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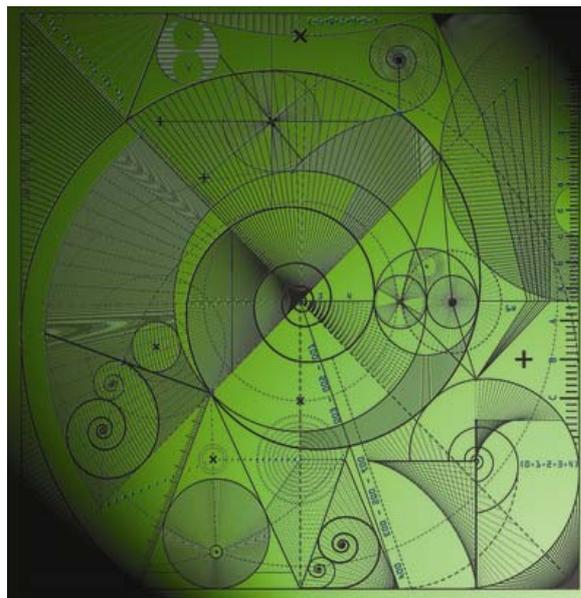
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by

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## Abstract

As low carbon technologies become more pervasive, distribution network operators are looking to support the expected changes in the demands on the low voltage networks through the smarter control of storage devices. Accurate forecasts of demand at the single household level, or of small aggregations of households, can improve the peak demand reduction brought about through such devices by helping to plan the appropriate charging and discharging cycles. However, before such methods can be developed, validation measures are required which can assess the accuracy and usefulness of forecasts of volatile and noisy household level demand. In this paper we introduce a new forecast verification error measure that reduces the so called “double penalty” effect, incurred by forecasts whose features are displaced in space or time, compared to traditional point-wise metrics, such as Mean Absolute Error and  $p$ -norms in general. The measure that we propose is based on finding a restricted permutation of the original forecast that minimises the point wise error, according to a given metric. We illustrate the advantages of our error measure using half-hourly domestic household electrical energy usage data recorded by smart meters and discuss the effect of the permutation restriction.

## 1 Introduction

As many countries progress towards a low carbon economy, the increased penetration of low-carbon technologies (LCTs) may produce new risks to the security and robustness of the electricity networks. The decarbonisation of transport and heating (for instance through the uptake of electric vehicles and heat pumps) is likely to increase the demand on the network, whilst household microgeneration increases the prospect of a two-way flow of electricity on the network as consumers become suppliers and feed back into the grid. In short, electricity demand is likely to increase and become more unstable, particularly at the low voltage (LV) level [6]. In

response to these new challenges, the UK government is aiming to help network operators and suppliers prepare for a low carbon economy through initiatives such as the £500m low carbon network fund (LCNF) [19] and the roll-out of smart meters to every home in the UK by 2020 [18]. Smart meters are advanced energy meters with 2-way communication capability which record high resolution (typically half hourly) energy consumption. These detailed patterns of energy demand provide an opportunity to improve understanding of energy consumption habits, to design smarter interventions for energy reduction, and to produce accurate forecasts of energy demand at the LV level. Such accurate household, or small aggregations of households, level forecasts can help distribution network operators to better understand and anticipate network demand and therefore improve LV network management and planning. Forecasts can also be combined with network storage devices to improve peak demand reduction. As part of the New Thames Valley Vision<sup>1</sup> LCNF project, storage devices are being considered to help alleviate high demand on the LV network at peak times. Simple set point control is the most common and simplest way of controlling battery storage but often fails to reduce peak demand [23]. However, accurate household level forecasts can optimise the use of the battery by helping to plan the appropriate charging and discharging of the storage device [25, 13]. Up until recently, the majority of load forecasting has been at the medium voltage (MV) to high voltage (HV) substation level, where demand is more smooth and regular [12, 1, 22]. However, at the low voltage network to household level, demand is volatile, noisy and typically consists of many different types of behaviour, such as frequent but irregular peaks [2]. Hence, forecast methods developed for the MV and HV level may not be appropriate for the household level. To produce and test the accuracy of household level forecast demands, appropriate forecast verification methods are required.

Forecast verification hinges on the ability of quantitative measures to assess the similarity between forecasts and observations, what [16] refers to as forecast *quality*. Hence measure-orientated approaches based on point-wise comparisons, such as mean absolute error (MAE) and root mean square error (RMSE), can often lead to spurious conclusions, see [3], [4] and [9]. In particular, an observed feature that is forecasted accurately in terms of size and amplitude but displaced in time, incurs a “double penalty” [10]. Thus, as we illustrate in this paper, it can be difficult for skilled, plausible forecasts to out-perform even a flat forecast that provides almost no informative value, particularly when the data is volatile and noisy. This problem has long been understood in the meteorology community. Consequently, a large number of alternative verification strategies have been proposed; see [4] for a review. The class of distribution-orientated approaches [17, 3] offer many insights but require large quantities of data and increased computational effort [3].

One approach for calculating displacement errors, also pioneered in meteorology, has been to formulate errors using an optimal distortion of the original field, i.e. *smooth* changes in position and amplitude that minimise the misfit between data and forecast [9]. Although such verification methods have been widely developed, they have limited appeal in the setting that we are primarily interested — volatile, noisy and irregular data. In this case, it may be more appropriate to use verification measures that deform the forecast *discontinuously*. To some extent such techniques are employed in ‘fuzzy’ verification techniques for high-resolution weather forecasting [7]. These typically compare the average state of ‘events’ occurring within a neighbourhood of interest. For real-valued variables, such as the amount of rainfall or wind intensity, events are defined relative to some threshold. In essence, these methods produce new fields for both the observed and forecasted data, which are then compared using a traditional point-wise metric. Such measures

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<sup>1</sup><http://www.thamesvalleyvision.co.uk/>

are both scale and threshold dependent, thus one must consider a matrix of errors that captures both of these variations.

Many algorithms and metrics have been developed to measure the similarity of time series, e.g. Dynamic Time Warping (DTW), longest common subsequence, edit distance on real sequences and edit distance with real penalty [5]. Often these algorithms are applied in information retrieval and data mining techniques to measure the cost of morphing one time series into another. Dynamic time warping is one of the most popular techniques for measuring time series similarity and has been successfully applied in automatic speech recognition algorithms [14]. DTW measures the difference between sequences which may vary in time or speed by stretching the time series through duplication of local points. The difference in the deformed time series is then calculated using a standard  $L_p$  metric. A more recent method called the Move-Split Merge (MSM) metric is similar to DTW except that duplicated and deleted values incur a fixed cost [21]. For time series matching methods, although suitable for comparing series with the same (but perhaps stretched) shape in time, they are biased toward preserving ordering and therefore in the context of energy demand are not flexible enough to cope with the natural irregularities in household behaviour. Additionally, DTW and MSM will tend to underestimate the cost for repeated peaks by simply merging/duplicating the local peaks with little or no penalty incurred for the inaccurate repetition. The additional complications and restrictions introduced by such techniques make them unsuitable for measuring the errors of household level forecasts. This motivates the development of a new forecast error measures, the topic of this paper.

