

**A NUMERICAL MODEL OF
SEDIMENT DEPOSITION
ON SALTMARSHES**

by

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Abstract

A one-dimensional model of sediment transport and deposition over a saltmarsh is developed by simplifying a three-dimensional mass balance equation for the sediment and integrating the resultant equation over the depth of the flow. The sediment is assumed to be distributed with a uniform concentration through a column of water above which the water is clear. The sediment in the column is assumed to settle out at a fixed rate and the height of the front between the clear and the water containing sediment is used as the dependent variable. The resulting equation for the height of this front is solved numerically for one tidal period using the box scheme. From this solution the position at which the front intersects the marsh surface is approximated which enables the depth of sediment deposited by one tide to be evaluated. The results obtained for a special case are compared with an analytic solution. The model is then used to investigate how varying the settling velocity and the dimensions of the saltmarsh affect the distribution of the deposited sediment.

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Notation

A	amplitude of tide [m]
b	height of saltmarsh above mean sea-level [m]
b'	non-dimensionalised b
C	fractional volume of sediment in fluid
C_0	fractional volume of sediment in tidal water
D	diameter of sediment particle [m]
E_s	absolute error in the depth of sediment deposited [m]
H	height of water above saltmarsh [m]
H'	non-dimensionalised H
j	label of space grid points
j_c	node before x_c^P
j_E	node before x_E
k	mode number of fourier decomposition of y
L	length of saltmarsh [m]
M	number of space steps
N	half the number of time steps
n	exponent or time level
P	number of time steps in characteristic solving routine
p	time level in characteristic solving routine
R	rate of deposition on saltmarsh [ms^{-1}]
r_M	gradient of line of best fit for graph of error against M
r_N	gradient of line of best fit for graph of error against N
S	depth of sediment deposited on the saltmarsh [m]
S_j^n	approximation to depth of sediment deposited on the marsh at (x_j, t_n) [m]
S'	non-dimensionalised S
T	tidal period [s]
t	time [s]
t_E	time at which no sediment is suspended above a point on the marsh [s]
t'	non-dimensionalised t
t_n	time at n th time level [s]
t_p	time at p th time level of characteristic solving routine [s]
t_0	phase of tide at which water covers the saltmarsh [s]
t'_0	non-dimensionalised t_0
u	horizontal fluid velocity [ms^{-1}]
u'	non-dimensionalised u

u_i	fluid velocity components [ms^{-1}]
$[uy]_j^n$	approximation to $uy(x_j, t_n)$ [m^2s^{-1}]
v_p	particle settling velocity [ms^{-1}]
v'_p	non-dimensionalised v_p
v_0	settling velocity of single smooth sphere [ms^{-1}]
w	vertical fluid velocity [ms^{-1}]
x	horizontal coordinate [m]
x'	non-dimensionalised x
x_c	position of characteristic through x_E [m]
x_c^p	approximation to $x_c(t_p)$ [m]
$x_c^{p,(k)}$	k th iteration of x_c^p [m]
x_j	position of j th node [m]
x_E	position at which no sediment is suspended above the marsh [m]
x_i	components of position vector [m]
y	height of column of sediment in water
y'	non-dimensionalised y
y_c	value of y on characteristic x_c [m]
$y_{c'}$	approximation to $y(x_c^p, t_n)$ [m]
y_j^n	approximation to $y(x_j, t_n)$ [m]
z	vertical coordinate [m]
α	label of characteristics
ϵ_i	sediment mixing coefficients [m^2s^{-1}]
η	dynamic viscosity of fluid [Nsm^{-2}]
θ	parameter in box scheme
λ	the Courant number
ξ	amplitude of fourier mode
ρ	density of fluid [kgm^{-3}]
σ	density of sediment [kgm^{-3}]
τ	truncation error
ϕ	parameter in box scheme
ω	angular frequency of tide [s^{-1}]
Δt	time step [s]
Δt_c	time step in characteristic solving routine [s]
Δx	space step [m]
$\Delta x'$	space step for first step in box scheme for ebb tide [m]

Chapter 1

Introduction

1.1 What is a Saltmarsh?

Coastal saltmarshes are relatively flat areas of land which are regularly flooded by the sea; they occur high in the intertidal zone, mainly in temperate and high latitudes on low energy coasts [Allen and Pye 1992]. Their occurrence is controlled by the coastal geography since deposited sediment can only accumulate where the wave action is small. Hence saltmarshes tend to be found only in sheltered areas like bays and estuaries or on the lee side of spits and barrier islands. An exception to this is where a major river deposits fine sediment which forms a large and shallow region close to the shore which reduces the intensity of the incoming waves, for example, the Mississippi Delta.

There are several processes which affect the development of saltmarshes. Firstly, they need a source of sediment: this is usually from the suspended material in the tidal water which periodically floods the marshes. The sediment will only be deposited when the fluid velocities are small. Hence a saltmarsh will only develop

near the top of the intertidal region which is only covered at slack water. The presence of vegetation on the marsh will affect the growth of the marsh in two ways; it provides both an effective method for trapping and binding the sediment carried by the water and also acts as a source of sediment itself. The vegetation will only form once the saltmarsh is high enough relative to the mean sea-level, but once this occurs the marsh will often grow more rapidly as the plants trap the suspended sediment and add decaying organic matter.

The tidal regime is also important; the periods between flooding allow the marsh to dry binding the sediment together. If the marsh is left exposed for long enough plant life can develop strengthening it even more. A change in relative sea-level can have a similar effect as this will also vary the length of time for which the marsh is flooded. The wind and wave climate can also play a vital role. Strong waves can erode large parts of the marsh and so a period of stormy weather can change the shape and height of the marsh significantly.

1.2 Why are Saltmarshes Important?

Saltmarshes can be seen along much of the coast of Britain, some 20% of the coastline of England and Wales, mostly in or around estuaries [Brampton 1992]. These saltmarshes have long been of interest to ecologists and conservationists because of the wide variety of plant and animal life they sustain. Many of the species found on the saltmarshes are believed to be exclusive to this environment [Doody 1992]. As a result some 80% of these saltmarshes have been declared Sites of Special Scientific Interest [Brampton 1992]. Saltmarshes are also of importance in agriculture, providing seasonal grazing and sites for reclamation of land for

farming arable crops.

Until recently however they have been of little interest to coastal engineers, but in the past few years there has been much attention paid to global warming and the resultant threat of rising sea-levels and a stormier climate on the lowlying coastal regions of Britain. The danger of loss of land to the sea has brought about a need to improve the coastal defences around much of the British Isles.

Traditional coastal defence tactics such as the construction of sea walls can be expensive and cheaper methods of protecting the land are being considered. One such tactic is the use of the natural features of the coastline as a defence mechanism. The most important feature of the saltmarsh in this respect is the way in which they dissipate much of the incoming wave energy so that little remains at the landward end [Brampton 1992]. This enables the land beyond to be protected from the sea by a much smaller and therefore cheaper wall. To use saltmarshes as an effective aid to coastal defence, without damaging their ecological value, requires an understanding of the ways in which the marsh develops and how human interference may affect this. This project uses a simplified one-dimensional model of sediment transport over the marsh to simulate numerically the deposition of sediment on the marsh by the tide.

Chapter 2

Theory

2.1 The Generalised Mass Balance Equation for Suspended Sediment

The generalised mass balance equation for suspended sediment is

$$\frac{\partial C}{\partial t} + \sum_{i=1}^3 \frac{\partial}{\partial x_i} (u_i C) = \sum_{i=1}^3 \frac{\partial}{\partial x_i} \left(\epsilon_i \frac{\partial C}{\partial x_i} \right) + \frac{\partial}{\partial z} (v_p C) \quad (2.1)$$

where C is the fractional volume of sediment in the fluid, although any other method of expressing concentration could be used, t is the time, u_i are the components of the fluid velocities in the coordinate directions x_i , with $x_3=z$ vertically upwards, v_p is the particle settling velocity, defined as positive downwards, and ϵ_i are the sediment mixing coefficients.

2.1.1 Settling Velocities

The settling velocity of a single smooth spherical particle in a stagnant unbounded fluid for small particle Reynolds numbers, $Re = \rho D v_0 / \eta$, is given by Stokes' Law

$$v_0 = \frac{1}{18} \frac{(\sigma - \rho)gD^2}{\eta} \quad (2.2)$$

where v_0 is the settling velocity of the single particle, σ is the particle density, ρ is the fluid density, g is the acceleration due to gravity, D is the particle diameter and η the dynamic viscosity of the fluid.

Richardson and Zaki(1954) find by experiment that the relationship between settling velocity for an array of particles and a single particle is given by

$$v_p = v_0(1 - C)^n \quad (2.3)$$

where n is a positive exponent dependent on the particle Reynolds number. Maude and Whitmore(1958) find theoretically that $2.33 \leq n \leq 4.65$ and generate a curve of the relationship between n and the Reynolds number which agrees well with the experimental results of Richardson and Zaki, as shown by Allen(1985).

