

A FLUX LIMITER BASED ON FROMM'S SCHEME

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ABSTRACT

A brief survey of Flux Limiters used with well known schemes to make them Total Variation Diminishing (TVD) leads to a definition of a limiter based on Fromm's scheme. Numerical results are given for this scheme on a number of test problems.

§1 INTRODUCTION

In the numerical modelling of sharp shocks or steep fronts on fixed grids the appearance of oscillations when other than first order methods are applied can be an embarrassment to the modeller, particularly if these trigger instabilities or correspond to non-physical behaviour of the system. First-order methods are invariably insufficiently accurate for the purpose here and lead to unacceptable smearing, but higher order methods although giving better resolution induce oscillations or wiggles. Often some kind of post-processing will remove the wiggles but such techniques are often somewhat ad hoc.

Over the last twenty five years a number of second-order schemes have been devised which, together with the addition of artificial viscosity to moderate the wiggles and to prevent instabilities, have become standard in treating such problems. More recently, in order to improve the methods and in particular the treatment of discontinuities, schemes have been advocated which give higher order accuracy in smooth regions but which use criteria other than polynomial accuracy to model sharp variations in the solution. These criteria rely on conservation, as is standard, but also introduce monotonicity preservation and total variation diminishing (TVD), which essentially substitute second order accuracy in regions of high curvature where the matching of a few terms of a Taylor series is of no particular significance. Such schemes have become known as TVD schemes, after Harten [2].

In this report we review a rather direct method of producing TVD schemes using the idea of Flux Limiters. The form of these limiters for some standard schemes is surveyed and particular attention paid to Fromm's scheme [4], which is well known to produce less pronounced wiggles and to give good phase accuracy. This leads to a definition of a limiter based on Fromm's scheme, giving an algorithm which is second-order over a greater range of data and possesses greater continuity than schemes using standard limiters.

In §2 and §3 limiters based on standard schemes are discussed and the Fromm-type limiter is introduced in §4. Numerical results are compared in §5.

§2 LIMITERS BASED ON THE LAX-WENDROFF SCHEME

In recent years several flux limiters have been proposed for the prevention of oscillations in the application of second order accurate numerical schemes for the approximate solution of non-linear hyperbolic equations. In ref. [1] Sweby surveyed and tested a number of these limiters on some standard test problems and presented a useful diagram on which limiters can be displayed, here shown as Fig. 1.

For a function u satisfying the non-linear equation

$$u_t + (f(u))_x = 0 \quad (2.1)$$

and a finite difference approximation u_k^n to u on a regular grid at a grid point $(k\Delta x, n\Delta t)$, define r to be the ratio of successive differences

$$r = \frac{\Delta u_{k+\frac{1}{2}}}{\Delta u_{k-\frac{1}{2}}} \quad (2.2)$$

where $\Delta u_{k+\frac{1}{2}} = u_{k+1} - u_k$.

The function $\phi(r)$ in Fig. 1 is the limiter whose general definition is given in ref. [1]. It is most simply described in relation to the Lax-Wendroff scheme [7] for equation (2.1) with $f(u) = au$ and $a > 0$, namely

$$u_k^{n+1} = u_k^n - v\Delta u_{k-\frac{1}{2}}^n - \alpha v\Delta[\Delta u_k^n] \quad (2.3)$$

where $v = \frac{a\Delta t}{\Delta x}$, $\alpha = \frac{1}{2}(1-v)$. The final term on the right hand side of (2.3) gives second order accuracy by an anti-diffusion term (compare ref. [1]).

When the limiter $\phi(r)$ is introduced this term becomes

$$- \alpha v\Delta[\phi(r)\Delta u_k] \quad (2.4)$$

Thus the Lax-Wendroff scheme corresponds to $\phi(r) \equiv 1$. In reference [1] Sweby shows that schemes which employ limiters $\phi(r)$ which lie in the region shaded in Fig. 1 are Total Variation Diminishing (TVD), a property which ensures that no spurious oscillations are generated by the scheme (see reference [2]).

Examples of limiters in use can readily be illustrated in Fig. 1. The TVD region is shown by the shaded area, while second order accuracy (corresponding to straight line segments of $\phi(r)$ through the point (1,1) - see ref. [1]) is shown by the overshaded area.

The piecewise linear function given by the lower boundary of the shaded TVD region in the figure (Minmod) is formally second order everywhere and is TVD. It is the most diffusive TVD limiter in practice but appears to be adequate in the presence of shocks. On the other hand, the piecewise linear function given by the upper boundary of the TVD region in Fig. 1 (Superbee) is a highly compressive limiter, second order accurate on the straight lines passing through the point (1,1), which reduces to first order accuracy elsewhere (for extreme values of r outside the interval $(\frac{1}{2}, 2)$) but stays TVD throughout. In reference [1] Sweby suggests that its most useful role is in the resolution of contact discontinuities.

Analytic forms of the limiter ϕ are, for Minmod

$$\phi(r) = \max[0, \min(r, 1)] \tag{2.5}$$

and, for Superbee

$$\phi(r) = \max[0, \max\{\min(2r, 1), \min(r, 2)\}] . \tag{2.6}$$

The limiter suggested by Van Leer [3] comes somewhere between the previous two. It is given by

$$\phi(r) = \frac{2r}{1+r} \tag{2.7}$$

and is shown in Fig. 1 by a curved dotted line.

§3 LIMITERS BASED ON OTHER SCHEMES

Limiters can be defined for other schemes. For the second order upwind scheme of Warming and Beam [8], (2.3) is replaced by

$$u_k^{n+1} = u_k^n - v\Delta u_{k-\frac{1}{2}}^n - \alpha v\Delta \{\Delta u_{k-1}^n\} \quad (3.1)$$

and the limited form of the last term becomes

$$- \alpha v\Delta\{\theta(r)\Delta u_{k-1}^n\}$$

where $\theta(r)$ is the limiter for the Warming-Beam scheme. From (2.2) we see that

$$\theta(r) = \phi(r)/r. \quad (3.2)$$

The diagram corresponding to Fig. 1 based on this limiter is given as Fig. 2 with the TVD region shaded. The Minmod and Superbee limiters are given respectively by the lower and upper boundaries of TVD, while the Van Leer limiter, shown as the intermediate dotted line, is

$$\theta(r) = 2/(1 + r) \quad (3.3)$$

The second order region is again the overshaded region in Fig. 2.

In the case of Fromm's scheme [4], which is the arithmetic mean of the schemes (2.3) and (3.1), namely,

$$u_k^{n+1} = u_k^n - v\Delta u_{k-\frac{1}{2}}^n - \alpha v\Delta[\frac{1}{2}\{\Delta u_k^n + \Delta u_{k-1}^n\}] \quad (3.4)$$

the appropriate flux limiter $\psi(r)$ enters the last term of (3.4) in the form

$$- \alpha v\Delta[\psi(r)\frac{1}{2}\{\Delta u_k^n + \Delta u_{k-1}^n\}]. \quad (3.5)$$

From (2.2) we then have

$$\psi(r) = 2\phi(r)/(1 + r) \quad (3.6)$$

and the diagram corresponding to Fig. 1, with the shaded regions, is shown in Fig. 3. The Minmod and Superbee limiters in this case are again the lower and upper boundaries of the TVD region, respectively, while the Van Leer Limiter is

$$\psi(r) = \frac{4r}{(1+r)^2} \quad (3.7)$$

and is shown again as a dotted line.

Note that (3.6) can be written

$$\psi(r) = \frac{2\phi(r)/r^{\frac{1}{2}}}{(r^{\frac{1}{2}} + r^{-\frac{1}{2}})} \quad (3.8)$$

from which we can readily deduce the symmetry condition

$$\psi\left(\frac{1}{r}\right) = \psi(r) \quad (3.9)$$

(c.f. [1] eqn. 3.18).

The three basic schemes considered above are applied in their non-limited form when ϕ, θ, ψ are all identically equal to 1. Both Minmod and Superbee use non-limited forms of both the Lax-Wendroff and Warming-Beam upwind schemes. However, neither of these limiters uses Fromm's scheme in its non-limited form. A limiter which does this is shown in Figs. 5, 6 and 7. We study this limiter in the next section.

§4 A LIMITER BASED ON FROMM'S SCHEME

In order to demonstrate the features of a limiter based on Fromm's scheme we introduce a change of variable. Let

$$s = \frac{r-1}{r+1} \quad r = \frac{1+s}{1-s} \quad (4.1)$$

and express $\psi(r)$ in terms of the new variable s as in Fig. 4.

Minmod is the lower boundary of the TVD region and Superbee is the upper boundary, both being symmetric about $s = 0$ and therefore expressible as functions of $|s|$, which in terms of the data u_k is given by

$$\frac{|\Delta u_{k+\frac{1}{2}} - \Delta u_{k-\frac{1}{2}}|}{|\Delta u_{k+\frac{1}{2}} + \Delta u_{k-\frac{1}{2}}|} \text{ or } \frac{\Delta^2 u_k}{|u_{k+1} - u_{k-1}|} \quad (4.2)$$

Van Leer's limiter is the dotted line in Fig. 4 given by

$$\psi(s) = 1 - s^2 \quad (4.3)$$

also symmetric about $s = 0$.

We now propose a limiter based on Fromm's scheme. The piecewise linear function shown in Fig. 8 (c.f. Fig. 4) corresponds to Fromm's scheme in its central sections and elsewhere is modified to keep it TVD. This limiter will be more compressive than Minmod but less compressive than Superbee with which, however, it shares the feature that second order accuracy is attained only for a limited range of values of s (and therefore r). This range is

$$-\frac{1}{2} \leq s \leq \frac{1}{2} \quad (4.4a)$$

or

$$\frac{1}{3} \leq r \leq 3 \quad (4.4b)$$

The corresponding bounds on r for Superbee are $\frac{1}{2}$ and 2 so that the new scheme is second-order accurate for a wider range of r . Also, unlike Minmod and Superbee, it has a smooth derivative at $r = 1$ (corresponding to $s = 0$), which strengthens the continuity of the scheme at inflection points. Within the second order region given by (3.4b) the scheme has the well-known features of Fromm's scheme, in particular a small relative phase error.

The form of the scheme outside the range (4.4a) is dictated by the shaded TVD region in Fig. 4. The limiter can be written conveniently in the form

$$\psi(s) = \begin{cases} 1 & |s| \leq \frac{1}{2} \\ 2(1 - |s|) & |s| > \frac{1}{2} \end{cases} \quad (4.5)$$

where $|s|$ is given by (4.2). In terms of r and $\phi(r)$ of the original Fig. 1 (q.v.) it is shown by the full line in Fig. 5.