Before sophisticated forecasting techniques for household electrical energy usage can be developed, we need to be able to quantitatively assess their veracity against data. However, we illustrate in this paper that the capricious nature of energy usage means that traditional point-wise measure-orientated approaches perform poorly at this task. Our main contribution is to suggest a new approach that allows for some flexibility in the timing of the forecast when computing the error while retaining some simplicity. Specifically, for each forecast we define the error to be the minimum error (with respect to an appropriate norm) over the set of all restricted spatial/temporal permutations of the forecast. We begin in Section 2 with a formal description of point-wise error measures, particularly the  $p$ -norm, we then introduce the ‘‘adjusted error’’ and illustrate its advantages using a simple, synthetic example. In Section 3, we apply our new measure to assess the accuracy of a hierarchy of daily forecasts of half-hourly electrical usage taken from individual household smart meter data. In Section 4, we present a detailed discussion of the effect of the ‘adjustment limit’, i.e. the maximum allowed permutation displacement. Finally, we draw conclusions and discuss the advantages and disadvantages of our method in Section 5.

## 2 Measuring Errors

### 2.1 Standard Error Estimates: The $p$ -Norm error

Let  $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$  and  $\mathbf{f} = (f_1, f_2, \dots, f_n)^T$  be the actual and forecasted data vectors respectively, such that each  $f_i$  is a prediction of the actual data  $x_i$  for  $i = 1, \dots, n$ . We focus on one-dimensional data (i.e. time-series), however the methods that we describe can be generalised to higher dimensions. Error measures can be described in terms of a vector function

$$E = F(\mathbf{f}, \mathbf{x}), \tag{2.1}$$

where  $F : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$  is some metric. In this paper we focus on the absolute  $p$ -norm,

$$E_p = \|\mathbf{f} - \mathbf{x}\|_p = \left( \sum_{i=1}^n |f_i - x_i|^p \right)^{1/p}, \quad (2.2)$$

for some  $p \geq 1$  see [8, p. 52]. For example, this type of error includes the Mean Absolute Error (MAE) and the root mean square (RMS) error, which are simply constant multiples of the 1-norm and 2-norm errors respectively.

## 2.2 The Adjusted Error

For the purpose of developing forecasts to plan the discharging and charging patterns of storage devices on the LV network, it is more important to predict peaks at approximately the correct times rather than not at all. As stated in Section 1, such forecasts incur a double penalty from point-wise error measures and may be judged incorrectly as poor forecasts. This motivates the idea that the error measure should allow for small, possibly discontinuous, displacements in time of the forecast values. We note that there exist many perfect matchings between the forecast values and actuals. Each match can be described by a permutation matrix  $P$ . To restrict the magnitude of the displacements of the forecast values, we impose an ‘adjustment limit’, denoted  $w \geq 0$ , on the permutations such that  $P_{ij} = 0$  for  $|i - j| > w$ . We define the *adjusted error* to be the solution to the minimisation

$$E^w = \min_{P \in \mathcal{P}} F(P\mathbf{f}, \mathbf{x}), \quad (2.3)$$

for the given metric  $F$ , where  $\mathcal{P}$  is the complete set of restricted permutations. The *adjusted  $p$ -norm error* is then

$$E_p^w = \min_{P \in \mathcal{P}} \|P\mathbf{f} - \mathbf{x}\|_p. \quad (2.4)$$

The adjusted error can be considered as a semimetric but not as a metric since in general it does not obey the triangle inequality. The error minimisation is a variant of the assignment problem, a well-known combinatorial optimisation problem that can be solved in polynomial time [15] using the ‘Hungarian method’, details of which can be found in [20]. To incorporate the adjustment limit into the algorithm, if  $|i - j| > w$  then we set  $|f_i - x_j|^p = \Omega$ , where  $\Omega$  is a large constant that effectively prevents such permutations. The method’s time complexity is  $O(n(m + n \log n))$  [24] where  $m$  is the number of potential error matches,  $|f_i - x_j|^p$  for  $i, j = 1, \dots, n$ . We note that this method is related to but distinct from the use of the Hungarian algorithm in Monge Type problems, (such as the Earth Mover’s distance) which redistributes the cumulative mass [11]. The adjusted error (2.4) does not subdivide or combine separate predictions but merely reorders them.

The adjustment limit  $w$  is a time-scale parameter that is problem dependent and has an important effect on the efficacy of our verification method. If  $w = 0$  then we recover the original  $p$ -norm error (2.2). Increasing  $w$  reduces the adjusted error, but a small error resulting from large displacements is not necessarily indicative of a good forecast. Thus the mean displacement, which can be obtained from the permutation matrix  $P$ , is an additional measure of accuracy that can be used to compare different forecasts. We discuss these points in detail in Section 4.

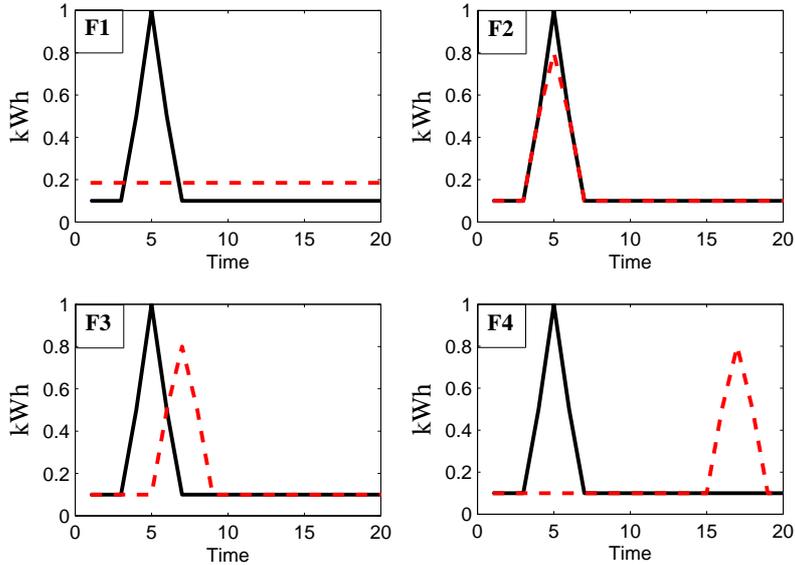


Figure 1: Four ‘Forecasts’ F1, F2, F3, F4 (Dashed line) together with the actual data (Solid Line) for a simplified example.

### 2.3 Simple Example

In this subsection we compare four qualitatively different forecasts of a simple energy load profile using the absolute and adjusted  $p$ -norm errors. The synthetic data, illustrated with solid black lines in each panel of Figure 1, consists of a single peak centred around  $t = 5$  with a constant background usage over a 20 time point domain.