Similarly Hallemeier(1981) suggests a scheme, based on experimental results, for modifying the settling velocity for non-spherical particles. This scheme has little effect on settling velocities in low Reynolds number cases.

2.1.2 Fluid Velocity Components

To solve equation 2.1 a knowledge of the fluid velocities is required, these can be obtained from analytical or numerical solutions of the equations of motion of the fluid or approximations to them. These fluid velocities will be specific to the problem being considered.

2.1.3 Sediment Diffusivities

The sediment mixing coefficients are related to the diffusivities for the momentum of the fluid. There are many ways of modelling turbulence and calculating diffusion coefficients, based on results from both theory, eg. Prandtl's mixing length theory [Graf 1971] or experiments, eg. Rajaratnam and Ahmadi(1981). Once again these results will be dependent on the problem being considered.

2.1.4 Boundary Conditions

In order to solve equation 2.1 in a finite region boundary conditions must be specified. These are based on the physical boundary conditions that no sediment may be transferred across the water surface. A known concentration profile may be specified at a boundary of the region. At a solid boundary, eg. the bed of the region, the rate of transfer is defined by the probabilities of a particle reaching the boundary being deposited and of a particle on the boundary being eroded. At a vertical boundary it can usually be assumed that there is no transfer of sediment. At the bed of the region the probabilities can be estimated in many ways James(1987) ignores the possibility of erosion and defines the probability of deposition by the complement of the erosion probability defined by Einstein(1950).

2.2 The Model

2.2.1 The Region of Investigation

The saltmarsh is assumed to be horizontal and alongside a body of tidal water, i.e. the sea or an estuary, straight enough and long enough that only variations in the

transverse direction need to be considered. Figure 2.1 shows the two-dimensional approximation to the region of investigation.

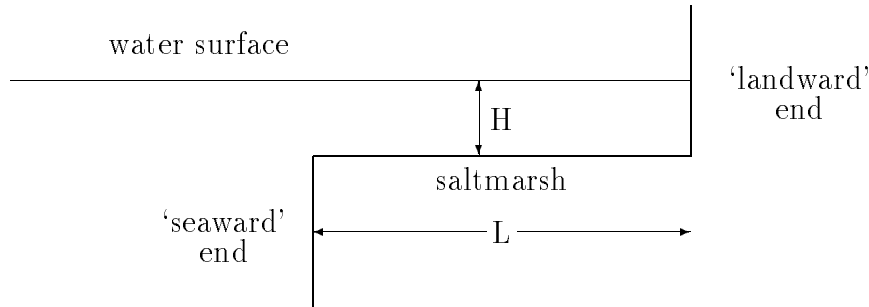


Figure 2.1: The region of investigation

The marsh is bounded at one end, hereafter known as the ‘seaward end’, by the body of water, which is the source of the sediment and at the ‘landward’ end, a distance L away, by a vertical barrier. It is assumed that the water surface remains horizontal across the marsh, this implies that the tide, when it reaches the level of the marsh, instantaneously covers it. The depth of the water, H , over the marsh is therefore uniform and a function of time only.

The simplifications made here allow the removal of one of the dimensions from equation 2.1. The mass balance equation for suspended sediment in two dimensions is

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x}(uC) + \frac{\partial}{\partial z}(wc) = \frac{\partial}{\partial x} \left(\epsilon_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial z} \left(\epsilon_z \frac{\partial C}{\partial z} \right) + \frac{\partial}{\partial z}(v_p C) \quad (2.4)$$

where x is the transverse direction and u and w are the velocities in the x and z directions, with the origin on the marsh at the seaward end.

2.2.2 Approximations to the Two-Dimensional Mass Balance Equation

In order to simplify the equations to be solved, further assumptions are made about the flow over the marsh; Firstly, it is assumed that there is no vertical motion of the fluid, i.e. $w = 0$. Secondly, that the flow is non-turbulent and hence, that the sediment mixing coefficients are zero. Finally, that there is no variation in the vertical direction of the horizontal velocity, u . These assumptions result in a one-dimensional nature of the flow over the marsh and the mass balance equation becomes

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x}(uC) - \frac{\partial}{\partial z}(v_p C) = 0. \quad (2.5)$$

It is further assumed that the settling velocity of each species of sediment is constant and that the water entering the region at the seaward end of the marsh has a uniform, constant in time, concentration of sediment, C_0 , distributed throughout its depth. The boundary condition at the bed of the region is approximated by assuming that there is no erosion from the bed and that all the suspended sediment reaching the bed is deposited on it.

2.2.3 Depth Integration of the One-Dimensional Mass Balance Equation

Equation 2.5, the one-dimensional mass balance equation can be integrated over the depth of the flow, $H(t)$, giving

$$\int_0^{H(t)} \frac{\partial C}{\partial t} dz + \int_0^{H(t)} \frac{\partial}{\partial x}(uC) dz - \int_0^{H(t)} \frac{\partial}{\partial z}(v_p C) dz = 0. \quad (2.6)$$

Using Leibniz's rule for differentiation under an integral sign and omitting

zero terms gives

$$\frac{\partial}{\partial t} \int_0^{H(t)} C dz - \frac{dH}{dt} C(H(t)) + \frac{\partial}{\partial x} \int_0^{H(t)} (uC) dz - \int_0^{H(t)} \frac{\partial}{\partial z} (v_p C) dz = 0. \quad (2.7)$$

Assuming that at (x, t) the sediment is distributed through a column of height y with a uniform concentration, i.e.

$$C = \begin{cases} C_0 & \text{if } 0 \leq z \leq y \\ 0 & \text{if } y < z \leq H \end{cases} \quad (2.8)$$

implies

$$\frac{\partial C}{\partial z} = -C_0 \delta(y) \quad (2.9)$$

where $\delta(y)$ is the Dirac delta distribution. The form of the concentration profile given by equation 2.8 also implies that at every point except $x = 0$ the concentration at the water surface is zero, so that for $x > 0$, $\frac{dH}{dt} C(H(t)) = 0$, which means that equation 2.7 becomes

$$\frac{\partial y}{\partial t} + \frac{\partial}{\partial x} (uy) + v_p = 0. \quad (2.10)$$

Equation 2.10 is the continuity equation for the suspended sediment that will be used in this project to provide a numerical simulation of the deposition of sediment on a saltmarsh. Only values of $y \geq 0$ have any physical significance in the solution of this problem.

2.2.4 The Velocity Profile over the Saltmarsh

In order to solve equation 2.10 the functional form of the velocity over the saltmarsh is required. This can be obtained by solving an equation of continuity for the fluid derived from a consideration of mass balance in the fluid.

Consider a region of the saltmarsh between x and $x + \Delta x$. Above this region there is a column of fluid, $V(t) = H(t)\Delta x$. The change in volume, ΔV , over the region in a time Δt is given by the net flow into the region in the same time, which is

$$\Delta V = \Delta H \Delta x = -H \Delta t (u(x + \Delta x) - u(x)) \quad (2.11)$$

$$\Rightarrow \frac{\Delta H}{\Delta t} = -H \frac{u(x + \Delta x) - u(x)}{\Delta x}. \quad (2.12)$$

Taking the limiting case of this as $\Delta x \rightarrow 0$ and $\Delta t \rightarrow 0$ gives the equation of continuity

$$\frac{\partial u}{\partial x} = -\frac{1}{H} \frac{dH}{dt}. \quad (2.13)$$

Applying the relevant boundary condition that the velocity is zero at the landward end of the saltmarsh, $x = L$, allows this equation to be solved to determine the velocity profile

$$u = \frac{L - x}{H} \frac{dH}{dt}. \quad (2.14)$$

2.2.5 The Water Height over the Saltmarsh

The depth of the water above the marsh is given by the height of the saltmarsh above mean sea-level subtracted from the height above mean sea-level of the water in the main channel, which is governed by the tidal regime. Assuming a sinusoidal tide the depth of water above the marsh is given by

$$H(t) = A \sin(\omega(t + t_0)) - b \quad (2.15)$$

where $b = A \sin(\omega t_0)$, is the height of the marsh above mean sea-level, A is the tidal amplitude, half the tidal range, and $T = \frac{2\pi}{\omega}$ is the period of the tide. The

time, t , is defined such that $t = 0$ when the water instantaneously covers the marsh. The marsh will be covered by the tide whilst

$$0 < t < \frac{\pi}{\omega} - 2t_0. \quad (2.16)$$

2.2.6 The Model Equations

In summary, the following equation is used to model the transport of sediment over the salt marsh,

$$\frac{\partial y}{\partial t} + \frac{\partial}{\partial x}(uy) + v_p = 0 \quad (2.17)$$

with the boundary condition that

$$y(0, t) = H(t) \quad \forall 0 \leq t \leq \frac{\pi}{2\omega} - t_0 \quad (2.18)$$

where the velocity and depth of water over the marsh are given by

$$u(x, t) = \frac{L - x}{H} \frac{dH}{dt} \quad (2.19)$$

$$H(t) = A \sin(\omega(t + t_0)) - b. \quad (2.20)$$

2.3 An Analytic Solution

The model equations given in Section 2.2.6 can be solved analytically for the case where $b = 0$, i.e. $H = A \sin(\omega t)$, by the method of characteristics described by Wood(1993).