In the scheme itself, (3.4)-(3.5), the limited term (3.5) can be written

$$\begin{aligned} & -\alpha v \Delta [\frac{1}{2} \{\Delta u_k^n + \Delta u_{k-1}^n\}] & |s| \leq \frac{1}{2} \\ & -\alpha v \Delta [\{\Delta u_k^n + \Delta u_{k-1}^n - |\Delta u_k^n - \Delta u_{k-1}^n|\}] & |s| > \frac{1}{2} \end{aligned} \quad (4.6)$$

which shows also that an extra diffusion term of the form

$$-\alpha v \Delta [\{\frac{1}{2} [\Delta u_k^n + \Delta u_{k-1}^n] - |\Delta u_k^n - \Delta u_{k-1}^n|\}] \quad (4.7)$$

is brought in effectively to suppress the oscillations when $|s| > \frac{1}{2}$.

The form of the limiter in terms of the ϕ function of (2.4) and Fig. 1 is

$$\phi(r) = \max[0, \min(2r, \frac{1}{2}(1+r), 2)] \quad (4.8)$$

(c.f. [1]), while in terms of the B-functions introduced by Roe and Baines [5] the new limiter has the form

$$B(b_1, b_2) = \begin{cases} \frac{1}{2}(b_1 + b_2) & \frac{1}{3} < \frac{b_1}{b_2} < 3 \\ 2 \min(b_1, b_2) & \text{otherwise} \end{cases} \quad (4.9)$$

if $b_1 b_2 > 0$ and zero otherwise.

§5 NUMERICAL RESULTS

We have carried out a number of numerical experiments using the Fromm-type limiter. The examples chosen were linear advection of a square wave and a Gaussian, and the well-known Sod shock tube problem [6].

For the advection of a square wave the new limiter is inferior to Superbee, giving rather surprisingly a less symmetric profile, particularly for low CFL numbers. For the Gaussian, however, although there is slightly more clipping of the peak than Superbee there is also less squaring off of the profile. The new limiter is clearly better than Minmod and slightly better than that of Van Leer. Results using Van Leer's limiter have better symmetry but for low CFL numbers Van Leer clips the peak much more.

For Sod's problem the results using the new limiter are better than those of Van Leer and are comparable with those using Superbee.

§6 CONCLUSION

The results from the Fromm-type limiter (4.8) indicate that in smooth regions the property of good relative phase error is preserved while the action of the limiter close to discontinuities prevents any oscillations arising. The placing of the scheme midway between Lax-Wendroff and Warming-Beam upwind ensures that the extremes of both schemes are avoided and that the limiter is invoked under less wide conditions (see (3.4b)). Second-order accuracy is therefore to be anticipated closer to sharp features of the solution than for other limited schemes with consequent better overall accuracy.

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- [8] WARMING, R.F. & BEAM, R.M. "Upwind second order difference schemes and applications in aerodynamics", AIAA J. 14, (1976).