The forecasts, illustrated with dashed lines, consist of a flat forecast (F1) (corresponding to the average usage) and a single peak centred around three different times (F2–F4) with the correct background usage. In the context of using the forecasts to reduce peak demand via a storage device, F2 is a very good forecast, F3 is reasonable and both F1 and F4 are poor. Planning the control of a storage device using the F2 forecast will enable a large reduction in peak demand and F3 should still facilitate moderate peak load shedding due to the expectation of a peak at approximately the correct time. F1 and F4 however would provide no peak load shedding due to the inaccuracy in forecasting the peak demand. The absolute and adjusted  $p$ -norm errors, with  $p = 4$ , for each of the forecasts illustrated in Figure 1 are presented in Table 1.

We have used the 4-norm, rather than the more common 2-norm, because we want to penalise large errors (i.e. missed peaks) much more than small errors. Different values of  $p$  yield qualitatively similar results. Table 1 illustrates the following:

- **Absolute 4-norm error.** The good forecast, F2, has the smallest error while the flat forecast, F1, has smaller error than both the poor forecast, F4, *and* the reasonable forecast, F3. This illustrates the double penalty effect present in point-wise error measures.
- **Adjusted 4-norm error,  $w = 1$  and  $w = 2$ .** The reasonable forecast, F3, error is reduced to about 95 % and 58% of the, F1, flat forecast error for adjustment limit  $w = 1$  and  $w = 2$

Error	Forecast			
	F1	F2	F3	F4
Absolute Error	0.82	0.20	0.99	1.00
Adjusted Error ( $w = 1$ )	0.82	0.20	0.79	1.00
Adjusted Error ( $w = 2$ )	0.82	0.20	0.48	1.00
Adjusted Error ( $w = 3$ )	0.82	0.20	0.20	1.00

Table 1: Comparison of the error measurements given by the different norms for the 4 different forecasts F1–F2 described in the main text.

respectively. The F1, F2 and F4 forecast errors are the same for both the adjusted and the absolute measures—displacing the forecast values does not change the errors.

- **Adjusted 4-norm error,  $w = 3$ .** The good F2 forecast and the reasonable forecast, F3, errors are equal. However, we can still distinguish F2 as the better forecast with this method by considering the mean displacement. F2 has zero mean displacement of the forecast values (implying the minimum permutation is achieved by the forecast) whereas F3 has a mean displacement of 0.6 grid points over the 20 forecasted values.

In summary, the synthetic example illustrates how the adjusted  $p$ -norm error can give a more accurate representation of the forecast usefulness than the standard  $p$ -norm error.

### 3 Application to Household Energy Load Forecasting

As shown in the previous section, for volatile, noisy and irregular data, standard point-wise measures may not be adequate for assessing the accuracy of a forecast. Although many forecast methods have been developed and calibrated for smoother higher voltage demands (see for instance [1]), their accuracy when applied to household or LV level demand cannot be assessed until an appropriate error measure has been established. Once suitable benchmarks are developed both old and new forecasting methods can be tested, compared and other techniques, such as clustering, can be applied to improve the forecasts. In this section we consider the standard and the adjusted 4-norm error in order to compare the performance of three simple forecasting methods applied to half-hourly domestic household electrical energy usage data. The data was collected by household smart meters as part of the Ofgem managed Energy Demand Research Project (EDRP) trial run by Scottish and Southern Energy (SSE)<sup>2</sup>. A wide variety of energy usage behaviours are observed between households and individual household demand is both volatile and noisy. However there are daily, weekly and seasonal patterns that could potentially be exploited by forecasting methods. Such forecasts can have a positive impact on network operations and planning. Figures 2(a)–(c) illustrate a week’s worth of half hourly electrical energy usage profiles in kilowatt-hours (kWh) for three representative UK households. Household (a) consumes most of their energy during one or two peak periods at regular daily intervals. Thus we would hope to be able to forecast their usage fairly accurately. Household (b) has irregular peak demands that are smaller than the other households, but they maintain a fairly constant

<sup>2</sup>See <http://www.ofgem.gov.uk/sustainability/edrp/Pages/EDRP.aspx> for further details

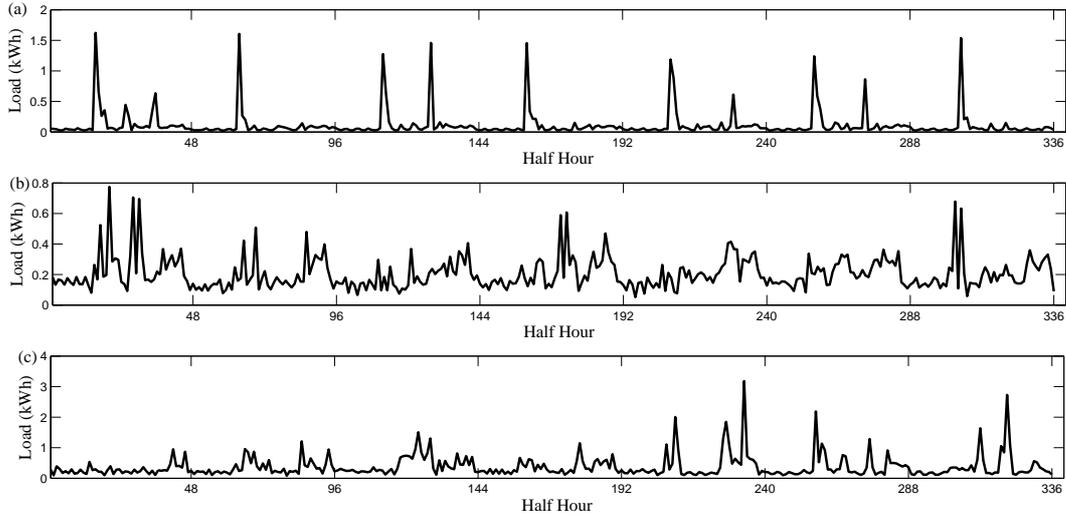


Figure 2: Example of half hourly smart meter electrical energy usage (in kWh) for household (a), household (b) and household (c) as described in the main text.

background usage. Household (c) is the most volatile, having large irregular peak demands and periods of low usage. We would expect this household’s energy usage to be difficult to forecast. The average daily energy usage for households (a), (b) and (c) is 5.51 kWh, 9.89 kWh and 18.12 kWh respectively.

Our household energy usage data-set consists of 10 weeks of half-hourly kWh records (3360 in total) for each of the three households. Each forecast generates an unsupervised rolling daily prediction from midnight over the course of the 10<sup>th</sup> week with access to the full data-history of each household separately. Our aim is to assess validation techniques and consequently the forecast methods that we implement are chosen to form a clear hierarchy. The three methods by which each of the daily forecasts are generated are as follows:

1. **Flat** forecast: The average usage over the previous 7 days, used as the forecast for all time periods.
2. **Last Week (LW)** forecast: The usage on the same day of the previous week.
3. **Averaged Adjustment (AA)** forecast: A combination of a historic average and baseline usage. A detailed description can be found in A.