From equations 2.19 and 2.20 we have that

$$u = (L - x)\omega \cot(\omega t) \quad (2.21)$$

and from equation 2.17 we have

$$u \frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = - \left(y \frac{\partial u}{\partial x} + v_p \right). \quad (2.22)$$

The characteristics of this equation are given by

$$\frac{dx}{dt} = u = \frac{L - x}{H} \frac{dH}{dt} \quad (2.23)$$

$$\Rightarrow \int_0^x \frac{1}{L - x} dx = \int_{H_0}^H \frac{1}{H} dH \quad (2.24)$$

where $H_0 = A \sin \alpha$, i.e. at $x = 0$, $\alpha = \omega t$. Hence the equation of the characteristic lines (see figure 2.2) is

$$x = L \left(1 - \frac{\sin \alpha}{\sin(\omega t)} \right). \quad (2.25)$$

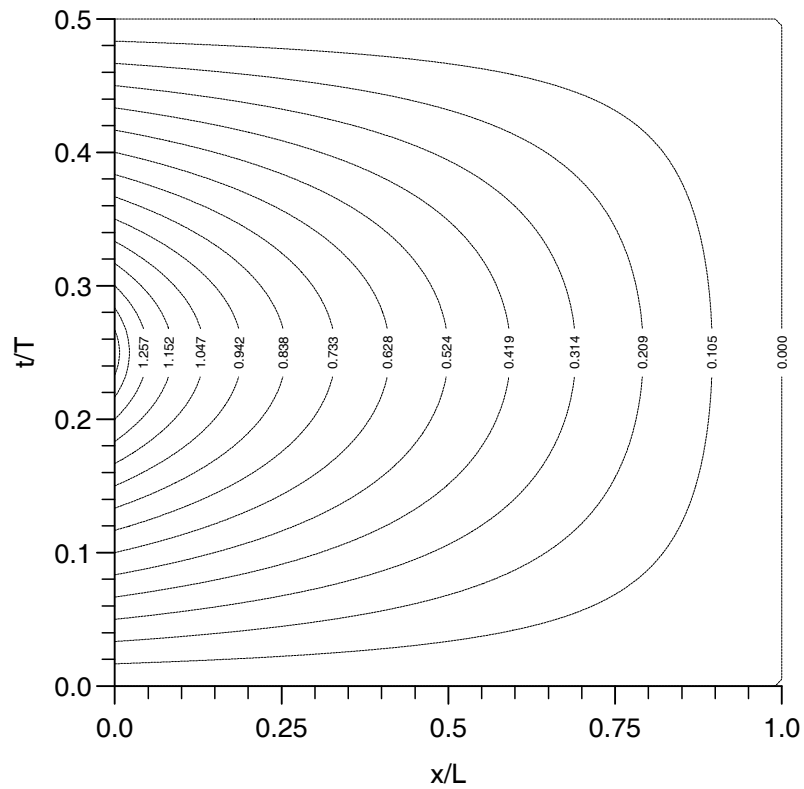


Figure 2.2: Characteristics of model equations of section 2.2.6 with $b = 0$. Annotation shows values of α

On the characteristic given by equation 2.25 we have that

$$\frac{dy}{dt} = - \left(y \frac{\partial u}{\partial x} + v_p \right) \quad (2.26)$$

which implies from equation 2.21 that

$$\frac{dy}{dt} - (\omega \cot \omega t)y = -v_p. \quad (2.27)$$

Multiplying equation 2.27 by a factor $\operatorname{cosec}(\omega t)$ gives

$$\frac{d}{dt} \left(\frac{y}{\sin(\omega t)} \right) = -\frac{v_p}{\sin(\omega t)}. \quad (2.28)$$

Given that, for $0 \leq t \leq \frac{\pi}{2}$ at $x = 0$, $\alpha = \omega t$ and $y = A \sin \alpha$, integration of equation 2.28 gives

$$y = A \sin(\omega t) \left[1 - \frac{v_p}{A\omega} \ln \left(\frac{\tan \frac{\omega t}{2}}{\tan \frac{\alpha}{2}} \right) \right] \quad (2.29)$$

where α is given by equation 2.25.

The rate of deposition, $R(x, t)$, of sediment on the saltmarsh is given by

$$R = \begin{cases} C_0 v_p & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.30)$$

so, in order to be able to determine the depth of sediment deposited on the marsh the time, $t_E(x)$, at which $y = 0$ is needed. The position, $x_E(t)$, at which $y = 0$ can be found from equation 2.29 by substituting in an expression for α from equation 2.25, giving

$$x_E(x) = L \left\{ 1 - \frac{\sin \left[2 \arctan \left\{ e^{-\frac{A\omega}{v_p}} \tan \left(\frac{\omega t}{2} \right) \right\} \right]}{\sin(\omega t)} \right\}. \quad (2.31)$$

Rearranging and using trigonometric identities gives

$$t_E(x) = \frac{2}{\omega} \arctan \left\{ \sqrt{\frac{[e^{\frac{A\omega}{v_p}} (1 - \frac{x}{L}) - 1]}{[1 - e^{-\frac{A\omega}{v_p}} (1 - \frac{x}{L})]}} \right\}. \quad (2.32)$$

It must be noted that for $x > L[1 - e^{-\frac{A\omega}{v_p}}]$ the expression does not hold as there is no time for which $y = 0$ at these x values.

From equation 2.30 the depth of sediment, S , deposited by one tidal sequence is given by

$$S(x) = \int_0^{\frac{\pi}{\omega} - 2t_0} R(x, t) dt \quad (2.33)$$

which gives

$$S(x) = C_0 v_p t_E(x). \quad (2.34)$$

The depth of sediment deposited is shown in figure 2.3 for two different values of v_p .

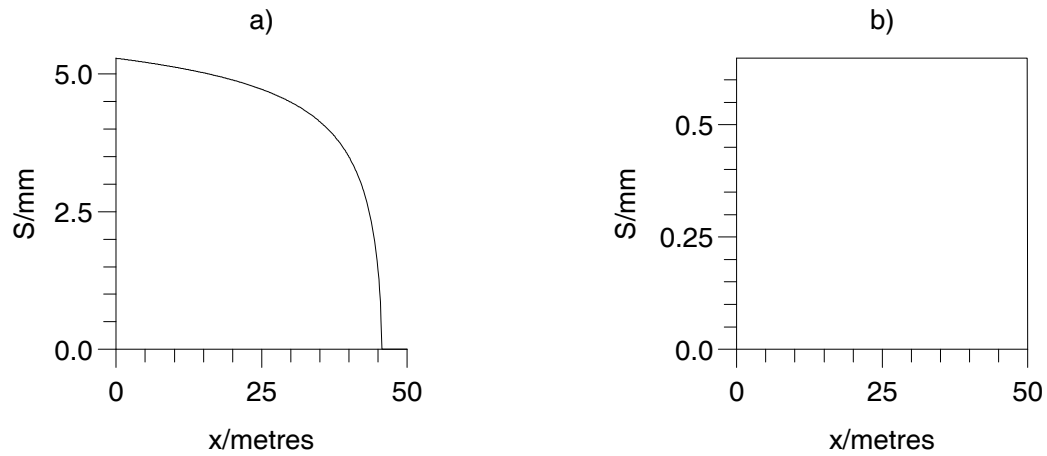


Figure 2.3: The depth of sediment deposited by one tide when $A = 5m$, $L = 50m$, $T = 12hrs$ and $C_0 = 1 \times 10^{-3}$ with a) $v_p = 3 \times 10^{-4} ms^{-1}$ and b) $v_p = 3 \times 10^{-5} ms^{-1}$.

Chapter 3

The Numerical Model

The tidal period over which equation 2.17 from section 2.2.6 is to be solved can be split into two distinct parts. The first is whilst the tide is rising and water, full of sediment from the channel, is coming into the region over the saltmarsh. The second part is after the tide has turned and the water, with some sediment remaining, is flowing out of the region. The equation could be solved approximately by the numerical method of characteristics which would involve the repeated numerical solution of the characteristic equation 2.23. This could be done by using either an explicit method, which would cause problems because of the need to satisfy the stability condition of the method or by using an unconditionally stable method which would involve the repeated solution of a non-linear equation which could be time consuming. For these reasons the numerical method of characteristics was not used to solve the equations in general. However, the method is helpful in the solution of the equations during the ebb tide, where it is used once at each time level to provide one height, y , of the column of sediment at the new time level. Apart from this the box scheme [Preissmann, 1961] is used to integrate the

equations numerically.