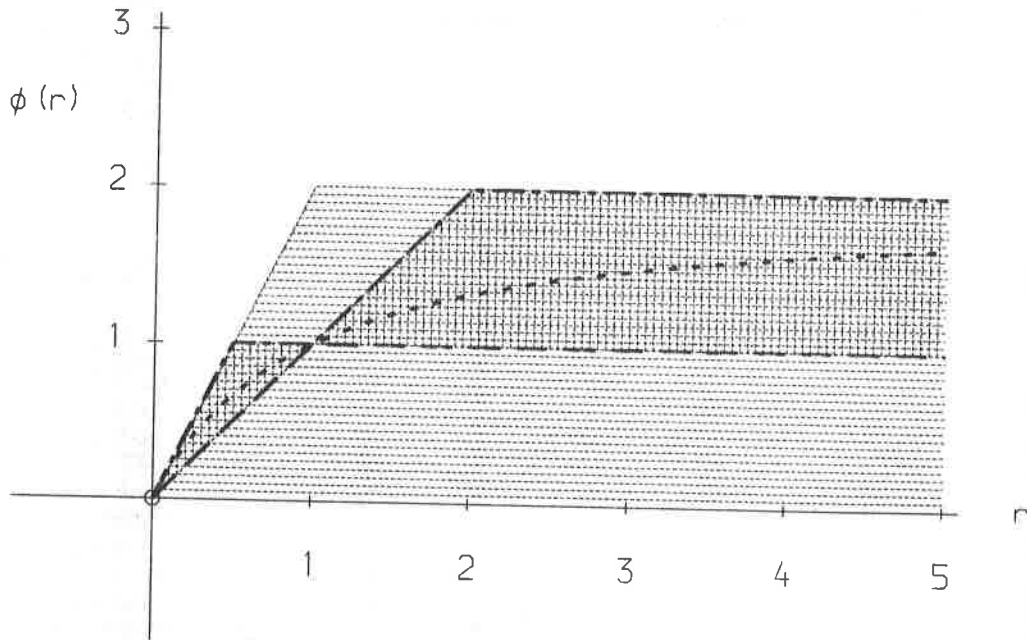


FIGURE 1

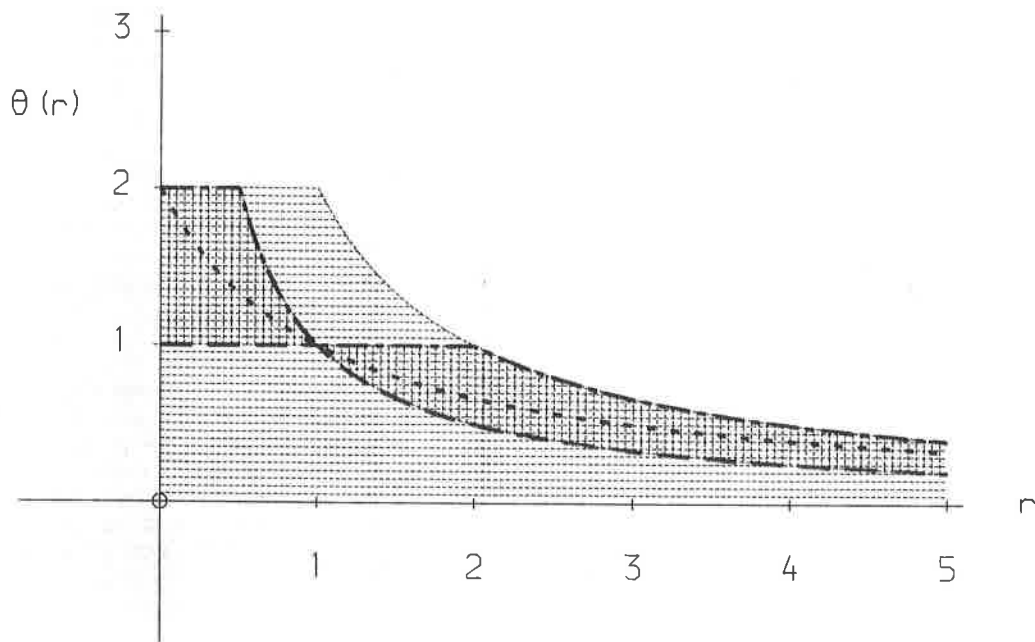


FIGURE 2

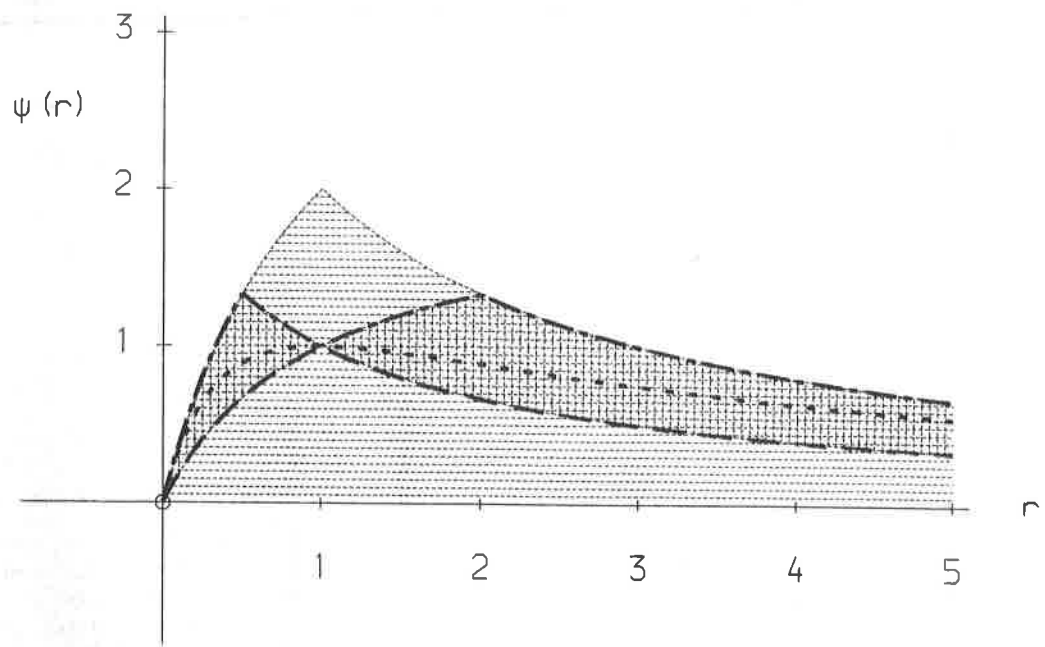


FIGURE 3

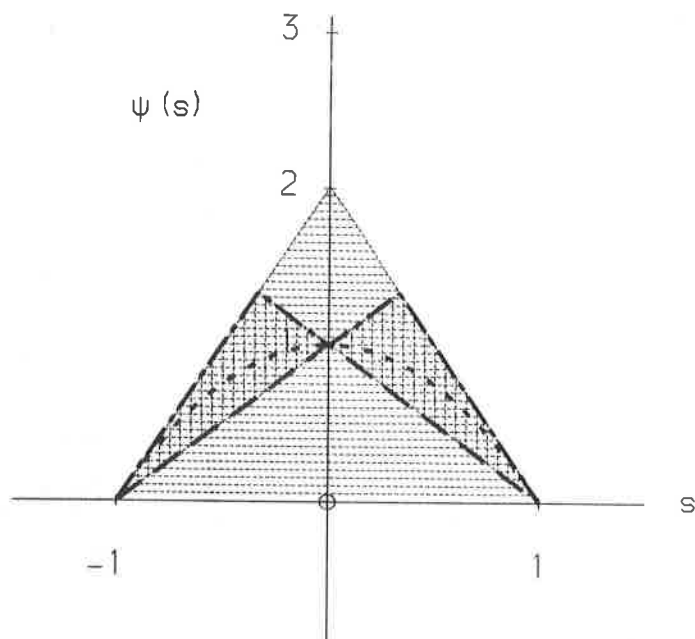


FIGURE 4

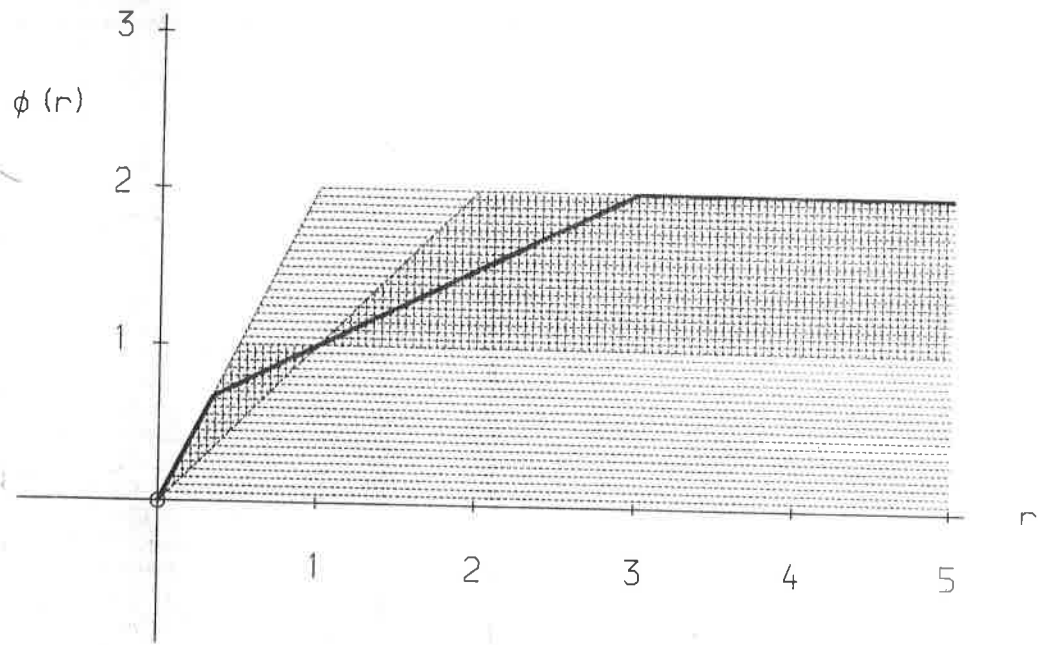


FIGURE 5

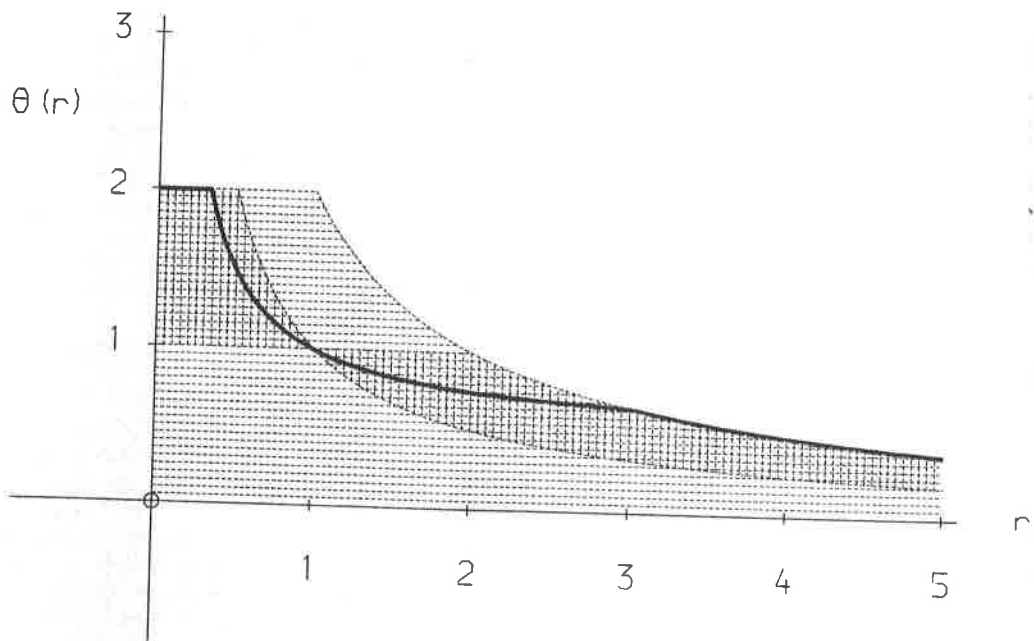


FIGURE 6

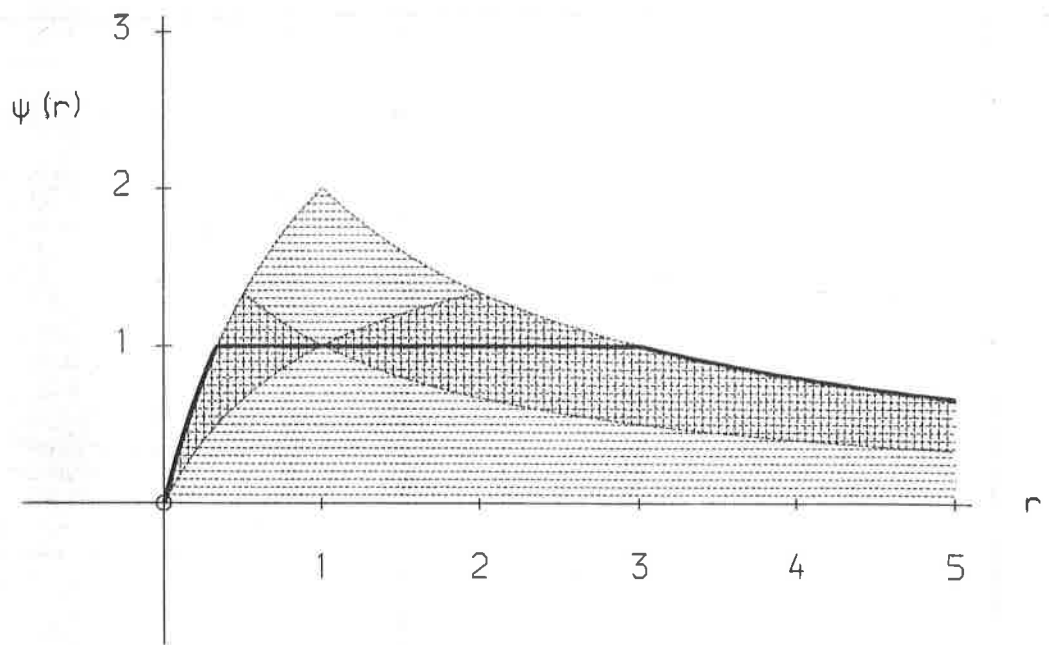


FIGURE 7

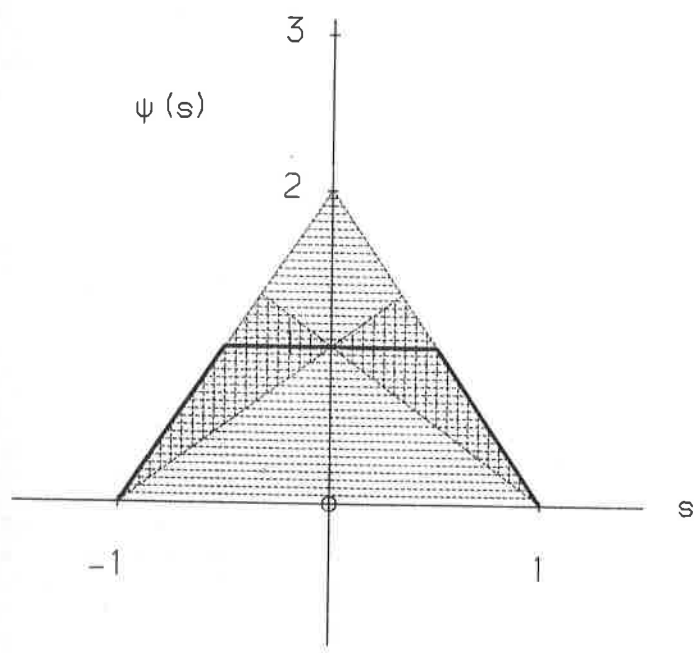
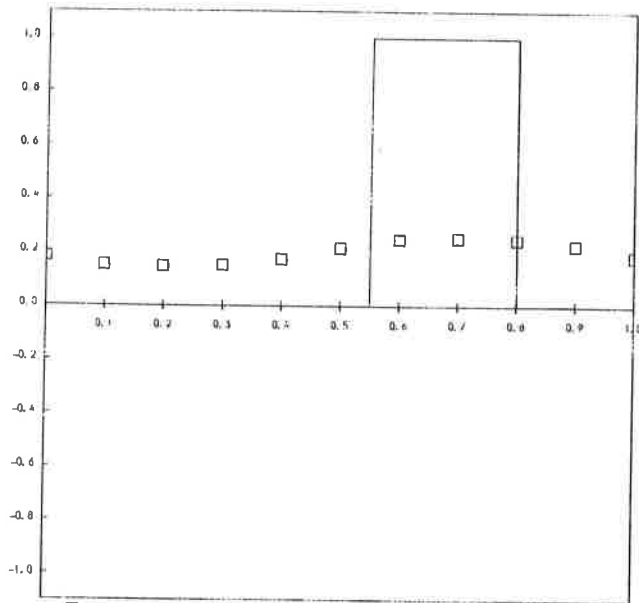