A snap shot of a single day’s data from each household and the corresponding forecasts are illustrated in Figure 3.

Clearly the flat forecast provides little informative value, while the LW forecast is innately realistic but performs poorly for irregularities in week to week behaviour. The AA forecast is subjectively better than the other forecasts but volatility still reduces its performance.

As in the simple example described in Section 2.3, we compare the absolute and adjusted p-norm errors with  $p = 4$  in order to penalise larger peaks to a greater extent than smaller

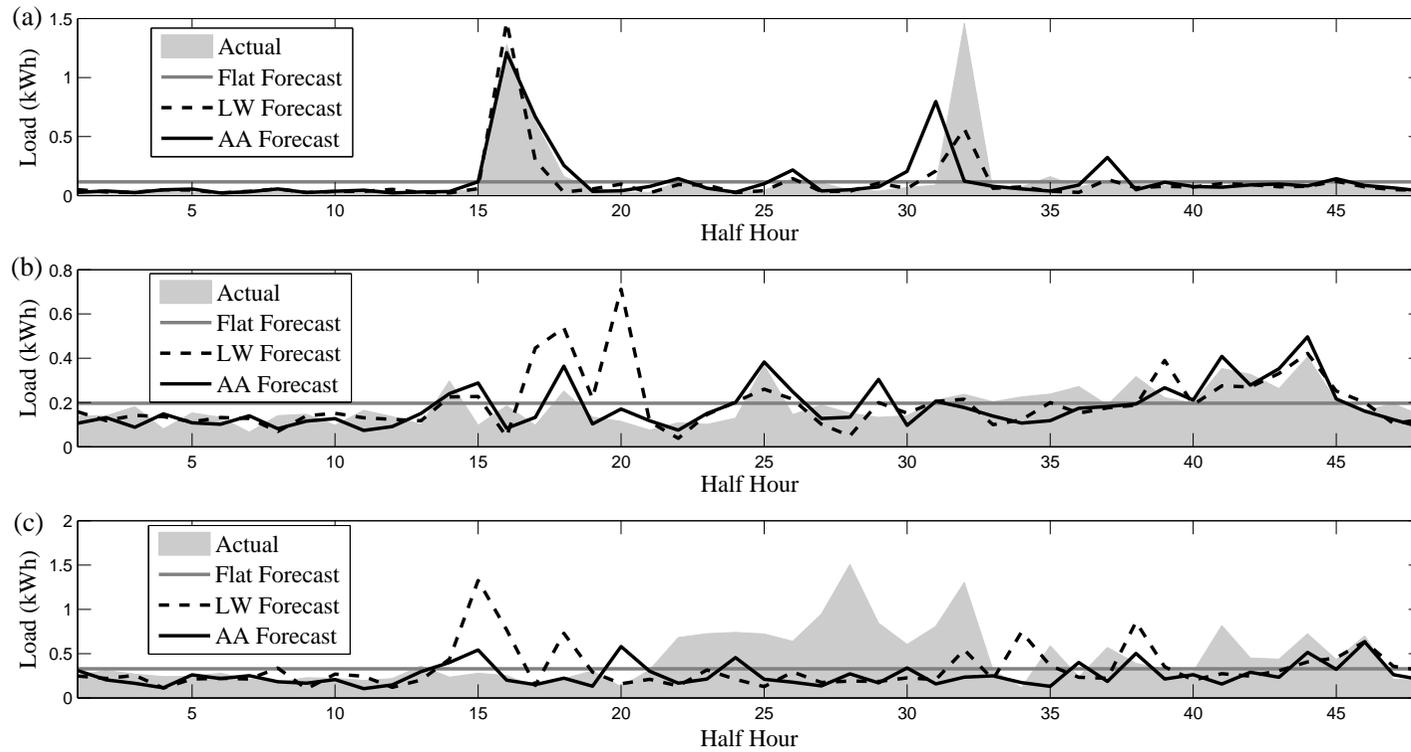


Figure 3: Forecasted usage on Wednesday of the final week of the data set of each household described in text. Plot shows actual usage (shaded area) together with forecasts for household (a), (b) and (c) using the AA (black line), LW (dashed line) and Flat (gray line) forecast methods.

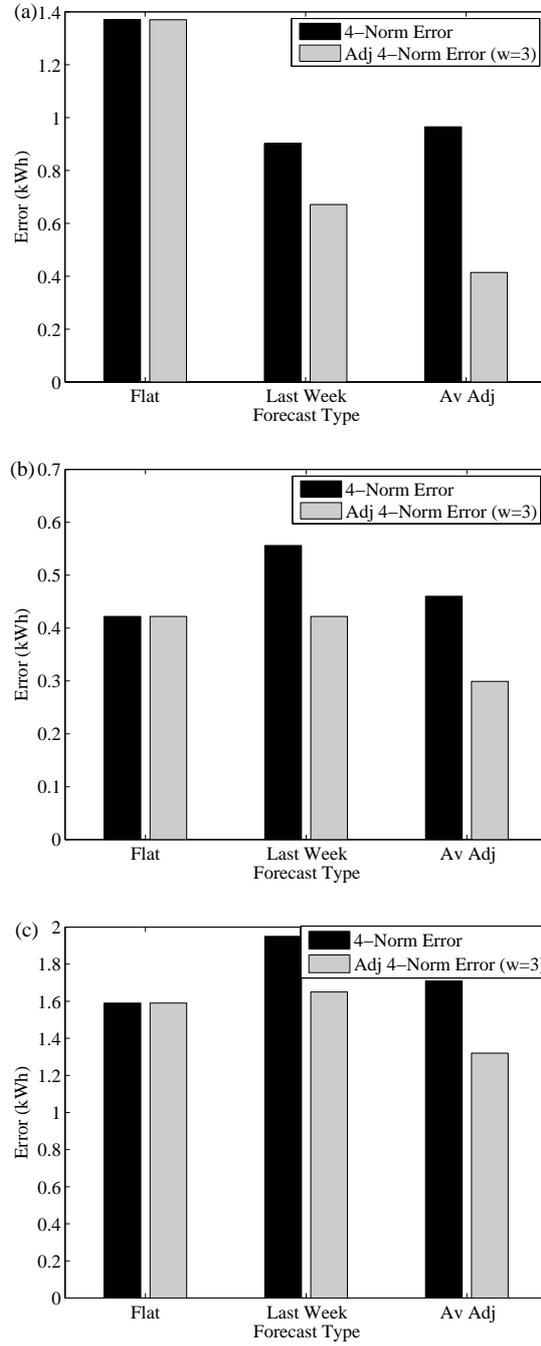


Figure 4: Panels (a)–(c) correspond to households (a)–(c) respectively. Each panel depicts the daily averages of the 4-norm (Black) and adjusted 4-norm (Gray) errors for the three forecasts.

peaks. We use  $w = 3$  as the adjustment limit, hence forecasts can be displaced up to one and a half hours *either side* of their original forecast time. The effects of  $w$  are considered in more detail in Section 4. Because the forecasts produce rolling daily predictions, we calculate the  $i^{\text{th}}$  day's errors for each measure,  $e_i$ , and then use the mean absolute error,

$$\langle E \rangle = \frac{1}{7} \sum_{i=1}^7 e_i, \quad (3.5)$$

to compare forecasts.