3.1 Discretisation of the Problem

The saltmarsh is divided into M equal sections of length $\Delta x = L/M$ with $x_j = j\Delta x$. The time discretisation is done by using $2N$ equal time steps, $\Delta t = (\frac{\pi}{2\omega} - t_0)/N$, with $t_n = n\Delta t$. The solution $y(x_j, t_n)$ is approximated by y_j^n and $[uy]_j^n = u(x_j, t_n)y_j^n$. The depth of sediment deposited, $S(x_j, t_n)$, on the marsh is approximated numerically by S_j^n .

3.2 Properties of the Box Scheme

When the box scheme is applied to equation 2.17 the time derivatives are approximated by a weighted average of the finite difference form at two spatial points

$$\frac{\partial y}{\partial t} \approx \frac{(1-\phi)}{\Delta t}(y_j^{n+1} - y_j^n) + \frac{\phi}{\Delta t}(y_{j+1}^{n+1} - y_{j+1}^n), \quad (3.1)$$

and the space derivatives are replaced by a weighted average of the finite difference forms at two time levels

$$\frac{\partial}{\partial x}(uy) \approx \frac{(1-\theta)}{\Delta x}([uy]_{j+1}^n - [uy]_j^n) + \frac{\theta}{\Delta x}([uy]_{j+1}^{n+1} - [uy]_j^{n+1}) \quad (3.2)$$

where θ and ϕ are user defined parameters between zero and one.

By substitution of Taylor series expansions into equations 3.1 and 3.2 it can be seen that the errors in these approximations are

$$\frac{\Delta t}{2} \frac{\partial^2 y}{\partial t^2} + \phi \Delta x \frac{\partial^2 y}{\partial x \partial t} + O(\Delta t^2) + O(\Delta x^2) \quad (3.3)$$

and

$$\frac{\Delta x}{2} \frac{\partial^2(uy)}{\partial x^2} + \theta \Delta t \frac{\partial^2(uy)}{\partial x \partial t} + O(\Delta t^2) + O(\Delta x^2) \quad (3.4)$$

respectively.

This gives a finite difference approximation to the continuity equation for the suspended sediment, equation 2.17 of the form

$$\begin{aligned} & \frac{(1-\phi)}{\Delta t} (y_j^{n+1} - y_j^n) + \frac{\phi}{\Delta t} (y_{j+1}^{n+1} - y_{j+1}^n) + \\ & \frac{(1-\theta)}{\Delta x} ([uy]_{j+1}^n - [uy]_j^n) + \frac{\theta}{\Delta x} ([uy]_{j+1}^{n+1} - [uy]_j^{n+1}) + v_p = 0 \end{aligned} \quad (3.5)$$

with a truncation error, τ , given by

$$\tau = \frac{\Delta t}{2} \left(\frac{\partial^2 y}{\partial t^2} + 2\theta \frac{\partial^2(uy)}{\partial x \partial t} \right) + \frac{\Delta x}{2} \left(\frac{\partial^2(uy)}{\partial x^2} + 2\phi \frac{\partial^2 y}{\partial x \partial t} \right) + O(\Delta t^2) + O(\Delta x^2) \quad (3.6)$$

Equation 2.17 can be partially differentiated with respect to t giving

$$\frac{\partial^2 y}{\partial t^2} = - \frac{\partial^2(uy)}{\partial x \partial t} \quad (3.7)$$

and with respect to x giving

$$\frac{\partial^2(uy)}{\partial x^2} = - \frac{\partial^2 y}{\partial x \partial t}. \quad (3.8)$$

Substituting these into equation 3.6 gives

$$\tau = \frac{1}{2}(1-2\theta)\Delta t \frac{\partial^2 y}{\partial t^2} + \frac{1}{2}(1-2\phi)\Delta x \frac{\partial^2(uy)}{\partial x^2} + O(\Delta t^2) + O(\Delta x^2). \quad (3.9)$$

This equation shows that the box scheme is second-order accurate with $\phi = \theta = 0.5$ and first order accurate otherwise.

The stability of the box scheme can be examined using the von Neumann technique described by Wood(1993) on the finite difference scheme for the homogeneous, constant coefficient equation obtained by assuming that the velocity, u ,

is constant and neglecting the settling velocity, v_p , in equation 2.17

$$\frac{\partial y}{\partial t} + u \frac{\partial y}{\partial x} = 0. \quad (3.10)$$

Substituting the Fourier mode

$$y_j^n = \xi_k^n e^{ijk\Delta x}, \quad (3.11)$$

where k is the mode number and $i = \sqrt{-1}$, into the finite difference scheme and cancelling the term $\xi_k^n e^{ijk\Delta x}$ gives

$$(\xi - 1)[1 - \phi + \phi e^{ik\Delta x}] + \lambda[e^{ik\Delta x} - 1](1 - \theta + \xi\theta) = 0 \quad (3.12)$$

where $\lambda = u \frac{\Delta t}{\Delta x}$.

For stability of the scheme we require that $|\xi| \leq 1$, which with manipulation of equation 3.12 gives a stability condition for the scheme

$$(1 - 2\theta)\lambda + (1 - 2\phi) \leq 0 \quad (3.13)$$

This condition shows that the box scheme is unconditionally stable for $\phi, \theta \geq 0.5$.

For $\theta < 0.5$ and $\phi > 0.5$ we need that

$$u \frac{\Delta t}{\Delta x} \leq \frac{(2\phi - 1)}{(1 - 2\theta)}. \quad (3.14)$$

3.3 The Flood Tide

During the period of time in which the tide is flooding the marsh the height of the column of sediment at the seaward boundary is known from equation 2.18. This means that if the box scheme is used, stepping along the region from the seaward end, then there is only one unknown in each equation. Since the differential

equation is linear in y , i.e. the velocity, u , is independent of y , the equation produced by discretising using the box scheme is linear and can be solved easily.

Equation 2.17 is discretised using the box scheme with the parameters set at $\theta = \phi = 0.5$. Rearranging the finite difference form, equation 3.5 to solve for y_{j+1}^{n+1} explicitly gives

$$y_{j+1}^{n+1} = \frac{[y_j^n - y_j^{n+1} + y_{j+1}^n] \Delta x + [[uy]_j^n - [uy]_{j+1}^n + [uy]_j^{n+1}] \Delta t - 2v_p \Delta t \Delta x}{[\Delta x + u(x_{j+1}, t_{n+1}) \Delta t]} \quad (3.15)$$

This equation is then solved for $0 \leq j \leq M - 1$ to step along the region from the seaward end for each time step $0 \leq n \leq N - 1$, until the turn of the tide.

A numerical approximations x_E^n to $x_E(t_n)$ is obtained by using linear interpolation between the two points across which the sign of y_j^n changes. At each time level there will be a j_E such that $y_{j_E}^n > 0$ and $y_{j_E+1}^n < 0$ and x_E^n is defined by

$$x_E^n = j_E \Delta x + \frac{y_{j_E}^n \Delta x}{y_{j_E}^n - y_{j_E+1}^n} \quad (3.16)$$

The approximation to the depth of sediment deposited, S_j^n , is updated at each time level using the following algorithm

$$S_j^{n+1} = \begin{cases} S_j^n + C v_p \Delta t & \text{if } x_j < x_E^{n+1} \\ S_j^n + \frac{(x_E^n - x_j) C v_p \Delta t}{x_E^n - x_E^{n+1}} & \text{if } x_E^{n+1} \leq x_j < x_E^n \\ S_j^n & \text{otherwise} \end{cases} \quad (3.17)$$

3.3.1 The Initial Time Step

Initial values of y are given, at $t = 0$, $y = 0$. Initial values of uy are not so obvious; initially, from equation 2.14, u is infinite and the product, uy , is not so simply evaluated.

Two methods for dealing with the product uy were tried. Firstly $[uy]_j^0$ was set to zero for all j . The second method was based on the assumption that for very

small t the horizontal velocities are large compared to the settling velocities of the sediment. This means that for very small t the column of sediment in the water is approximately the same depth, H , as the water. Then, using the expression for the velocity from equation 2.19 the product uy , for small t , is given by

$$uy = \frac{L - x}{H} \frac{dH}{dt} \cdot H = (L - x) \frac{dH}{dt} \quad (3.18)$$

Both these expressions for uy were tried for the initial time step in the numerical scheme applied to the problem described in section 2.3 and the results compared with the analytic solution given there.