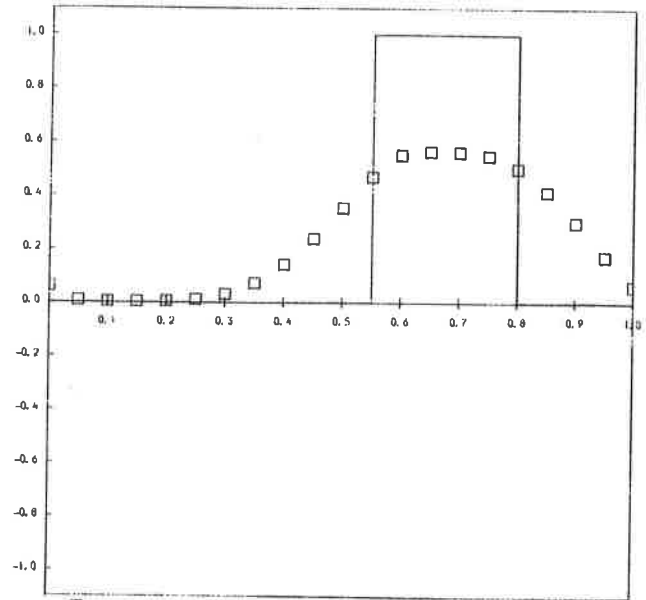
FIGURE 8



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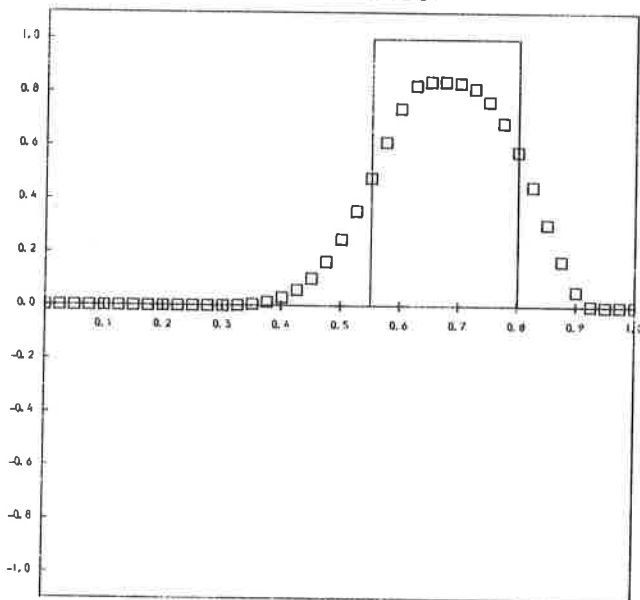
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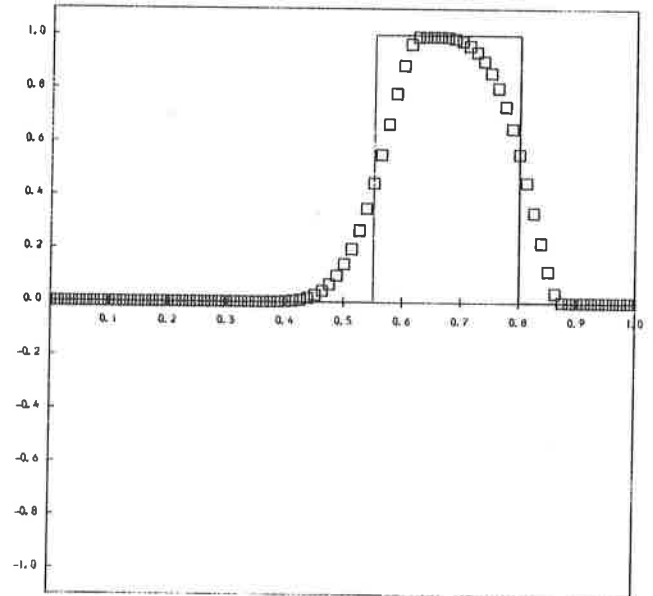
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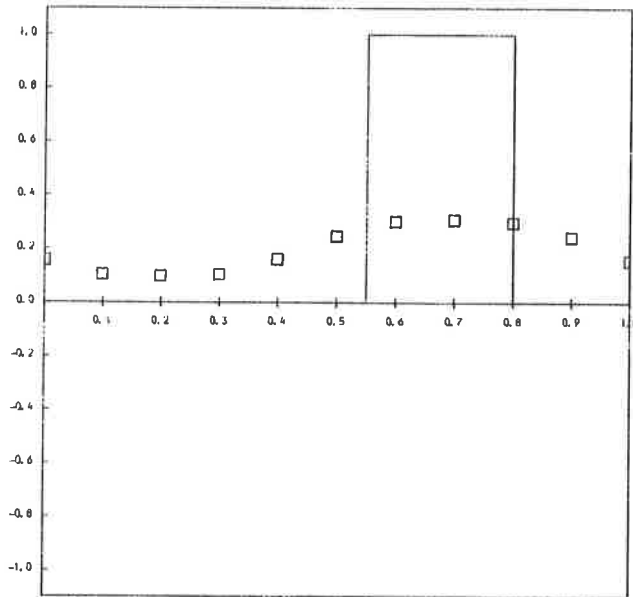
ENGQUIST-OSHER SCHEME

FROMM BASED LIMITER

LINEAR ADVECTION EQUATION

SQUARE WAVE DATA

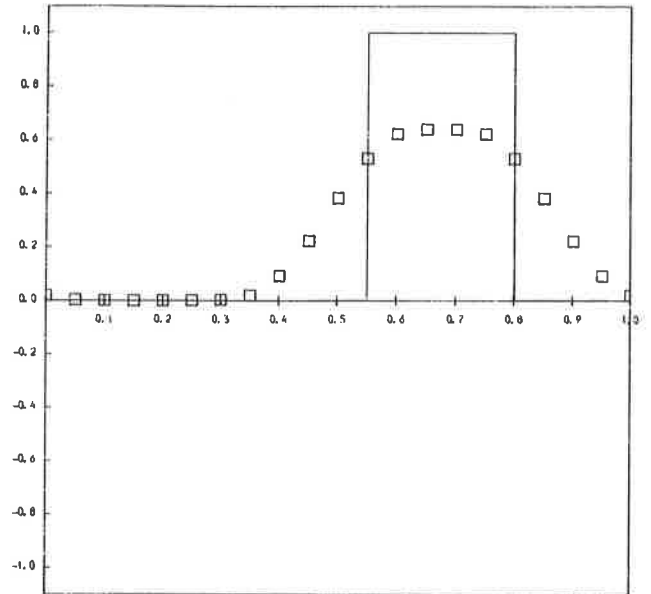
PERIODIC BOUNDARY CONDITIONS



T= 6.3000 (126 STEPS)

$\Delta x = 0.1000$

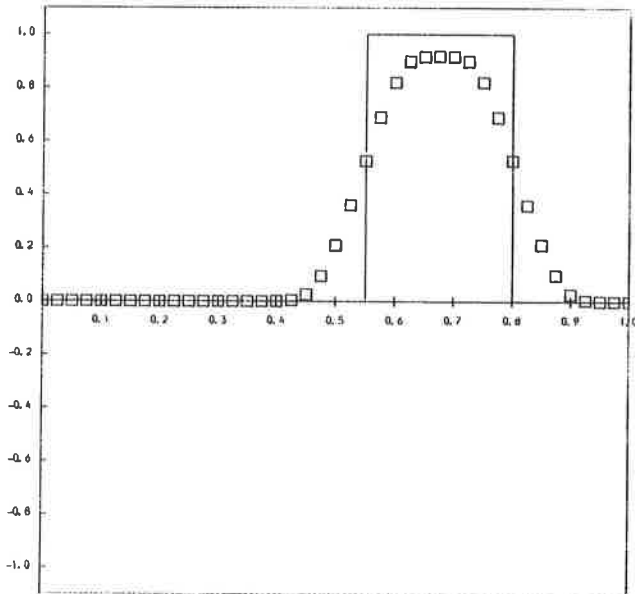
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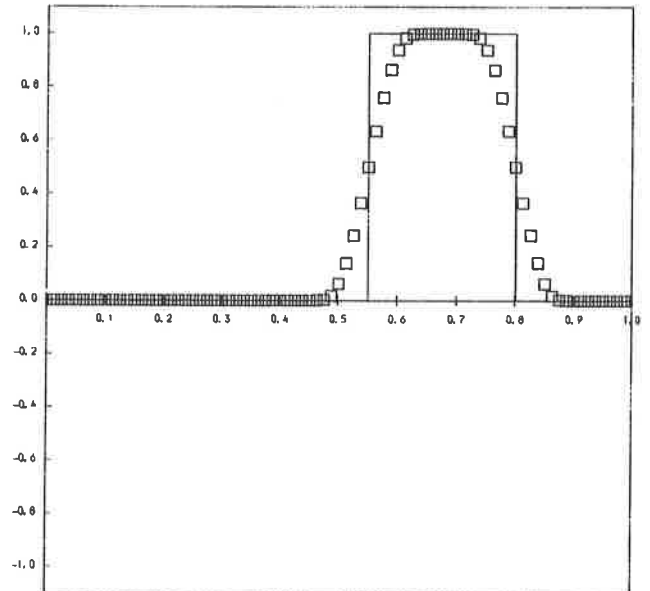
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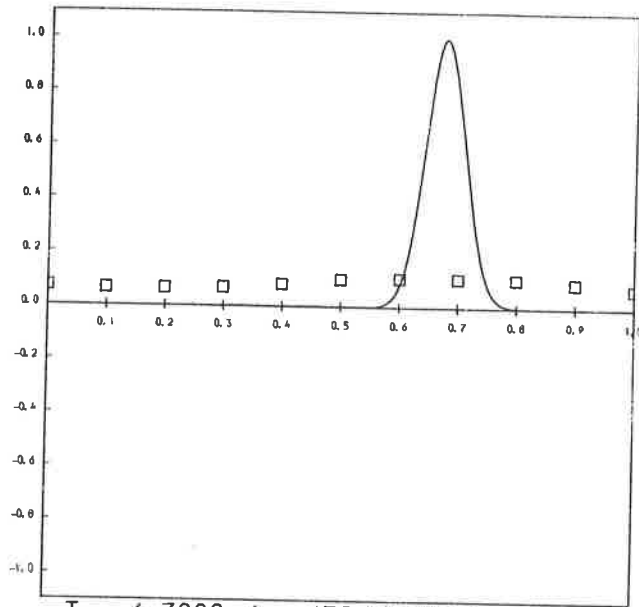
ENGQUIST-OSHER SCHEME

FROMM BASED LIMITER

LINEAR ADVECTION EQUATION

SQUARE WAVE DATA

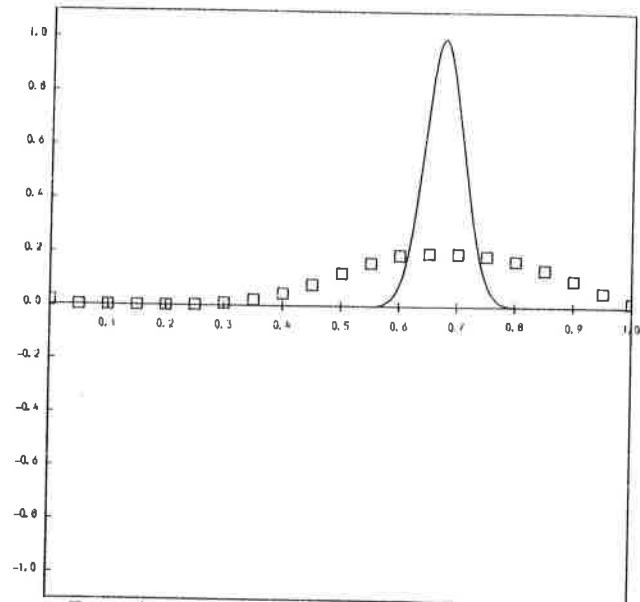
PERIODIC BOUNDARY CONDITIONS



T= 6.3000 (630 STEPS)