The daily mean errors of each forecast method are shown in Figures 4(a)–(c) for each household respectively. The black bars show the daily-mean 4-norm error and the gray bars show the daily-mean adjusted 4-norm error. Focusing first on the 4-norm errors, we note that the flat forecast out-performs the other forecasts for both households (b) and (c). Additionally, the AA forecast is beaten by the LW forecast for household (a). Clearly these results do not agree with the proposed forecasting hierarchy. In particular, we know that the flat forecast reproduces none of the daily household usage patterns. By ignoring peaks altogether, the flat forecast avoids the double penalty and can appear better than more sophisticated forecasts, but is clearly of no use for control or scheduling purposes.

We now consider the 4-norm adjusted errors, illustrated with gray bars in panels (a)–(c) of Figure 4. We note that the adjusted norm does not change the flat forecast errors, but reduces all of the LW and AA errors. The AA forecast is now the most successful forecast for all households with a marked improvement for household (a) in particular. This can be attributed to the regular peak demands observed in the data being forecasted close to when they actually occur and the absence of the double penalty in the adjusted error measure. Relative to the flat forecast errors, the improvement in the errors for the AA forecast decreases from households (a) through to (c), owing to the relative increase in volatility respectively. The magnitude of the errors for household (c) are by far the largest and the relative difference between methods is the smallest, indicating that forecast sophistication only introduces marginal relative improvements as volatility increases.

To illustrate that our results hold more generally, we consider the 4-norm and adjusted 4-norm errors of the three forecast methods applied to the usage data of 600 individual domestic households. As in the example above the data set for each household consists of half hourly electrical energy usage over a 10 week period, collected by smart meters during the EDRP trial. Using the Flat, LW and AA methods, a rolling daily forecast of each household's energy usage was produced for the final week of each data set. Figure 5 shows the mean daily difference between the flat forecast errors and the 4-norm and 4-norm adjusted errors (with  $w = 3$ ) for both the LW and AA forecasts. The horizontal-axis represents the mean daily difference between the 4-norm errors of the Flat forecast and the LW (or AA) forecast and the vertical-axis represents the mean daily difference between the adjusted 4-norm errors of the Flat forecast and the adjusted 4-norm errors of the LW (or AA) forecast. The diagonal line indicates where the mean 4-norm and mean adjusted 4-norm errors are equal. Since the adjusted 4-norm error is always smaller than the 4-norm error, no forecasts can occupy the area below the line.

The three occupied quadrants of the graph establish a 3 cluster segmentation of the forecasts in terms of their accuracy:

1. Points in the lower-left quadrant represent forecasts whose mean 4-norm and mean adjusted 4-norm errors are larger than or equal to the mean flat forecast errors. We refer to these

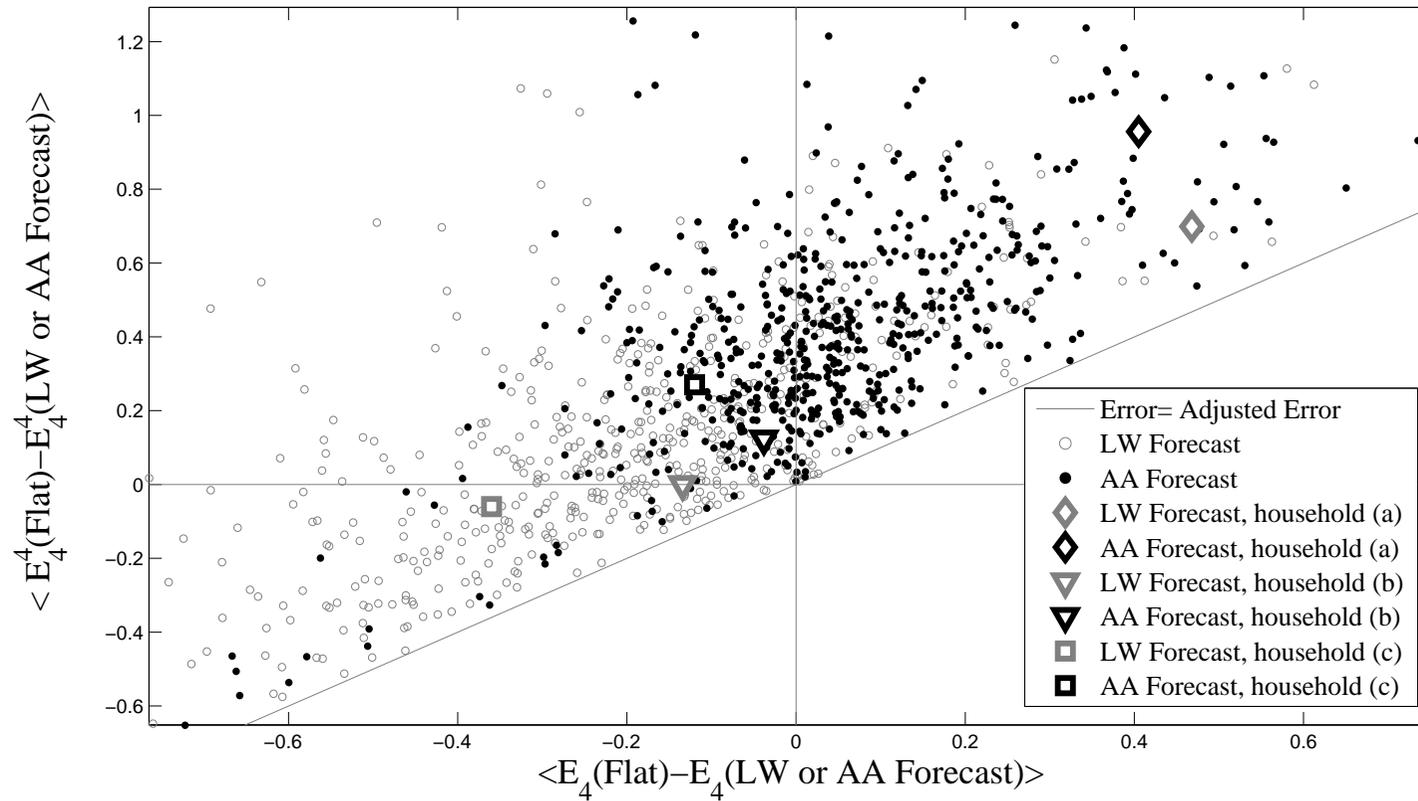


Figure 5: Mean daily difference in the adjusted 4-norm forecast errors of the flat and LW (unfilled circles) or AA (filled) forecasts versus the mean daily difference in 4-norm forecast errors of the flat and LW (unfilled circles) or AA (filled) forecasts. Also included are the data for the Households (a) (Diamonds), (b) (Triangles) and (c) (Squares).

forecasts as *Poor*.