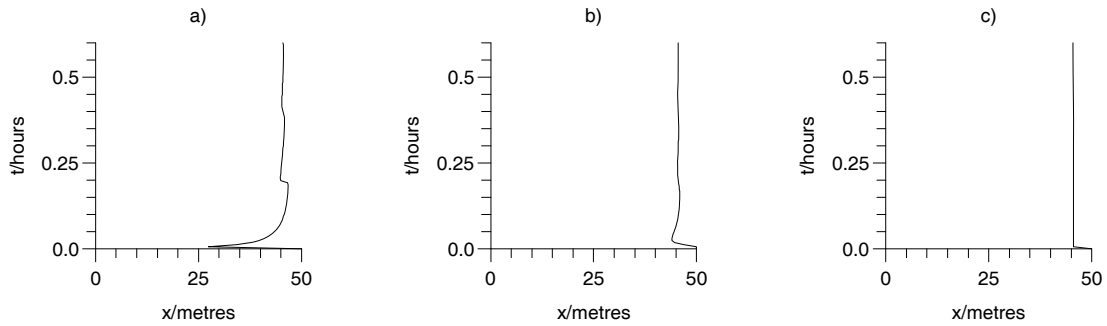


Figure 3.1: The position of x_E for $A = 5m$, $v_p = 3 \times 10^{-4}$ and $T = 12hrs$ obtained from; a) the numerical method with initial conditions $uy(x, 0) = 0$, b) the numerical method with initial conditions $uy(x, 0) = (L - x) \frac{dH}{dt}$ and c) the analytic solution.

It can be seen by from figure 3.1 that neither of these initial conditions gave results which agreed well with the analytic solution and so both methods were

ruled out.

However, the product uy only appears in the space-derivative in equation 2.17 and so if the box scheme is used with $\theta = 1.0$ instead of $\theta = 0.5$ then an expression for $uy(x, 0)$ is not needed. In order to preserve the second order accuracy of the method for the other time steps, $\theta = 1.0$ is used only for the first time step and for subsequent time steps the original $\theta = 0.5$ is used. This method can be seen to provide sufficiently accurate results by comparing the solution obtained with the analytic solution.

Using this method the box scheme for the first time step becomes

$$y_{j+1}^{n+1} = \frac{[y_j^n - y_j^{n+1} + y_j^{n+1}]\Delta x + 2[uy]_j^{n+1}\Delta t - 2v_p\Delta t\Delta x}{\Delta x + 2u(x_{j+1}, t_{n+1})\Delta t} \quad (3.19)$$

3.3.2 The Landward Boundary

When the model is run with the smaller particle sizes in the range for which the scheme is required here, the approximation y_M^n to the height of the column of sediment at the landward end of the region is positive. The analytic solution gives $y \rightarrow -\infty$ as $x \rightarrow L$ and this implies that at the very end of the region there can be no sediment suspended above the marsh. For this reason, if the approximation $y_M^n > 0$ then the numerical model sets its value to zero. If the value was set to a large negative number to match the analytic solution more closely, the linear interpolation used to find the approximation x_E^n to x_E would be biased towards the point X_{M-1} . In the cases where this problem at the landward end occurs, the solution to the equation has a very steep gradient near the end of the region and so the linear interpolation between the two points is not a valid method of obtaining the position of x_E^n . Since the deposition on the marsh is defined by

the position of x_E^n it is more important to have this value accurately evaluated than to have the height of the column at the end of the region accurately defined because only values of $y > 0$ have any physical significance in this problem and the value of y_M^n affects the solution at subsequent time levels only at x_M and therefore does not affect the solution in the interior of the region.

3.4 The Ebb Tide

During the ebb tide the box scheme cannot be used quite as simply as during the flood tide. Since there is no boundary data no values are known at the new time level and it is therefore necessary to evaluate at least one of the heights, y_j^{n+1} , at the new time level by some other method. If a position towards the landward end of the region is chosen then this new value can be used like a boundary condition and the box scheme can be used to step towards the seaward end from this point in the same way as it is used during the flood tide.

Several things need to be considered when calculating this ‘boundary value’, the most important of which is that in order to get a complete solution for the part of the marsh which still has sediment suspended above it, the first value at the new time level must be in the region where there is no sediment suspended above the marsh.

One possible method of obtaining the first value at the new time level is by using the implicit three point scheme obtained from the box scheme by setting the parameters to $\theta = 0$, $\phi = 1$, remembering that we are now stepping in the

negative x direction and that u is negative this gives a scheme,

$$y_{j-1}^{n+1} = \frac{y_{j-1}^n \Delta x + ([uy]_{j-1}^n - [uy]_j^n) \Delta t - v_p \Delta x \Delta t}{\Delta x} \quad (3.20)$$

There are two main problems with this method, firstly the explicit scheme given in equation 3.20 is only conditionally stable which will mean that $\Delta x, \Delta t$ will have to be varied to provide stability. Secondly, once a value has been obtained at the new time level a check will have to be made to ensure that the value is negative and if not a new $\Delta x, \Delta t$ chosen to get a negative value.

An alternative method is to trace along the characteristic from a point at the old time level, n , where the solution is non-positive to the new time level and evaluate a numerical solution at this new point. This method guarantees that the value at the new time level will be negative and does not create the same problems over choice of Δx and Δt . It does however require the characteristic equation 2.23 to be solved numerically, which may itself create problems with stability conditions.

The method used in this project is to trace the characteristic from the point where the numerical solution is zero, x_E^n to the next time level, $n+1$ by numerically solving the characteristic equation

$$\frac{dx}{dt} = \frac{L - x}{H} \frac{dH}{dt} \quad (3.21)$$

and to simultaneously update the values of y on this characteristic from the equation

$$\frac{dy}{dt} = -(y \frac{\partial u}{\partial x} + v_p) \quad (3.22)$$

by use of a numerical method. To avoid problems with stability the trapezium rule was chosen because it is unconditionally stable.

Equations 3.21 and 3.22 are integrated from t_n to t_{n+1} using P time steps $\Delta t_c = \frac{\Delta t}{P}$. The position of the characteristic through x_E^n is given by $x_c(t)$ and is approximated by x_c^p at a time $t_p = t_n + p\Delta t_c$. The trapezium rule then gives

$$x_c^{p+1} = x_c^p + \frac{1}{2}\Delta t_c [u(x_c^{p+1}, t_{p+1}) + u(x_c^p, t_p)] \quad (3.23)$$

This equation is solved iteratively by making an initial guess

$$x_c^{p+1,(0)} = x_c^p + \Delta t_c u(x_c^p, t_p) \quad (3.24)$$

and then using the iteration

$$x_c^{p+1,(k+1)} = x_c^p + \frac{1}{2}\Delta t_c [u(x_c^{p+1,(k)}, t_{p+1}) + u(x_c^p, t_p)] \quad (3.25)$$

until two consecutive approximations differ by less than a specified tolerance.

If the value of y on the characteristic is approximated by y_c^p then application of the trapezium rule to equation 3.22 gives

$$y_c^{p+1} = \frac{y_c^p - \left[v_p + \frac{1}{2}y_c^p u_x(x_c^p, t_p) \right] \Delta t_c}{\left[1 + \frac{1}{2}\Delta t_c u_x(x_c^{p+1}, t_{p+1}) \right]} \quad (3.26)$$

With x_c^P and y_c^P known it is possible to use the box scheme to integrate along the region towards the seaward end. The first step must be done with a steplength defined by $\Delta x' = x_c^P - x_{j_c}$ where j_c is such that $x_{j_c} \leq x_c^P < x_{j_c+1}$. This means that an approximation to $y(x_c^P, t_n)$ is needed also. The value of $y(x_c^P, t_n)$ is approximated by $y_{c'}$ using linear interpolation between the two calculated values either side of it. If $j_c(t_{n+1}) = j_E(t_n)$ then the interpolation is done between $x_{j_c}^n$ and x_E^n , otherwise $x_{j_c}^n$ and $x_{j_c+1}^n$ are used.

The box scheme applied to the first horizontal step at each time level becomes

$$y_{j_c}^{n+1} = \frac{[y_{j_c}^n + y_{c'} - y_c^P]\Delta x' + [[uy]_{j_c}^n - u(x_c^P, t_n)y_{c'} - u(x_c^P, t_{n+1})y_c^P]\Delta t - 2v_p\Delta t\Delta x'}{[\Delta x' - u(x_{j_c}, t_{n+1})\Delta t]} \quad (3.27)$$

For subsequent horizontal time steps the box scheme is given by

$$y_{j-1}^{n+1} = \frac{[y_{j-1}^n + y_j^n - y_j^{n+1}]\Delta x + [[uy]_{j-1}^n - [uy]_j^n - [uy]_j^{n+1}]\Delta t - 2v_p\Delta t\Delta x}{(\Delta x - u(x_{j-1}, t_{n+1})\Delta t)}. \quad (3.28)$$

If $y(0, t_{n+1}) > 0$ then the position of x_E^{n+1} and the depth of deposited sediment are evaluated in the same way as for the flood tide.

3.4.1 The Seaward Boundary

If the characteristic being traced goes out of the region, i.e. $x_c \leq 0$ then the integration of the characteristic equation stops, the time at which $x_c = 0$ and the value of y_c are approximated using linear interpolation between t_p and t_{p+1} . In this case or if $y_0^{n+1} < 0$ then the method evaluates the time, t_s , at which $y(0, t) = 0$ by using linear interpolation along the time axis at $x = 0$, updates the deposited sediment for the shortened time step, $t_s - t_n$ and stops.