$\Delta x = 0.1000$

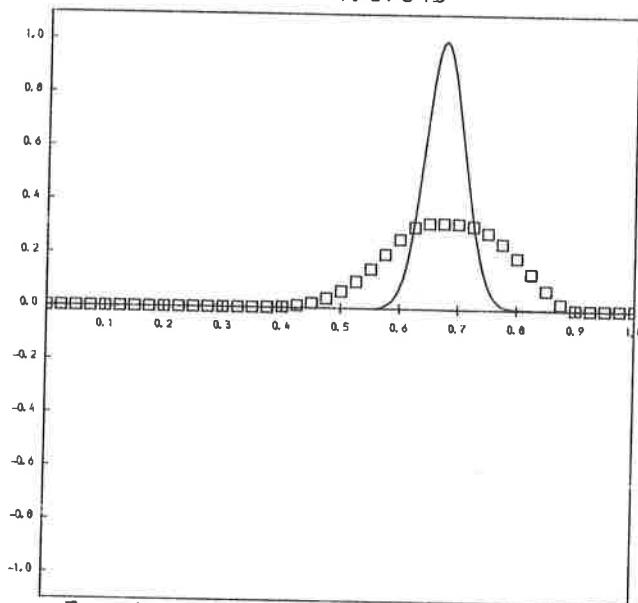
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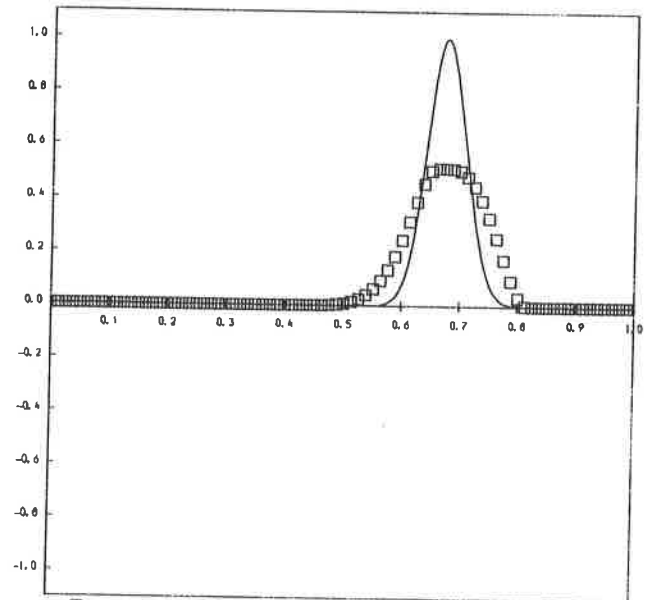
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T= 6.3000 (5040 STEPS)

$\Delta x = 0.0125$

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MESH RATIO= 0.10

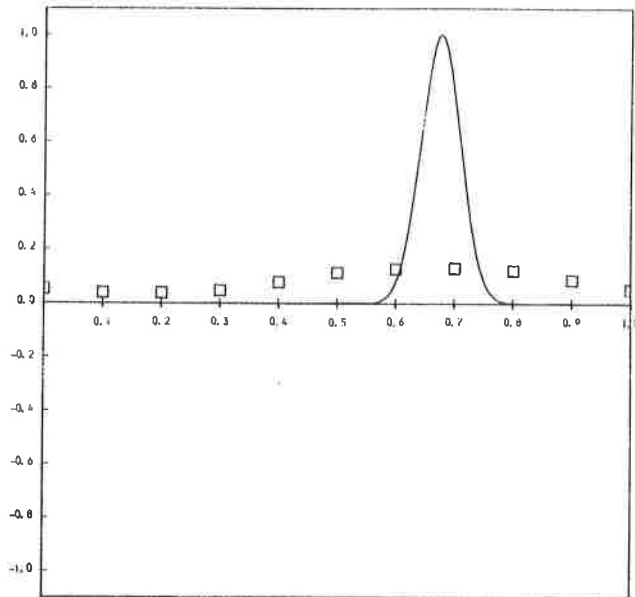
ENGQUIST-OSHER SCHEME

FROMM BASED LIMITER

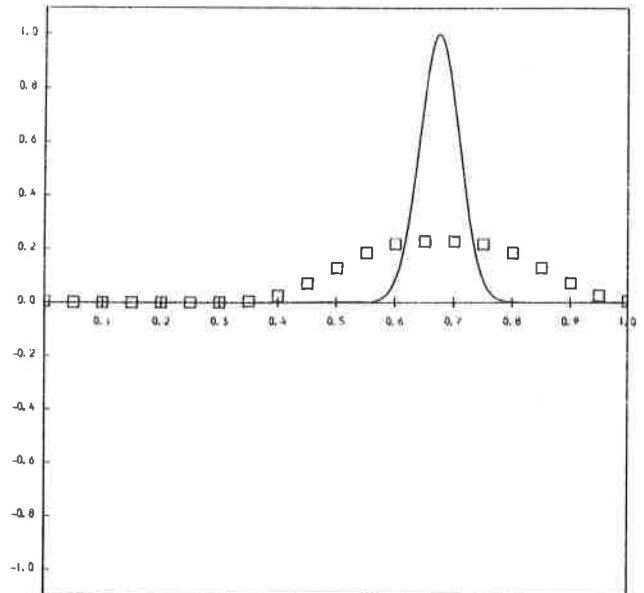
LINEAR ADVECTION EQUATION

GAUSSIAN DATA

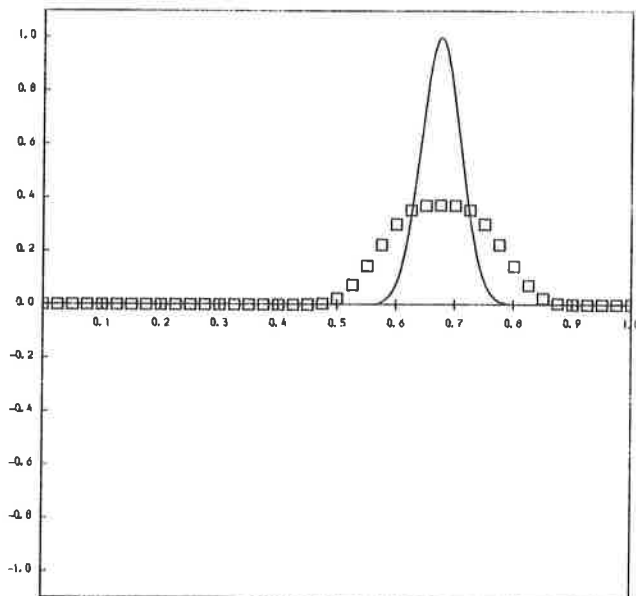
PERIODIC BOUNDARY CONDITIONS



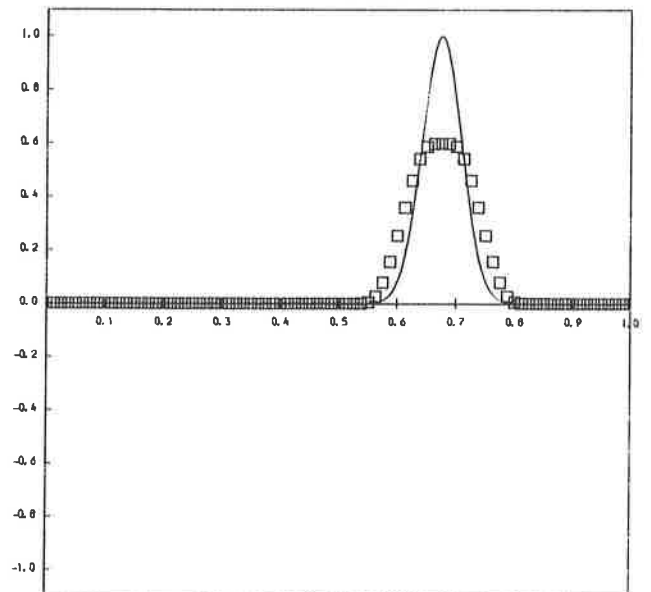
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 GLOBAL ERROR = 1.05080



T= 6.3000 (504 STEPS)
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T= 6.3000 (1008 STEPS)
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MESH RATIO= 0.50

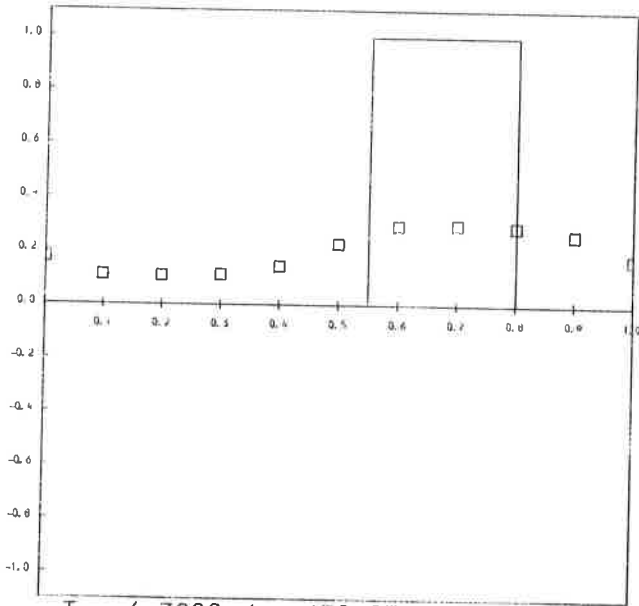
ENGQUIST-OSHER SCHEME

FROMM BASED LIMITER

LINEAR ADVECTION EQUATION

GAUSSIAN DATA

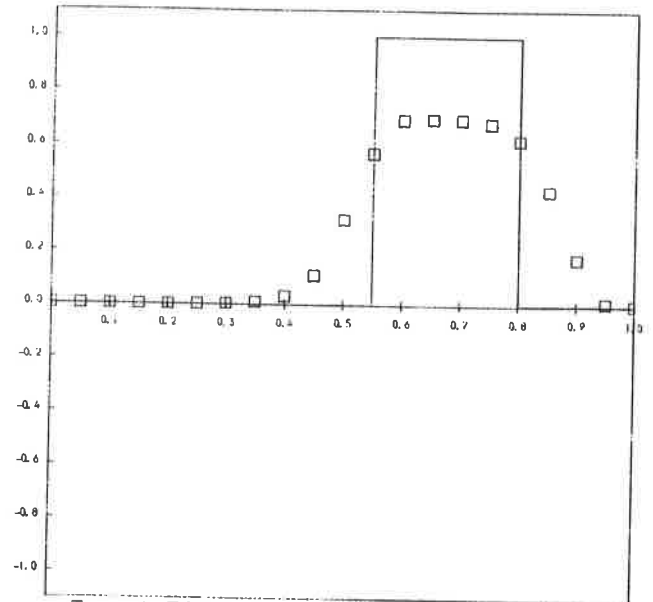
PERIODIC BOUNDARY CONDITIONS



T= 6.3000 (630 STEPS)