2. Points in the upper-left quadrant represent forecasts whose mean flat forecast error is smaller than the mean 4-norm error forecast but larger than the mean adjusted 4-norm error. Since the small temporal re-alignment has reduced the error compared to the 4-norm error we refer to these forecasts as *Good after adjustment*.
3. Points in the top right quadrant represent forecasts whose mean 4-norm and mean adjusted 4-norm errors are both smaller than the mean flat forecast errors. We refer to these as *Good* forecasts.

The plot shows that the AA forecasts (filled circles) are in general superior to the LW forecasts (unfilled circles). The majority of the AA forecast are either good (360) or good after adjustment (208). Only 32 of the AA forecast are poor whereas 225 of the LW forecasts are poor. For the LW method, only 105 are good forecasts and just less than half (270) are good after adjustments. Of the 600 households, the LW forecast only out-performs the AA forecasts for 30 households in the 4-norm but for 46 households in the adjusted 4-norm. In Figure 5 we also include the data for the LW and AA forecasts of households (a), (b) and (c). In terms of our accuracy classification both the LW and AA are good forecasts for household (a) whereas the AA forecast is good after adjustment for households (b) and (c) while the LW forecast is poor for households (b) and (c). The large proportion of forecasts that are good after adjustment are particularly important. If only the 4-norm is used as an accuracy measure then these forecast methods could potentially be mistakenly rejected, despite their improved performance with respect to the adjusted norm.

## 4 The adjustment limit

In this section we analyse the adjusted error in more detail. The choice of the adjustment window,  $w$ , is largely subjective and application specific. In section 3 we chose  $w = 3$  based on the assumption that a reasonable forecast of household electrical energy usage should only misplace a peak by a maximum of an hour and a half. Other criteria, such as requiring the forecast to out-perform the flat forecast, can also be used to inform on a suitable adjustment limit. For a given application it may be necessary to consider the error as a function of  $w$ , as described in this section, in order to make a more informed decision on the size of the adjustment window. For smart control algorithms utilised in storage devices it is preferable to forecast a peak earlier rather than later, this guarantees that the battery is fully charged and thus able to more efficiently reduce the actual peak. Ideally then there should be a bias in the adjustment window toward forecasting earlier peaks. This is not considered in this paper but is entirely feasible. One simply is required to penalise adjustments which shift relatively larger forecasted peaks to earlier times.

Figure 6 displays the mean adjusted 4-norm error for each of the households introduced in Section 3 for the AA and LW forecast for different values of  $w$ , illustrated in panels (a) and (b) respectively. Each curve is a monotonically decreasing function of the adjustment limit. The black markers on the graph of each line shows where the forecast error equals the error of the flat forecast (The forecasts for household (a) have smaller errors than the flat forecast in these examples hence the absence of a marker). For all households, in order to outperform the flat forecast the AA forecast must use  $w \geq 1$ , whereas the LW forecast must use  $w \geq 4$ . As we increase  $w$ , large reductions in the adjusted error indicate that large peaks in the forecast are

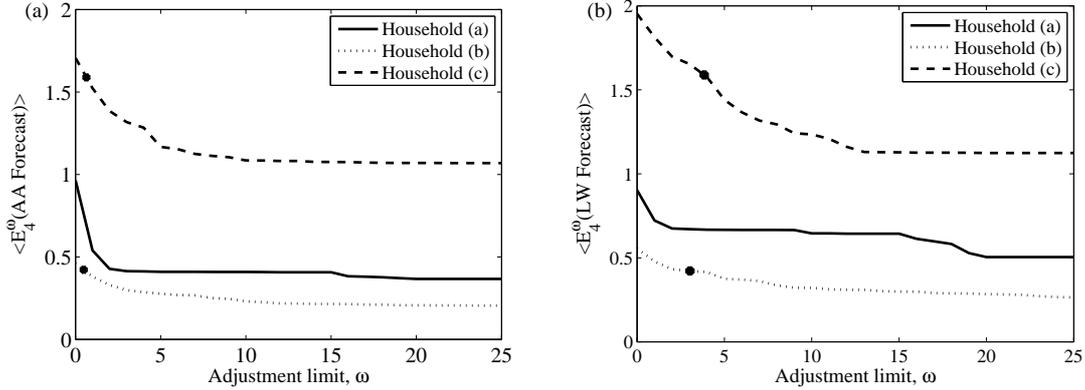


Figure 6: The mean adjusted errors for (a) the AA forecast and (b) the LW forecast for the usage of three different households (a) (solid line), (b) (dotted) and (c) (dashed) as a function of  $w$ . The black marker on each line shows where the forecast errors equal the errors of the flat forecast.

being matched to the actuals. We focus on the AA forecast for our analysis, similar results hold for the LW forecast. As we increase  $w$  from 0 to 2 there are large decreases in the adjusted error of the forecast for household (a) due to the closeness (within 3 half hours) of the peaks in the forecast and actual usage. Moderate decreases in the forecast errors are also observed for household (c), although even with  $w = 20$  (shifts of  $\pm 10$  half hours) the errors are relatively large compared with the errors in the forecasts for households (a) and (b). Household (b) has a slow rate of reduction as  $w$  increases. As shown in Section 3, the general behaviour of household (b) can be forecasted accurately and so the slow reduction is likely to be due to the matching of the small daily irregularities.

The adjusted error decreases with increasing  $w$  but this is likely to simultaneously increase the mean displacement of the forecast positions. Smaller displacements are more desirable as they indicate a closer proximity of the features of the forecast with the actuals. To fully describe the accuracy of a forecast we must consider both the mean displacement and the adjusted error of the forecast. As shown for the synthetic example in Section 2.3, the mean displacement can be used to distinguish between the accuracy of two forecasts with the same adjusted error. Since we are primarily interested in the displacement of the peak loads, we consider a weighted mean displacement. Suppose that the forecast at point  $i$ ,  $f_i$ , is matched to the actual at  $j$ , and  $d_i = |i - j|$  is the forecast displacement then we define the average displacement for each day as

$$\hat{D} = \frac{\sum_{i=1}^{48} f_i^4 d_i}{\sum f_i^4}. \quad (4.6)$$

The power of 4 ensures that our measure is representative of larger peaks.