3.4.2 The Final Time Step

If the numerical solution continues until $t = \frac{\pi}{\omega} - 2t_0$ without giving $y_0^{n+1} < 0$ then the method forces $y_0^{2N} = 0$ and interpolates between x_E^{2N-1} and zero to evaluate the deposition for the final time step.

Chapter 4

Numerical Results and Discussion

4.1 Testing

The numerical method was tested by using it to solve the problem described in section 2.3 and comparing the results with the analytic solution given there.

The testing was done with the parameters of the problem set as; $A = 5m$, $L = 50m$, $T = 12hrs$ and $v_p = 3 \times 10^{-4}ms^{-1}$ and $v_p = 3 \times 10^{-5}ms^{-1}$, corresponding to particle diameters of approximately $20\mu m$ and $5\mu m$. These two values were chosen because they give solutions with different forms, (see figure 2.3) both of which the method must be able to reproduce.

4.1.1 A Visual Comparison

Figure 4.1 shows the numerical solution for the depth of sediment deposited on the marsh, this compares visually well with the analytic solution, figure 2.3.

The numerical solution does smooth the corner, at $x = L$, for the case with $v_p = 3 \times 10^{-5} m s^{-1}$, which corresponds to the numerical solution underestimating x_E . This is because the linear interpolation used to find x_E underestimates the steepness of the gradient in the solution for y at the very end of the region.

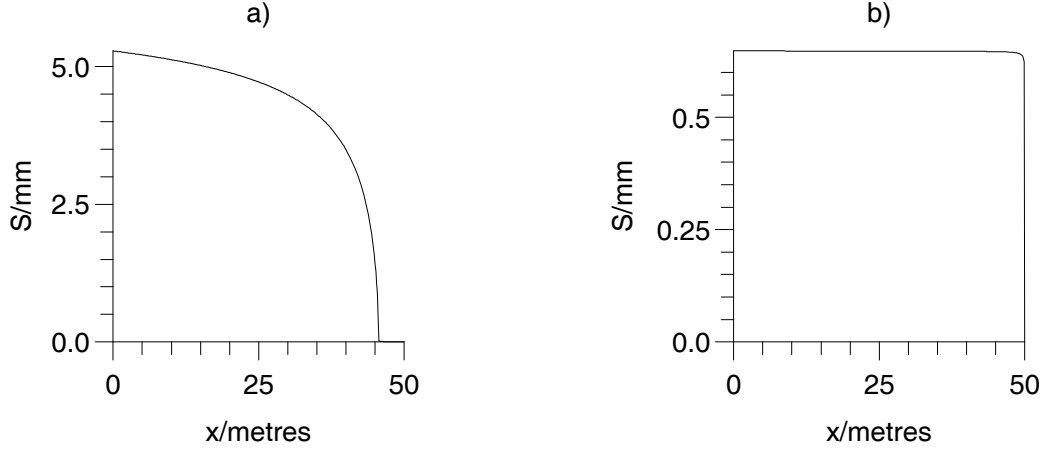


Figure 4.1: The depth of sediment deposited by one tide as evaluated by the numerical model when $A = 5m$, $L = 50m$, $T = 12hrs$ and $C_0 = 1 \times 10^{-3}$ with a) $v_p = 3 \times 10^{-4} m s^{-1}$ and b) $v_p = 3 \times 10^{-5} m s^{-1}$, for $M = N = 400$.

Because of the high curvature of y , on the integration for the ebb tide small oscillations which are not in the analytic solution are generated by the scheme, see figure 4.2.

This phenomena is not uncommon when second order schemes are used to solve hyperbolic problems in which there is a steep gradient and since the oscillations remain small and the depth of sediment deposited is dependent on the position of x_E which is not affected by these oscillations no action needs to be

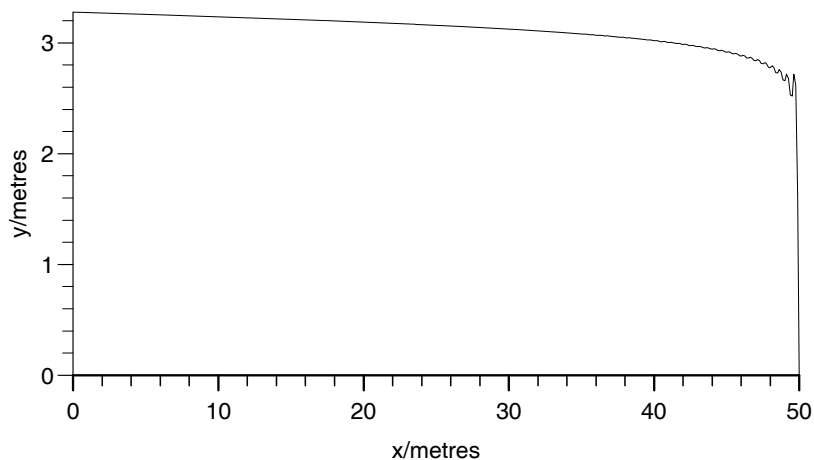


Figure 4.2: The numerical solution for y at $t = 4.5hrs$ when $A = 5m$, $L = 50m$, $T = 12hrs$ and $v_p = 3 \times 10^{-5}ms^{-1}$ with $M = N = 400$.

taken to counter this.

4.1.2 Error Measurements

Measurements of the error between the numerical solution of the problem and the analytic solution of the problem were made. An averaged absolute error in the depth of deposited sediment, E_s , is defined by

$$E_s = \frac{1}{M+1} \sqrt{\sum_{j=0}^M (S_j^{2N} - S(x_j))^2} \quad (4.1)$$

The error in the solution was evaluated for varying time and space discretisations, i.e. M, N , for the parameters given above and the results are shown in table 4.1.

The results in table 4.1, shown graphically in figure 4.3, bring out some important points about the error in the depth of sediment deposited; Firstly, it

N	M	Error, E_s/mm	
		$v_p = 3 \times 10^{-4}ms^{-1}$	$v_p = 3 \times 10^{-5}ms^{-1}$
50	50	1.9657×10^{-3}	7.4086×10^{-4}
50	100	1.1740×10^{-3}	4.6134×10^{-4}
50	200	8.1780×10^{-4}	2.9941×10^{-4}
50	400	5.9296×10^{-4}	2.0041×10^{-4}
50	800	4.4190×10^{-4}	1.3744×10^{-4}
100	50	2.0946×10^{-3}	6.4394×10^{-4}
100	100	7.2483×10^{-4}	3.5656×10^{-4}
100	200	4.4250×10^{-4}	2.0693×10^{-4}
100	400	3.0412×10^{-4}	1.2698×10^{-4}
100	800	2.1305×10^{-4}	7.9361×10^{-5}
200	50	2.3575×10^{-3}	6.4530×10^{-4}
200	100	7.0094×10^{-4}	3.4750×10^{-4}
200	200	2.6464×10^{-4}	1.8575×10^{-4}
200	400	1.5609×10^{-4}	1.0268×10^{-4}
200	800	1.0442×10^{-4}	5.8160×10^{-5}
400	50	2.3432×10^{-3}	6.7564×10^{-4}
400	100	7.2911×10^{-4}	3.5625×10^{-4}
400	200	2.6778×10^{-4}	1.8822×10^{-4}
400	400	9.2843×10^{-5}	9.9209×10^{-5}
400	800	5.3609×10^{-5}	5.2819×10^{-5}
800	50	2.1998×10^{-3}	6.8404×10^{-4}
800	100	7.3507×10^{-4}	3.6360×10^{-4}
800	200	2.3386×10^{-4}	1.9122×10^{-4}
800	400	7.6897×10^{-5}	9.9516×10^{-5}
800	800	3.0227×10^{-5}	5.2401×10^{-5}

Table 4.1: Error in the numerical solution of the deposited sediment with $A = 5m$, $T = 12hrs$ and $C = C_0$ for $v_p = 3 \times 10^{-4}ms^{-1}$ and $v_p = 3 \times 10^{-5}ms^{-1}$ for varying values of N and M .

