exchange rates for the G7 countries and for different measures of fundamentals. The concept of Granger-causality simply means that, for two stochastic variables \( x_t \) and \( y_t \), \( y_t \) Granger-causes \( x_t \) if past values of \( y_t \) improve the forecasts of \( x_t \) beyond the information contained in past values of \( x_t \). We have already seen that, if two variables are cointegrated, then at least one of them adjusts towards the long-run relationship implying Granger-causality. But if two \( I(1) \) variables are not cointegrated, there may still be information content if we model them in first-differences. Engel and West (2005) find little evidence in favour of cointegration between exchange rates and fundamentals, and hence run Granger-causality tests in first differences by estimating:

\[
\Delta f_t = \sum_{i=1}^{d} \zeta_i \Delta f_{t-i} + \sum_{i=1}^{d} \xi_i \Delta s_{t-i} + \varepsilon_t, \tag{45}
\]

where \( d \) is the maximum lag-length that can be determined by a variety of methods. The Granger-causality test then consists of a joint test of significance for the \( \xi_i \) coefficients (using an \( F \)-test or a Likelihood Ratio (LR) test). They perform the test for both variables, that is, Granger-causality from \( \Delta s_t \) to \( \Delta f_t \) and from \( \Delta f_t \) to \( \Delta s_t \). They find that exchange rates help predict fundamentals for a number of cases, but confirm that fundamentals are of little use in predicting the exchange rate. This is then still compatible with the asset approach to exchange rates.

The question is, however, why can’t fundamentals predict exchange rates? This is explained by an analytical finding that, under plausible conditions, models like (18) actually predict that the exchange rate will behave as a near-random walk. This is the case if the discount factor \( 1 - \gamma \) approaches unity and fundamentals are \( I(1) \) processes. Under these conditions, they prove that the log of the exchange rate approximates a process which, for finite samples, is practically indistinguishable from the random walk. It is not surprising, hence, that the NER appears to be well approximated by a random walk.

Engel, Mark and West (2007) claim that this theorem renders forecast comparisons relative to the random walk unreliable. So they propose a number of alternative ways

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40 See Cheung, Chinn and Garcia-Pascual (2005) for a general assessment. Rogoff and Stavrakeva (2008) also show that exchange rate forecasts are not robust to the choice of forecasting window (essentially the choice of \( \tau \)).

41 Properly speaking, this is a test for Granger non-causality, i.e. the null hypothesis is non-causality.

42 Sarno and Sojli (2009) directly test the implications of the model by estimating the discount factor using survey data on expectations. They find point estimates in the range of 0.98-0.99, which supports the Engel and West (2005) hypothesis.
of model evaluation, including in-sample fit, the role of monetary policy in affecting expectations, the response by exchange rates to news about future fundamentals, models that account for observed exchange rate volatility as well as models that allow for survey forecasts in determining the future value of the fundamentals. Their conclusions are less pessimistic than the prevailing literature, claiming that fundamentals-based models, when analyzed from different angles, are not as bad as previously thought.\footnote{See Rogoff’s (2007) reply to Engel, Mark and West (2007) for further discussion of these points.}

4.2.3 Where Do We Stand?

The large empirical literature on the link between fundamentals and exchange rates, despite having started with rather nihilistic results, has proven useful in understanding the dynamics of exchange rates. Several facts have been accumulating since the seminal work of Meese and Rogoff. Without attempting to be exhaustive, some of these facts can be summarized in the following points:

- Exchange rate models appear to beat the random walk at forecast horizons beyond one year, although these results are not fully robust and shorter horizon forecasts still remain problematic.

- The in-sample fit of exchange rate models is satisfactory although, as Engel, Mark and West (2007) maintain, is not a reliable benchmark.

- As stated by Rogoff (2001a), in periods of current and/or expected high inflation (especially for emerging markets) exchange rate models have proven extremely useful.

- Exchange rates and fundamentals appear to be linked in a way that is not incompatible with asset-price models. Specifically, exchange rates seem to contain significant information about the future evolution of fundamentals; a relation that Rossi (2007) finds to be quite robust.

- Fundamentals are good predictors of exchange rates for so-called “commodity currencies” pertaining to the Australian, Canadian, and New Zealand dollars (Chen and Rogoff, 2003). Because a significant share of the overall exports of each of these three countries is accounted for by commodities, the respective country’s major commodity export prices are highly correlated with exchange rate changes even in out-of-sample forecasts.

- There is ample evidence that news about macroeconomic fundamentals affect exchange rates in a manner that is compatible with theoretical predictions.

The important question is then to analyze the potential reasons for the failure of these models beyond mere statistical issues. Rogoff (2001a and 2001b) summarised the reasons for this failure. One cause for the failure is the link between exchange rates and prices, which leads to the famous PPP condition: this link is a very long run relationship, as claimed in Rogoff (1996) too, with the \textit{half-life} of PPP deviations being very persistent (3 to 4 years) in linear models. A second cause is the potential instability of the money demand function, which would make the link between money and prices less transparent. This adds to the well-reported changes in monetary policy stance in
most major industrialised countries: the current practice is to set interest rates, making money supply endogenous. A related point is that this change in policy stance led to a downward convergence process in inflation rates across major countries, making the effects of changes in monetary aggregates more difficult to detect. A final, perhaps more fundamental, cause has to do with the magnitude of market segmentation: even across industrialised countries, it is of an order consistent with international transactions costs for all GDP (not just the actually traded goods) averaging 25% or more, an argument emphasised by Obstfeld and Rogoff (2001).

In light of these points, and given the simplifying assumptions of standard theoretical models, one could indeed claim that they perform well empirically in spite of their simplicity. As a conclusion, Rogoff (2007) writes that “exchange rates remain a very tough nut to crack, even after the Great Moderation in macroeconomic variables. Right now, things still look pretty good if we can call the glass ten percent full.”

**Problems and Exercises**

1. *Government spending in the overshooting model.* Assume that we now introduce government spending \( G \) or \( \ln G = g \) in the model. The expression for aggregate demand (29) now becomes

\[
\ln D = u + \delta (s - p) + \gamma y - \sigma \nu + g,
\]

Analyse the impact on the exchange rate of an expansionary fiscal policy, \( dg \). Would this fiscal policy expansion induce NER overshooting?

2. *Dornbusch (1976) Part V.* In the sticky price model, suppose that we relax the assumption of fixed output and we allow short run changes in output to respond to changes in aggregate demand. The goods-market equilibrium condition becomes

\[
y = \ln D = u + \delta (s - p) + \gamma y - \sigma \nu,
\]

which can also be written as

\[
y = \mu [u + \delta (s - p) - \sigma \nu], \quad \mu = \frac{1}{1 - \gamma} > 0,
\]

and the price adjustment equation is now

\[
\dot{p} = \pi (y - \bar{y}),
\]

where \( \bar{y} \) denotes the full-employment level of output, so that inflation is now a function of the output gap. Find the equilibrium of the model and analyse the impact of a permanent unanticipated monetary expansion. Is there an overshooting effect?

3. *Forecasting exchange rates.* Using the long run forecasting equation (44) and assuming values for \( \lambda = [0.1, 0.4, 1.0] \), compute the 1, 2, 4, 8 and 16 quarters ahead forecast for the bilateral NER of Canada, Japan and the UK against the USD. Use the data provided in www.oup.co.uk/leonledesma-mihailov/Ch3.45

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44 Engel, Mark and West (2007) discuss this point at length and provide further references.

45 A Gauss 8.0 code is provided in the webpage.
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