Figure 7 shows the mean displacement of the AA and LW forecasts over the final week as a function of  $w$  for each household, together with a plot of the expected average displacement if the forecast was assigned randomly. (The random displacement is found by calculating the expected displacement for each of the 48 daily points within the adjustment limit, assuming any displacement is equally likely. The mean over the 48 daily points is then calculated.) We present

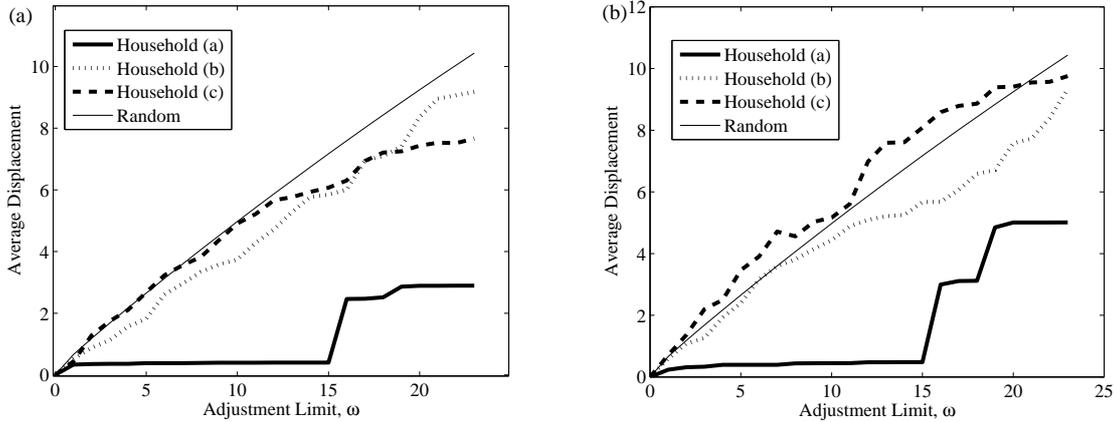


Figure 7: Average weighted displacement of forecasted points for different adjustment limits for (a) the AA forecast and (b) the LW forecast for the three different households (a) (solid line), (b) (dotted) and (c) (dashed). Also included is the expected displacement if forecast points were assigned randomly.

the results for the AA forecast, the LW forecast results are similar. The mean displacements of the forecasts for households (b) and (c) closely match the random displacement curve when  $w < 10$ . It is likely that the features of the forecasts are being matched to the irregular week to week behaviours of the households. As we showed in Section 3, the regular behaviour of household (b) is accurately forecasted but the small irregular demands are poorly forecasted. Household (c) has no regular week to week behaviour and is largely unpredictable. In contrast, household (a) has regular weekly behaviour and the peaks are accurately forecasted and therefore the mean displacement remains small for all  $w$  values. As the adjustment limit is increased beyond  $w = 15$  some of the afternoon and morning peaks are matched resulting in the small increase in the size of the average displacement.

Figures 6 and 7 together reveal extra information about the usage patterns and forecast accuracy for each of the different households. In particular, for household (a), sharp drops in the forecast error as  $w$  is increased from 0 to 2 indicate the forecast closely approximates the large features in the data. The small average displacements confirm that regular peaks are being matched. In contrast, for household (c) the large reduction in forecast error is likely to be the result of matching random, irregular behaviour as shown by the mean displacement being similar to a random assignment in Figure 7. Similarly we find that the small reduction in the adjusted error for household (b) as we increase  $w$  are mainly the consequence of matching the small irregular behaviour which are missed by the forecast.

## 5 Conclusions

As low carbon technologies become ubiquitous there are increased risks to the robustness and security of the low voltage (LV) electricity networks. The electrification of heating and transport is expected to increase network peak demand, while the increased uptake of more intermittent forms of generation such as photovoltaics is likely to increase network volatility. To effectively

manage the local networks it is vital that network operators understand how demand is changing and what practical solutions are available. Household smart meters are becoming an integral part of many government’s low carbon agenda and many countries aim to have a meter in every home within the next decade. Smart meters provide a valuable opportunity for detailed data analytics and in particular for forecasts at the individual and low voltage substation level. Accurate household level forecasts can also be utilised for planning the smart control of storage devices to reduce peak demands. However, before useful household level forecasts can be developed an appropriate verification measure must be established to assess the accuracy of such forecasts.

In this paper we suggest such a measure for assessing the success of forecasts of volatile and noisy data. A standard treatment of the accuracy is to consider the  $p$ -norm of the error, but due to the “double penalty” effect such measures have been shown to be inadequate, especially when attempting to forecast peaks and troughs in the data. Any successful forecast method requires a degree of flexibility in the spatial/temporal positioning of the peaks. Our proposed solution, the adjusted  $p$ -norm error, allows for limited permutations of the forecasted data, which reduces the penalty imposed on shifted peaks. This was illustrated with a simple synthetic example and then demonstrated on forecasts of real, high resolution household electrical energy usage.

To test the forecast measure, three forecast methods were applied to three separate households energy demand data with varying degrees of week to week regularity and hence forecastability. The forecasts ranged in skill with a clear hierarchy: an innately poor flat forecast, a poor, yet realistic ‘last week as this week’ forecast and an adjusted-average of previous week’s behaviour. We found that, with respect to a point-wise metric, the flat forecast could outperform many of the more realistic, informative forecasts. This was not the case with our new error measure. We also applied the measure to forecasts of 600 independent households which confirmed the ability of the new measure to successfully distinguish between the accuracy of the 3 forecast methods. In addition, we also considered what the effect of changing  $w$  has on the adjusted error and the average displacement of the matched forecasts. This offered further insights into the accuracies of the forecasts. In summary, in this paper we have presented a new method for verifying forecast accuracy which has shown to be effective and efficient for assessing the accuracy of shifted features of volatile and noisy data sets.

The new measure presented in this paper deforms the forecast in a discontinuous way, which may not be appropriate for all applications. For instance at for high voltage level demand which is more smooth and regular the standard point-wise measures will be adequate. However, when the data is volatile and irregular, the smoothness of the deformation may be less significant. Additionally, for any particular application the method can be applied to any suitable error measure and is very simple to implement with only a single control parameter,  $w$ .

In the context of utilising forecasts to control storage devices on the low voltage network, it is arguably more appropriate for the forecast measure to impose heavier penalties for peaks forecasted too early rather than too late. In future work we will investigate modifications to the adjustment window and consider their impact on peak reduction through implementation of smart control of storage devices. Additionally we will consider the accuracy of the more standard forecast methodologies used in higher voltage load forecasting with respect to our new measure to test their suitability for forecasting electricity demand at the household level.