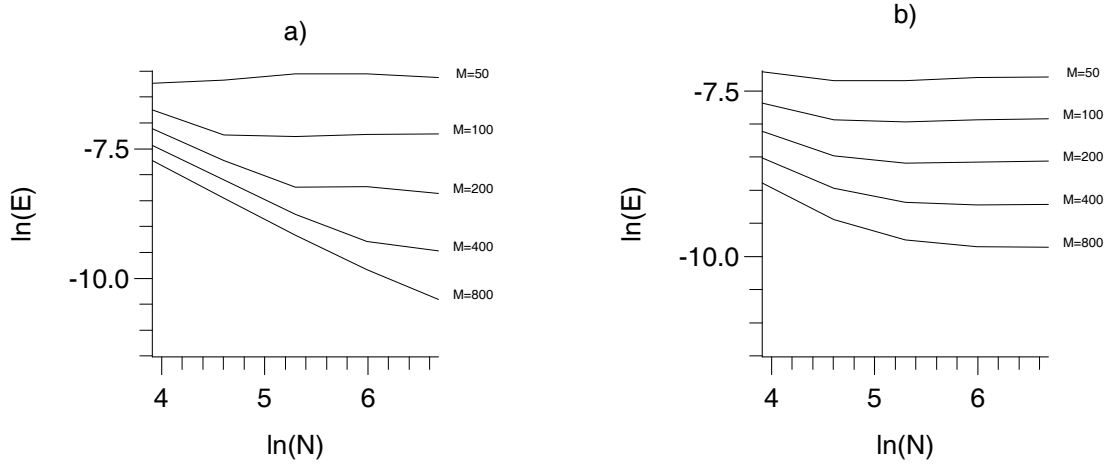


Figure 4.3: The variation of the error, E_s , with N for fixed values of M , with a) $v_p = 3 \times 10^{-4} ms^{-1}$ and b) $v_p = 3 \times 10^{-5} ms^{-1}$.

can be seen that for fixed M the error can not be reduced greatly, if at all, by increasing the value of N beyond that of M . Secondly, for a fixed value of N the error improves if M is increased, even beyond the value of N , although this effect is reduced for $M \gg N$. These two properties show that the error can be improved by reducing the time step only if the space step is already small, but that the error can be improved by reducing the space step, even for large time steps. From this it can be concluded that the error in the depth of deposited sediment is dominated by the error due to the space discretisation.

It is possible to get an estimate for the order of accuracy for the scheme by calculating the gradient of the graphs of $\ln(E_s)$ against $\ln(N)$ and $\ln(M)$. To estimate the order of accuracy in time the data for $M = 800$ was used as this will have the smallest contribution to the total error from the error due to the space

discretisation. Similarly to estimate the order of accuracy in space the data for $N = 800$ was used. The estimate was made by evaluating the gradient of the line of best fit through the data points using the method of least squares.

For $N = 800$ the gradient, r_M , of the line of best fit for $\ln(E_s)$ against $\ln(M)$ is $r_M = -1.5628$ for $v_p = 3 \times 10^{-4} \text{ms}^{-1}$ and $r_M = -0.9282$ for $v_p = 3 \times 10^{-5} \text{ms}^{-1}$.

For $M = 800$ the gradient, r_N , of the line of best fit for $\ln(E_s)$ against $\ln(N)$ is $r_N = -0.9730$ for $v_p = 3 \times 10^{-4} \text{ms}^{-1}$, the data for $v_p = 3 \times 10^{-5} \text{ms}^{-1}$ is not suitable for a straight line approximation and so was not used.

These results show that the scheme is at least first order in time and space, i.e.

$$E_s \approx O\left(\frac{1}{N}\right) + O\left(\frac{1}{M}\right) \quad (4.2)$$

$$= O(\Delta t) + O(\Delta x) \quad (4.3)$$

The scheme used is first order because although the second order box scheme is used to solve for y , linear interpolation which is only first order is used to calculate the approximations to x_E and hence the scheme becomes first order overall.

4.2 Experiments

The model was used to examine how the depth of sediment deposited on the marsh varies with the parameters A, b, v_p, L for a fixed tidal period $T = 12 \text{hrs}$.

4.2.1 A Non-dimensional Form

If A, b, v_p are held constant and L is varied the solution as a function of the variable x/L remains unchanged. This can be seen in figure 4.4 which shows the solution for three different values of L with the other parameters unchanged.

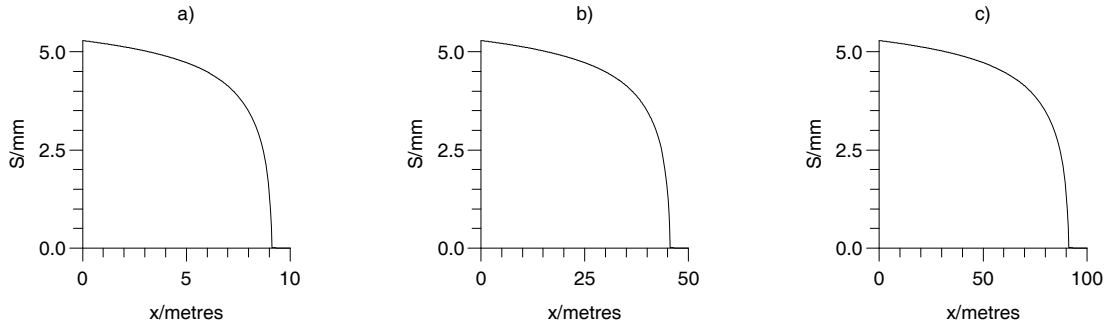


Figure 4.4: The depth of sediment deposited for $A = 5m, v_p = 3 \times 10^{-4}ms^{-1}$ and $T = 12hrs$ with a) $L = 10m$, b) $L = 50m$ and c) $L = 100m$.

Similarly, with $b = 0$ if A and v_p are varied such that the ratio A/v_p remains constant the solution changes only by the ratio of the two values of A . Figure 4.5 shows the depth of sediment deposited for three different values of A with the ratio A/v_p fixed.

These two results can be predicted from the model equations of section 2.2.6 by non-dimensionalising them by the following making the following changes of variable.

$$x' = \frac{x}{L} \quad (4.4)$$

$$y' = \frac{y}{A} \quad (4.5)$$

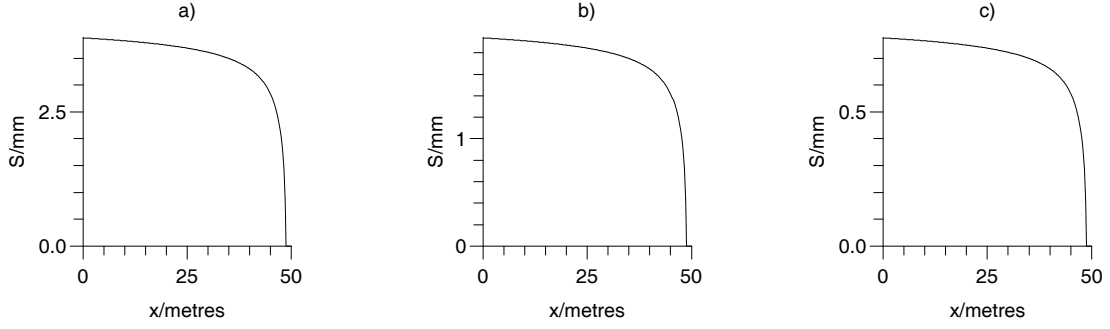


Figure 4.5: The depth of sediment deposited for $L = 50m$, $T = 12hrs$ with a) $A = 4m$ and $v_p = 4 \times 10^{-4}ms^{-1}$, b) $A = 2m$ and $v_p = 2 \times 10^{-4}ms^{-1}$ and c) $A = 1m$ and $v_p = 1 \times 10^{-4}ms^{-1}$.

$$t' = \omega t \quad (4.6)$$

$$H' = \frac{H}{A} \quad (4.7)$$

$$b' = \frac{b}{A} \quad (4.8)$$

$$t'_0 = \omega t_0 \quad (4.9)$$

$$S' = \frac{S}{A} \quad (4.10)$$

$$v'_p = \frac{v_p}{A\omega} \quad (4.11)$$

$$u' = \frac{u}{L\omega} \quad (4.12)$$

Substituting these into the model equations leads to a non-dimensional problem defined by the equation

$$\frac{\partial y'}{\partial t'} + \frac{\partial}{\partial x'}(u'y') + v'_p = 0 \quad (4.13)$$

with the boundary condition

$$y'(0, t') = H'(t') \quad \forall 0 \leq t' \leq \frac{\pi}{2} - t'_0 \quad (4.14)$$

where

$$u'(x', t') = \frac{1 - x' dH'}{H' dt'} \quad (4.15)$$

$$H'(t') = \sin(t' + t'_0) - b' \quad (4.16)$$

From this non-dimensional form of the equation it can be seen that the solution is dependent on two non-dimensional parameters, b/A and $v_p/A\omega$.

4.2.2 Distribution of Deposited Sediment

By holding A, b, L and T constant and varying v_p shows how the depth of sediment deposited on the marsh depends on the settling velocity. From figure 4.6 it can be seen that the sediment with small settling velocities is uniformly distributed over the saltmarsh and that as the settling velocity increases a gradient in the depth of sediment deposited develops across the marsh and the deposited sediment does not extend across the whole width of the marsh.

Figure 4.7 shows that increasing the ratio b/A reduces the depth of sediment deposited on the marsh for a given concentration and also causes sediment of a given settling velocity to be less evenly distributed over the marsh.

4.2.3 An Array of Particles

Sediment suspended in tidal water is not made up of particles of a unique size and settling velocity but particles of many different sizes. This model can be used to investigate this case because it has been assumed that each particle acts

independently and therefore the solution for each particle size is independent of the others.