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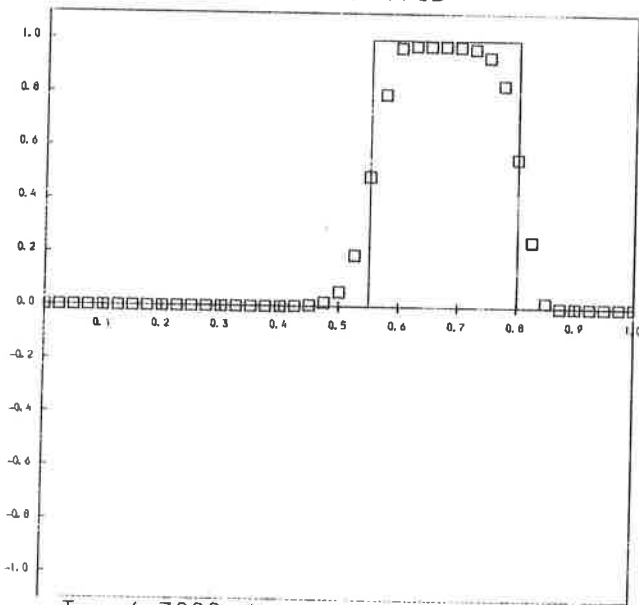
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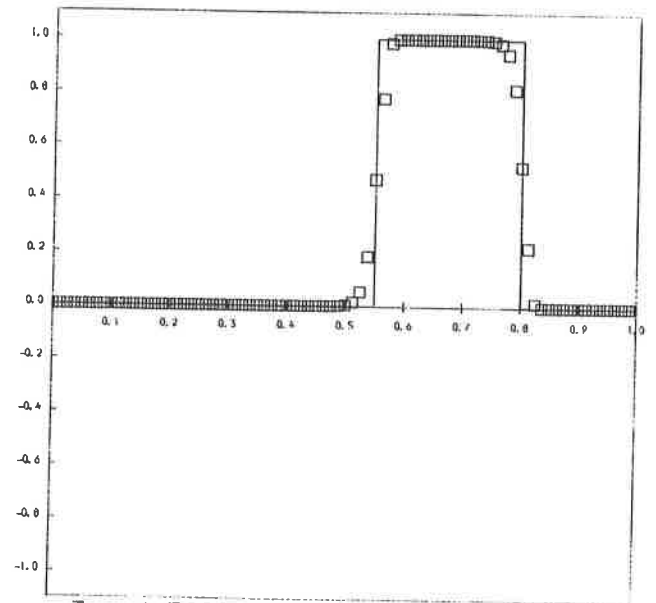
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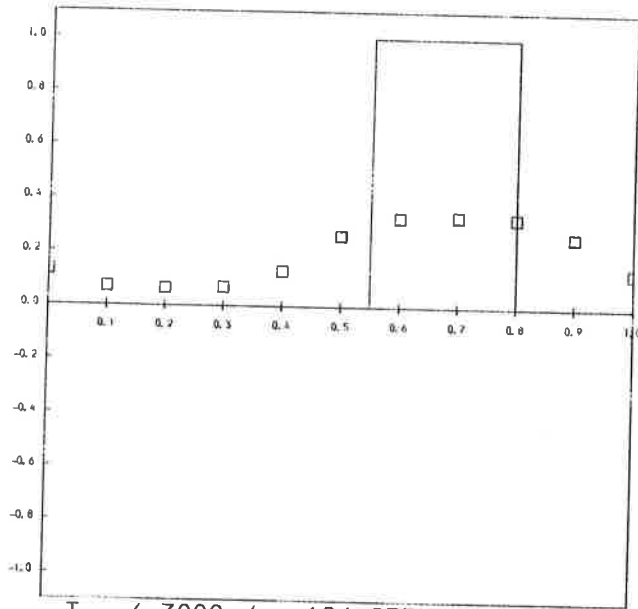
ENGQUIST-OSHER SCHEME

SUPERBEE LIMITER

LINEAR ADVECTION EQUATION

SQUARE WAVE DATA

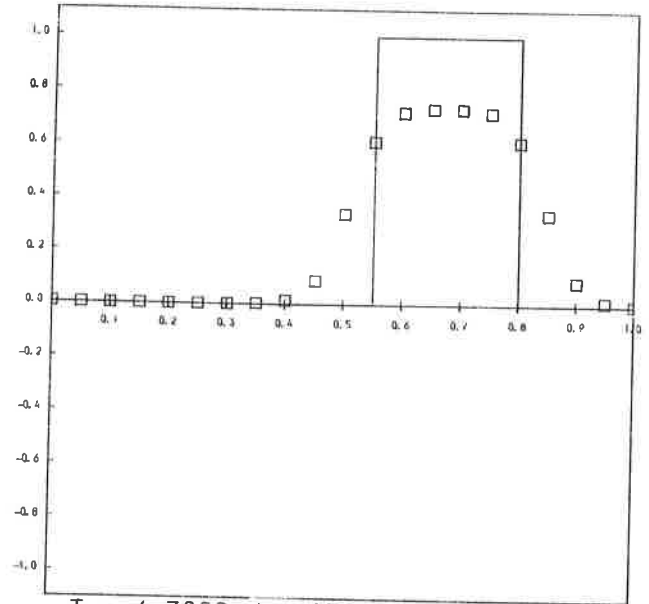
PERIODIC BOUNDARY CONDITIONS



T= 6.3000 (126 STEPS)

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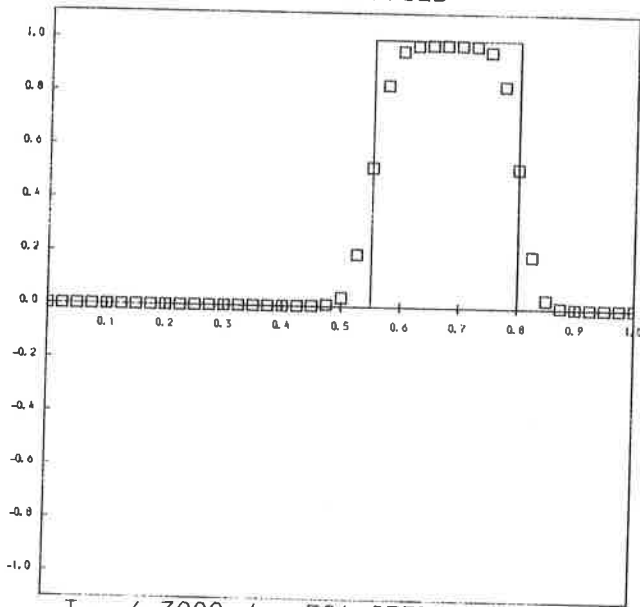
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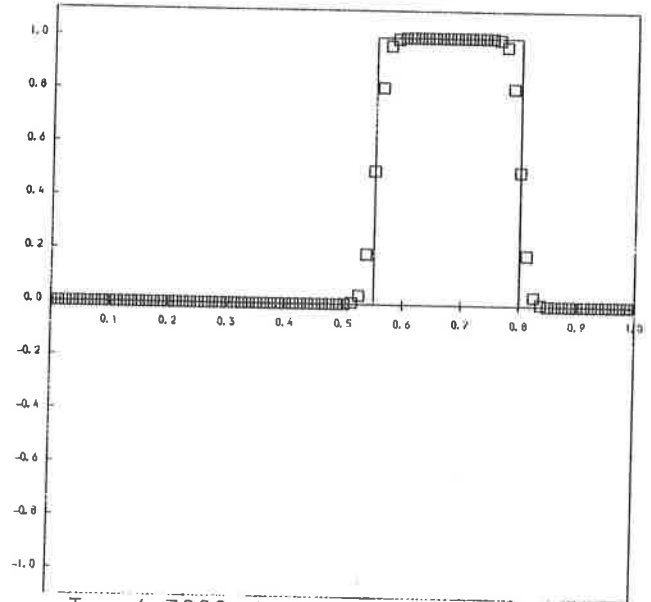
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GLOBAL ERROR = 0.10302



T= 6.3000 (1008 STEPS)

$\Delta x = 0.0125$

GLOBAL ERROR = 0.04544

MESH RATIO= 0.50

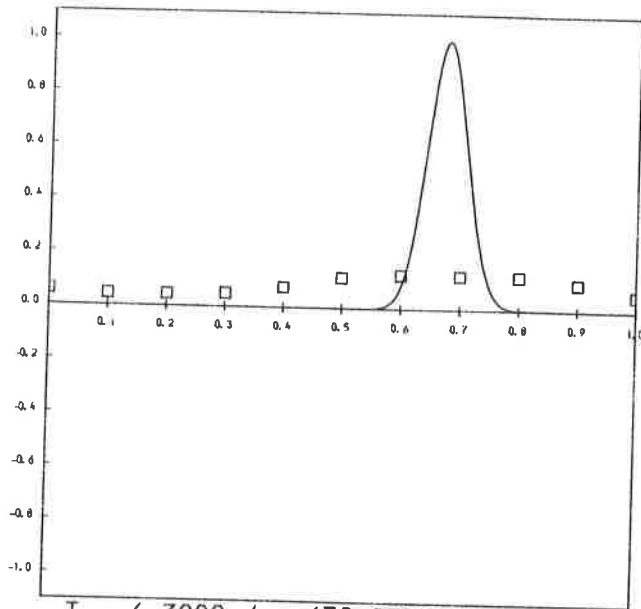
ENGQUIST-OSHER SCHEME

SUPERBEE LIMITER

LINEAR ADVECTION EQUATION

SQUARE WAVE DATA

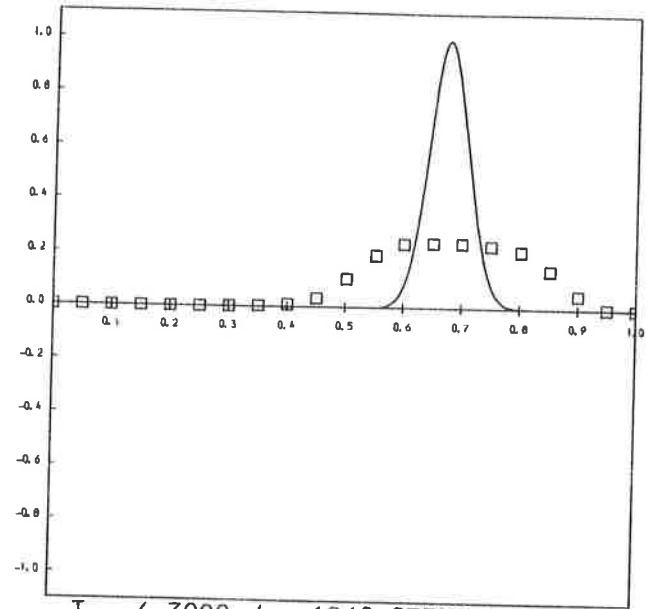
PERIODIC BOUNDARY CONDITIONS



T= 6.3000 (630 STEPS)

$\Delta x = 0.1000$

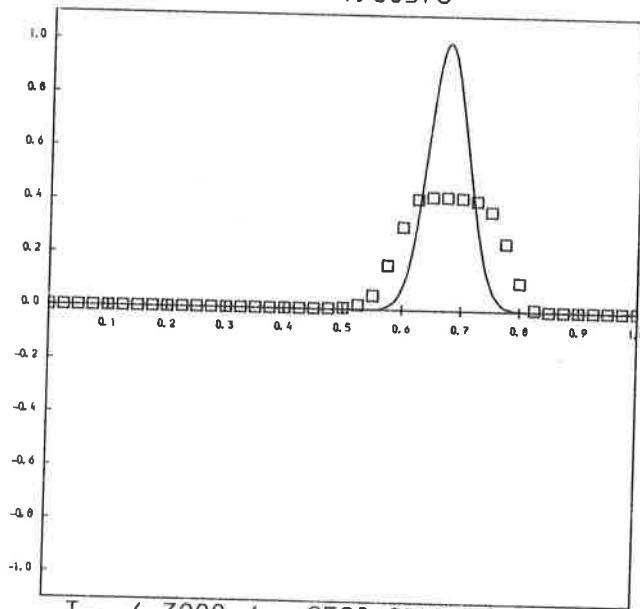
GLOBAL ERROR = 1.30578



T= 6.3000 (1260 STEPS)