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## A The Averaged Adjustment Forecast

In this appendix we briefly describe the Averaged Adjustment (AA) forecast as implemented in this report. For clarity, we show how we forecast for one particular day, the other days of the week are forecasted in an analogous way. We assume that we have  $N$  daily usage profiles of half hourly resolution of the  $d^{\text{th}}$  day of the week ( $d = 1, \dots, 7$ ) which we notate  $\mathbf{G}^{(k)} = (g_1^{(k)}, g_2^{(k)}, \dots, g_{48}^{(k)})^T$  for  $k = 1, 2, \dots, N$ , where  $\mathbf{G}^{(1)}$  is the previous week usage of the  $d^{\text{th}}$  day and  $\mathbf{G}^{(2)}$  is the usage over the  $d^{\text{th}}$  day from 2 weeks before etc. We create a base profile  $\mathbf{F}^{(1)} = (f_1^{(1)}, f_2^{(1)}, \dots, f_{48}^{(1)})^T$  where each half hour is defined to be the median value over all  $N$  half hours. We iteratively update the baseline profile using matching with each successive previous weeks data. This is performed as follows. Suppose  $\mathbf{F}^{(k)}$  is the current baseline for the  $k^{\text{th}}$  iteration ( $1 \leq k \leq N - 1$ ). We define  $\hat{\mathbf{G}}^{(k)} = \hat{P}\mathbf{G}^{(k)}$ , where  $\hat{P} \in \mathcal{P}$  is a permutation matrix such that

$$\|\hat{P}\mathbf{G}^{(k)} - \mathbf{F}^{(k)}\|_4 = \min_{P \in \mathcal{P}} \|P\mathbf{G}^{(k)} - \mathbf{F}^{(k)}\|_4. \quad (\text{A.7})$$

where  $\mathcal{P}$  represents the set of restricted permutations of the half hour loads (i.e. each half hour  $i$  moved to some half hour  $j$  where  $|i - j| \leq w$  and  $w$  is the deformation limit as described in Section 2.2). In other words,  $\hat{\mathbf{G}}^{(k)}$  is the usage from the previous week that minimises the deformed norm error between the baseline load usage and the usage of the current week  $\mathbf{G}^{(k)}$ . The new baseline is defined to be

$$\mathbf{F}^{(k+1)} = \frac{1}{k+1} (\hat{\mathbf{G}}^{(k)} + k\mathbf{F}^{(k)}). \quad (\text{A.8})$$

The process is repeated for each of the remaining weeks to give the final forecast  $\mathbf{F}^{(N)}$ . Hence, the forecast is defined to be an average of the initial baseline and permutations of the previous weeks

$$\mathbf{F}^{(N)} = \frac{1}{N+1} \left( \sum_{k=1}^N \hat{\mathbf{G}}^{(k)} + \mathbf{F}^{(1)} \right). \quad (\text{A.9})$$

## References

- [1] H.K. Alfares and M. Nazeeruddin. Electric load forecasting: literature survey and classification of methods. *International Journal of Systems Science*, 33:23–34, 2002.
- [2] Marek Brabec, Ondrej Kon Emil Pelik and Marek Mal. A nonlinear mixed effects model for the prediction of natural gas consumption by individual customers. *International Journal of Forecasting*, 24:659 – 678, 2008.
- [3] H.E. Brooks and C.A. Doswell III. A comparison of measures-oriented and distributions-oriented approaches to forecast verification. *Weather and Forecasting*, 11:288–303, 1996.

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<sup>3</sup>See <http://www.ofgem.gov.uk/Networks/Techn/NetwrkSupp/Innovat/ifi/Pages/ifi.aspx> for Innovation Funding Incentive reports.

- [4] B. Castati, L.J. Wilson, D.B. Stephenson, P. Nurmi, A. Ghelli, M. Pocernich, U. Damrath, E.E. Ebert, B.G. Brown, and S. Mason. Review forecast verification: current status and future directions. *Meteorological applications*, 15:3–18, 2008.
- [5] L. Chen and R. Ng. On the marriage of lp-norms and edit distance. *Proceedings of the Thirtieth international conference on very large databases*, 30:792–803, 2004.
- [6] F. M. Combrink and P. T. M. Vaessen. Low voltage, but high tension. [https://www.idc-online.com/technical\\_references/pdfs/electrical\\_engineering/LV-HT.pdf](https://www.idc-online.com/technical_references/pdfs/electrical_engineering/LV-HT.pdf), 2006. [Online; accessed 20-Jan-2013].
- [7] Elizabeth E. Ebert. Fuzzy verification of high-resolution gridded forecasts: a review and proposed framework. *Meteorological applications*, 15:51–64, 2008.
- [8] G. H. Golub and C. F. Van Loan. *Matrix Computations, third edition*. Johns Hopkins University Press, 1996.
- [9] Ross N. Hoffman, Zheng Liu, Jean-Francois Louis, and Christopher Grassotti. Distortion representation of forecast errors. *Monthly Weather Review*, 123:2758–2770, 1995.
- [10] Christian Keil and George C. Craig. A displacement and amplitude score employing an optical flow technique. *Weather and Forecasting*, 24:1297–1308, 2009.
- [11] E. Levina and P. Bickel. The earth mover’s distance is the mallows distance: some insights from statistics. *Proceedings. Eighth IEEE International Conference on Computer Vision*, 2:251–256, 2001.
- [12] I. Moghram and S. Rahman. Analysis and evaluation of five short-term load forecasting techniques. *IEEE Transactions on Power Systems*, 4:1484–1491, 1989.
- [13] A. Molderink, V. Bakker, M. G. C. Bosman, J. L. Hurink, and G. J. M. Smit. A three-step methodology to improve domestic energy efficiency. IEEE Innovative Smart Grid Technologies Conference 19-21 Jan 2010, 2010.
- [14] M. Muller. *Information Retrieval for Music and Motion*. Springer, 2007.
- [15] James Munkres. Algorithms for the assignment and transportation problems. *Journal of the Society for Industrial and Applied Mathematics*, 5:32–38, 1957.
- [16] A. H. Murphy. What is a good forecast? an essay on the nature of goodness in weather forecasting. *Weather and Forecasting*, 8:281–293, 1993.
- [17] A.H. Murphy and R.L. Winkler. A general framework for forecast verification. *Monthly Weather Rev.*, 115:1330–1338, 1987.
- [18] National Grid. UK Future Energy Scenarios 2012. <http://www.nationalgrid.com/NR/rdonlyres/C7B6B544-3E76-4773-AE79-9124DDBE5CBB/56766/UKFutureEnergyScenarios2012.pdf>, 2012. [Online; last accessed 20-Jan-2013].
- [19] Ofgem. Low carbon networks fund governance document v.5. <http://www.ofgem.gov.uk/Networks/ElecDist/lcnf/Pages/lcnf.aspx>, 2012. [Online; last accessed 10-Feb-2013].

- [20] Alexander Schrijver. *Combinatorial Optimization: Polyhedra and Efficiency*. Springer, 2002.
- [21] A Stefan, V. Athitsos, and G. Das. The move-split-merge metric for time series. *IEEE Tran. on Knowledge and Data Eng*, forthcoming.
- [22] James W. Taylor and Antoni Espasa. Energy forecasting. *International Journal of Forecasting*, 24:561 – 565, 2008.
- [23] P. R. Thomas. American electric power’s community energy storage. EPRI 4th Int. Conf. Integration of Renewable and Distributed Energy Resources, 2010.
- [24] N. Tomizawa. On some techniques useful for solution of transportation network problems. *Networks*, 1:33–34, 1971.
- [25] Y. Xu, L. Xie, and C. Singh. Optimal scheduling and operation of load aggregator with electric energy storage in power markets. North America Power Symposium, 2010.