The model can be used to solve the equations for a variety of particle sizes of varying concentrations which represent the sediment suspended in the water and the sum of the depth of sediment deposited for each size will give the total depth of sediment on the marsh. From the solution for each particle size and the total depth the proportion of each size particle in the deposited sediment at a point on the marsh can also be evaluated.

From the solutions obtained for individual particles it can be seen that distribution of sediment for a mixed sediment supply would be characterised by a greater depth of sediment deposited at the seaward end than the landward end of the marsh and that at the seaward end the sediment will have a higher proportion of the larger particles than at the landward end. These characteristics are in agreement with observations from measurements of the deposited sediment on saltmarshes in the Severn Estuary [Allen 1992] and the North Norfolk Coast [French and Spencer 1993].

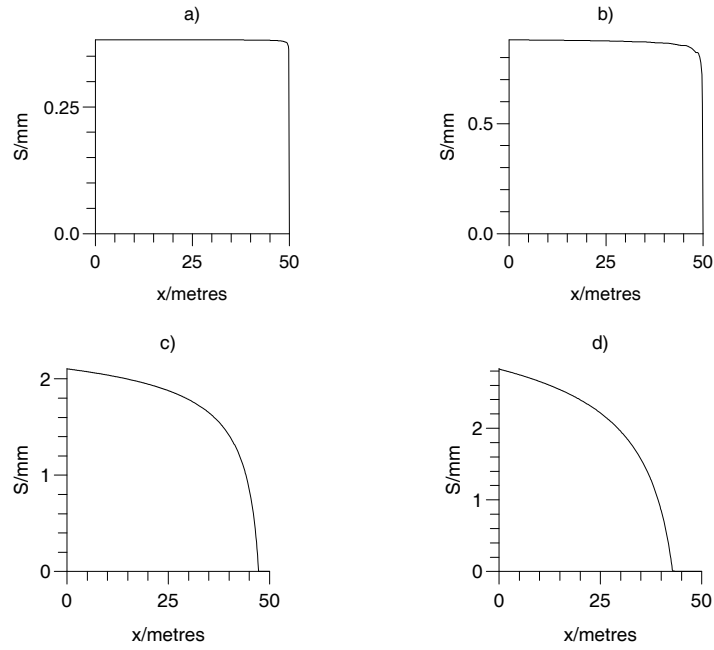


Figure 4.6: The depth of sediment deposited on the marsh for $A = 5m$, $b = 3m$, $L = 50m$, $T = 12hrs$ and $C_0 = 1 \times 10^{-3}$ with a) $v_p = 3 \times 10^{-5}ms^{-1}$, b) $v_p = 7 \times 10^{-5}ms^{-1}$, c) $v_p = 2 \times 10^{-4}ms^{-1}$ and d) $v_p = 3 \times 10^{-4}ms^{-1}$.

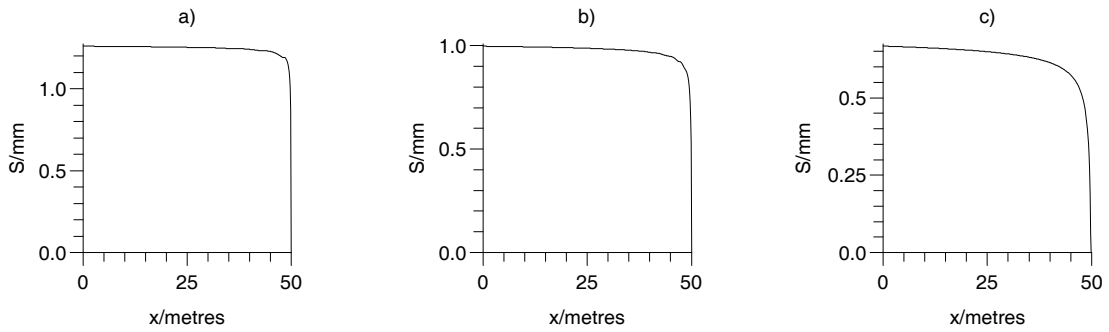


Figure 4.7: Depth of sediment deposited for $A = 5m$, $L = 50m$, $T = 12hrs$, $C_0 = 1 \times 10^{-3}$ and $v_p = 8 \times 10^{-5}ms^{-1}$ with a) $b = 2m$, b) $b = 3m$ and c) $b = 4m$.

Chapter 5

Conclusions

A simple one-dimensional mathematical model of sediment transport over a salt-marsh was developed from a generalised three-dimensional mass balance equation for suspended sediment by making a series of simplifying approximations regarding the flow of the water over the marsh, the suspension of sediment in the water and the topography of the marsh. Essentially, the marsh was assumed to be flat and uniform in the along channel direction, the flow was assumed to be non-turbulent and hence the diffusion of sediment in the fluid was ignored and the sediment was assumed to be distributed throughout a column of water with a uniform concentration. The height of this column of water was used as the variable to describe the quantity of sediment suspended above the marsh and an equation governing its variation in space and time was obtained by depth integration of the simplified mass balance equation. An analytical solution of this equation was obtained for a saltmarsh at mean sea-level to compare with the solution obtained from the numerical model.

The model equation was solved numerically by use of the box scheme [Preiss-

mann, 1961] during the flood tide and a combination of the box scheme and the numerical method of characteristics during the ebb tide. From the numerical solution the depth of sediment deposited on the marsh was evaluated using linear interpolation to approximate the solution between grid points.

The numerical solution for the depth of sediment deposited on the marsh was compared with the solution obtained from the analytic solution and the scheme was found to be approximately first order in time and space and the error term was dominated by the errors due to the space discretisation.

Once the model had been tested it was used to examine how the depth of sediment deposited on the marsh depends on the characteristics of the marsh and sediment. With $b = 0$ it was found that the form of the solution was dependent only on the non-dimensional parameter $v_p/A\omega$ and a non-dimensional form of the model equation was found which verified this.

The solution of this non-dimensional equation is dependent on the two parameters b/A and $v_p/A\omega$. Increasing b/A was found to reduce the depth of sediment deposited on the marsh and also to cause the sediment to be less evenly spread across the marsh. Increasing $v_p/A\omega$ caused a higher proportion of the sediment to be deposited on the marsh and also caused the sediment to be less evenly distributed on the marsh. These characteristics mean that sediment made up of different sized particles will be deposited on the marsh with a greater depth at the seaward end and that there will be a higher proportion of the larger particles at the seaward end than at the landward end. This pattern of sediment deposition is in agreement with observations on saltmarshes in the Severn estuary [Allen, 1992] and the North Norfolk Coast [French and Spencer, 1993].

References

- Allen, J.R.L. (1985) *Principles of Physical Sedimentology*, Allen and Unwin, London.
- Allen, J.R.L. (1992) Large-scale textural patterns and sedimentary processes on tidal salt marshes in the Severn Estuary, southwest Britain, *Sedimentary Geology*, **81**, 299-318.
- Allen, J.R.L. and Pye, K. (1992) Coastal saltmarshes: their nature and importance, in *Saltmarshes: Morphodynamics, Conservation, and Engineering Significance*, ed. J.R.L. Allen and K. Pye, Cambridge University Press.
- Brampton, A.H. (1992) Engineering significance of British saltmarshes, in *Saltmarshes: Morphodynamics, Conservation and Engineering Significance*, ed. J.R.L. Allen and K. Pye, Cambridge University Press.
- Doody, J.P. (1992) The conservation of British saltmarshes, in *Saltmarshes: Morphodynamics, Conservation and Engineering Significance*, ed. J.R.L. Allen and K. Pye, Cambridge University Press.
- Einstein, H.A. (1950) The bed-load function for sediment transport in open channel flows, *Technical Bulletin*, no. **1026**, United States Department of Agriculture, Soil Conservation Service, Washington D.C.
- French, J.R. and Spencer, T. (1993) Dynamics and sedimentation in a tide dominated beach barrier salt marsh, *Marine Geology*, **110**, 315-331
- Graf, W.H. (1971) *Hydraulics of Sediment Transport*, McGraw-Hill, New York.

Hallermeier, R.J. (1981) Terminal settling velocity of commonly occurring sand grains, *Sedimentology*, **28**, 859-865.

James, C.S. (1987) The distribution of fine sediment deposits in compound channel systems, *Water SA*, **13**, 7-14

Maude, A.D. and Whitmore, R.L. (1958) A generalised theory of sedimentation, *British Journal of Applied Physics*, **9**, 477-482.

Preissmann, A. (1961) Propagation des intumescences dans les canaux et rivières, *First Congress of the French Association for Computation, Grenoble*.

Rajaratnum N. and Ahmadi, R.M. (1981) Hydraulics of channels with flood plains, *Journal of Hydraulic Research*, **19**, 43-60

Richardson, J.F. and Zaki, W.N. (1954) Sedimentation and fluidisation, *Transactions of the Institute of Chemical Engineers*, **32**, 35-53.

Wood, W.L. (1993) *Introduction to Numerical Methods for Water Resources*, Oxford University Press.

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