$\Delta x = 0.0500$

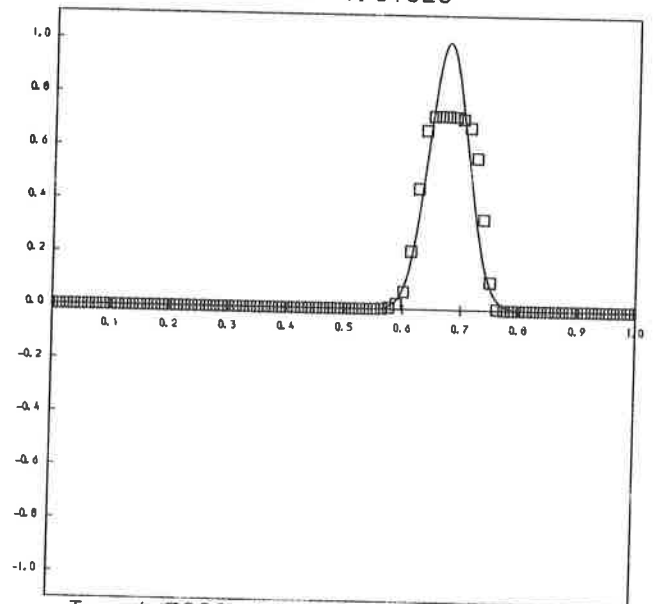
GLOBAL ERROR = 1.01323



T= 6.3000 (2520 STEPS)

$\Delta x = 0.0250$

GLOBAL ERROR = 0.67497



T= 6.3000 (5040 STEPS)

$\Delta x = 0.0125$

GLOBAL ERROR = 0.24389

MESH RATIO= 0.10

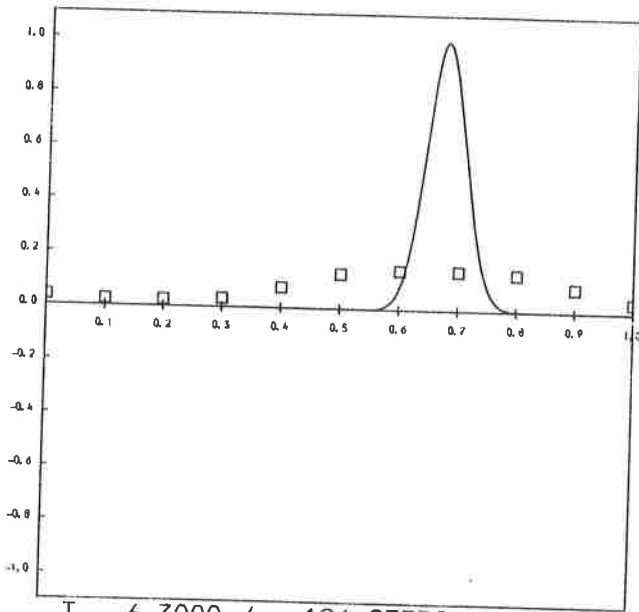
ENGQUIST-OSHER SCHEME

SUPERBEE LIMITER

LINEAR ADVECTION EQUATION

GAUSSIAN DATA

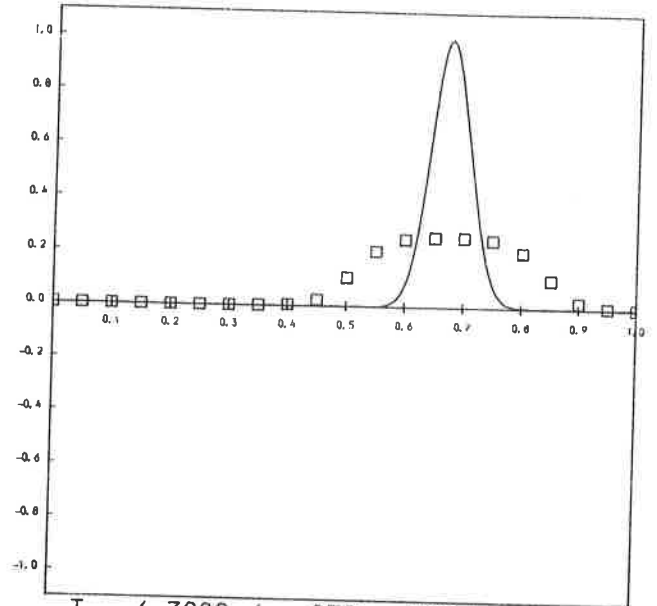
PERIODIC BOUNDARY CONDITIONS



T= 6.3000 (126 STEPS)

$\Delta x = 0.1000$

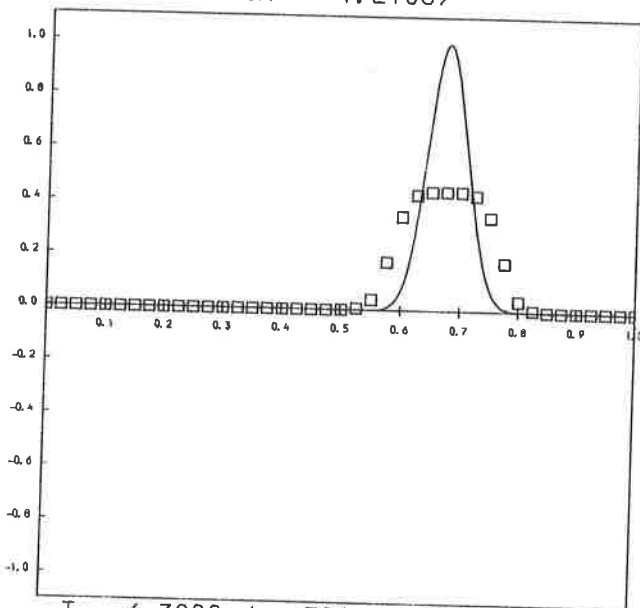
GLOBAL ERROR = 1.21069



T= 6.3000 (252 STEPS)

$\Delta x = 0.0500$

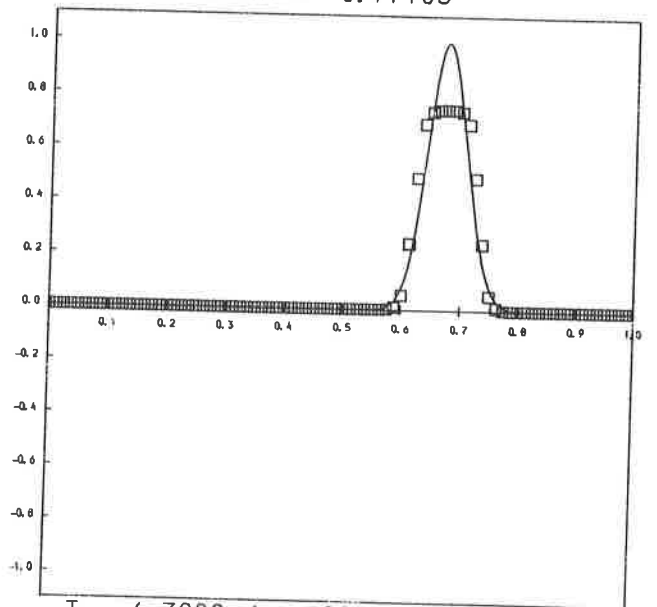
GLOBAL ERROR = 0.97103



T= 6.3000 (504 STEPS)

$\Delta x = 0.0250$

GLOBAL ERROR = 0.63107

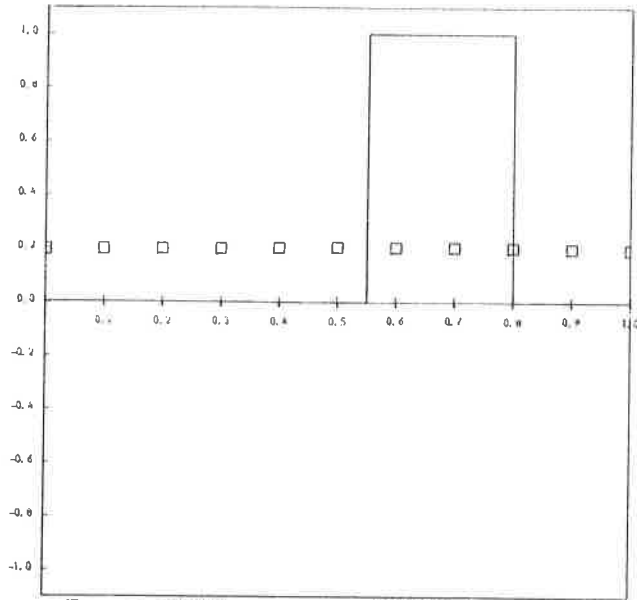


T= 6.3000 (1008 STEPS)

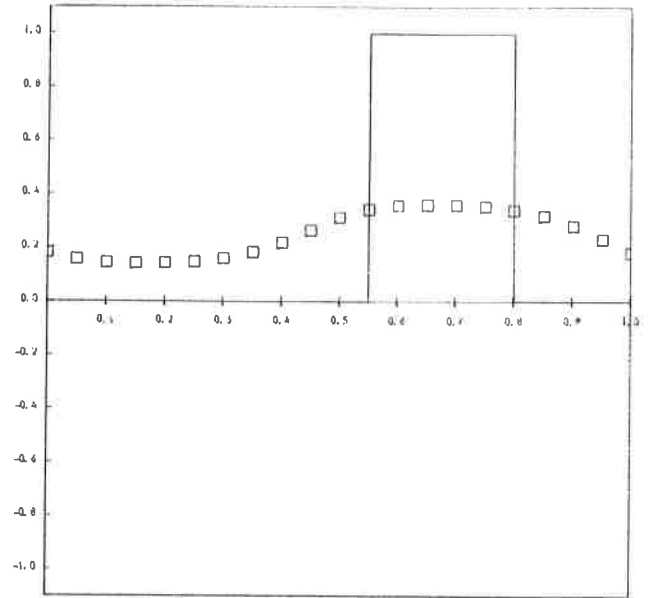
$\Delta x = 0.0125$

GLOBAL ERROR = 0.22309

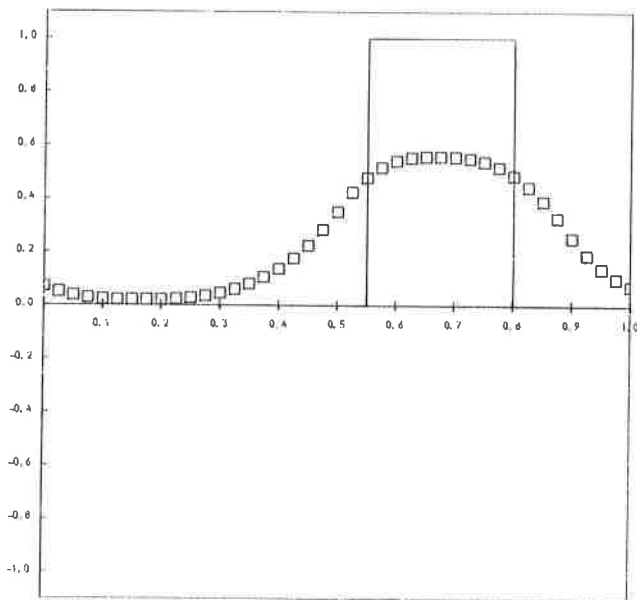
MESH RATIO= 0.50
 ENGQUIST-OSHER SCHEME
 SUPERBEE LIMITER
 LINEAR ADVECTION EQUATION
 GAUSSIAN DATA
 PERIODIC BOUNDARY CONDITIONS



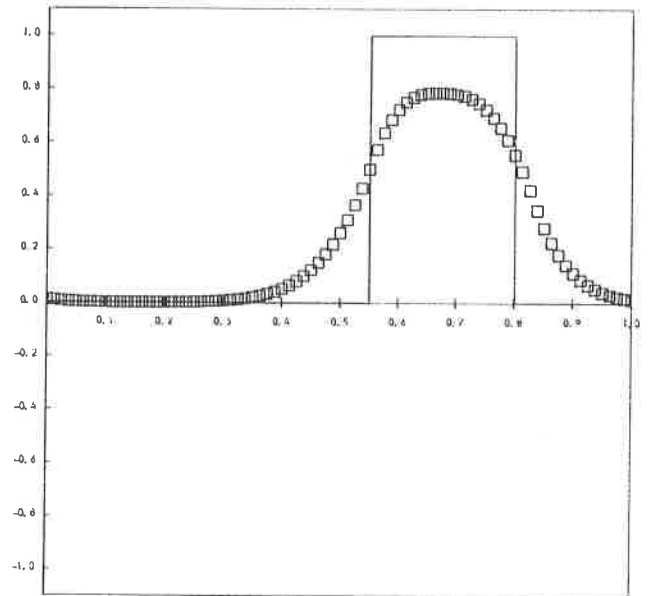
T= 6.3000 (630 STEPS)
 $\Delta x = 0.1000$
 GLOBAL ERROR = 1.38748



T= 6.3000 (1260 STEPS)
 $\Delta x = 0.0500$
 GLOBAL ERROR = 1.15201



T= 6.3000 (2520 STEPS)
 $\Delta x = 0.0250$
 GLOBAL ERROR = 0.82756



T= 6.3000 (5040 STEPS)
 $\Delta x = 0.0125$
 GLOBAL ERROR = 0.51720

MESH RATIO= 0.10

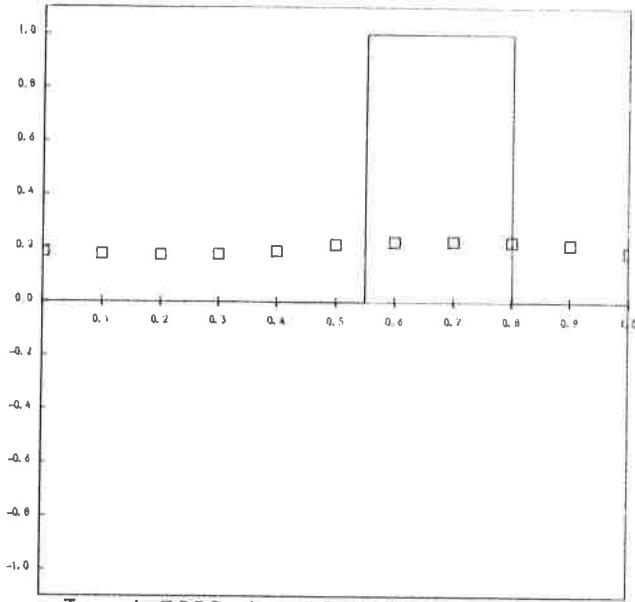
ENGQUIST-OSHER SCHEME

MINMOD LIMITER

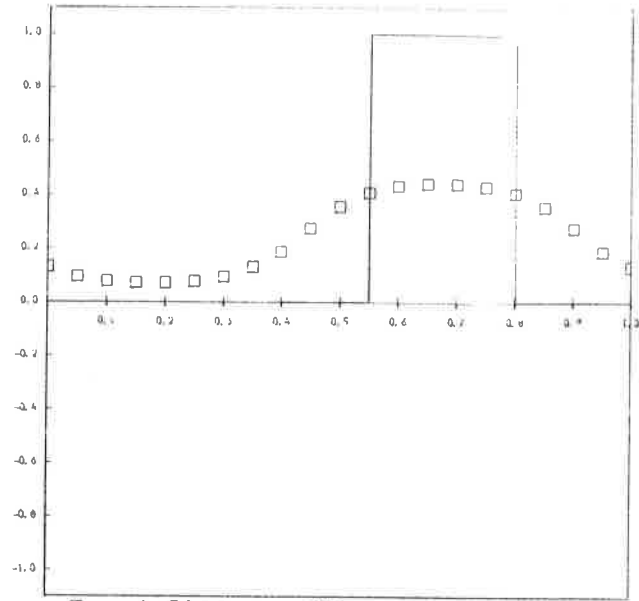
LINEAR ADVECTION EQUATION

SQUARE WAVE DATA

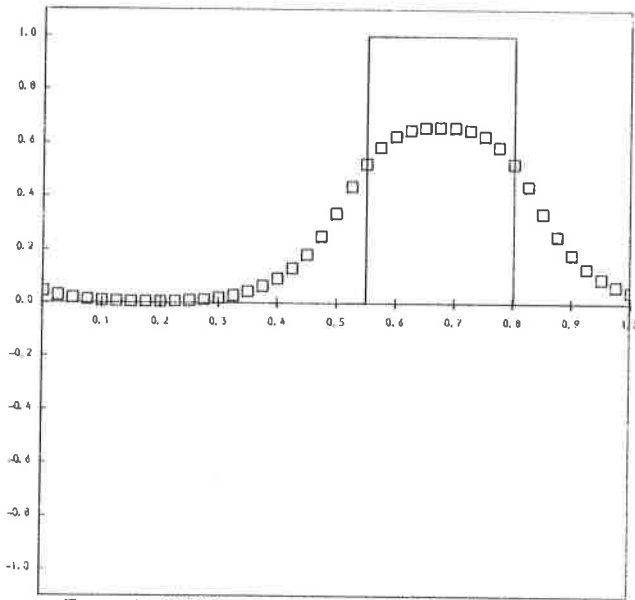
PERIODIC BOUNDARY CONDITIONS



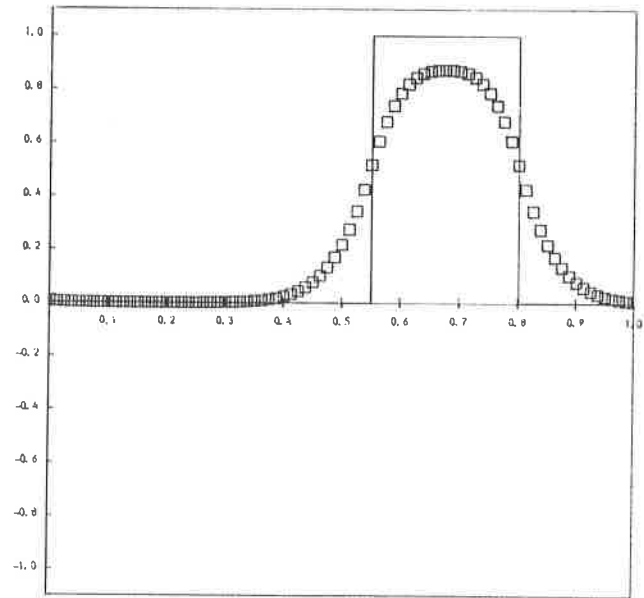
T= 6.3000 (126 STEPS)
 $\Delta x = 0.1000$
 GLOBAL ERROR = 1.32022



T= 6.3000 (252 STEPS)
 $\Delta x = 0.0500$
 GLOBAL ERROR = 0.96768



T= 6.3000 (504 STEPS)
 $\Delta x = 0.0250$
 GLOBAL ERROR = 0.66103



T= 6.3000 (1008 STEPS)
 $\Delta x = 0.0125$
 GLOBAL ERROR = 0.39693

MESH RATIO= 0.50

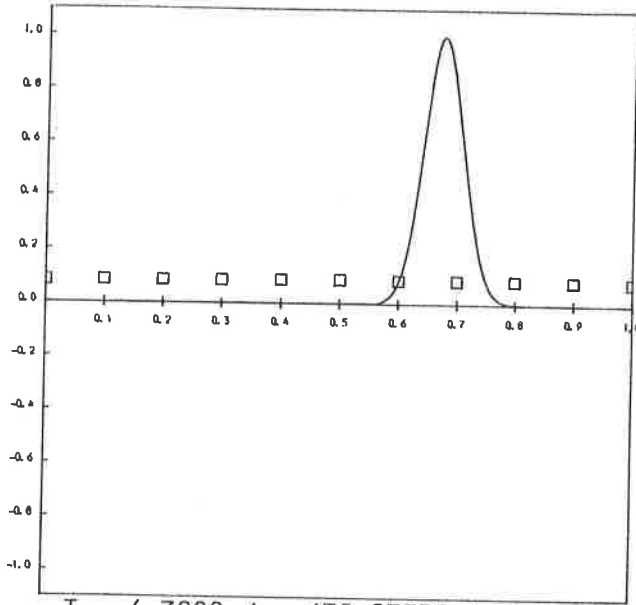
ENGQUIST-OSHER SCHEME

MINMOD LIMITER

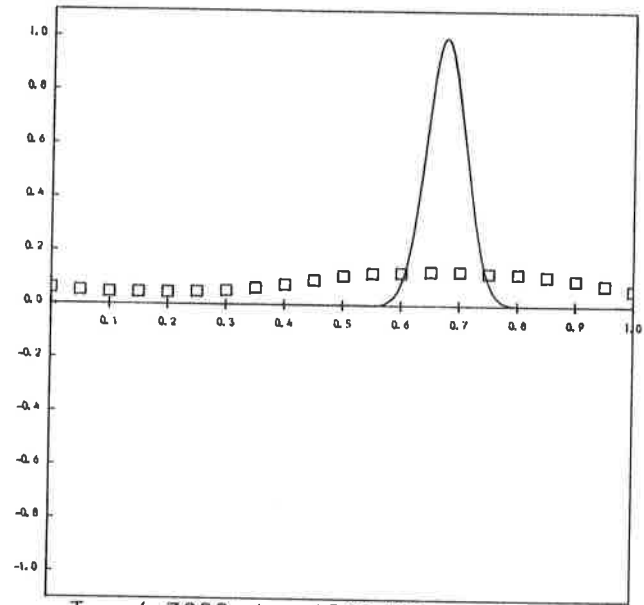
LINEAR ADVECTION EQUATION

SQUARE WAVE DATA

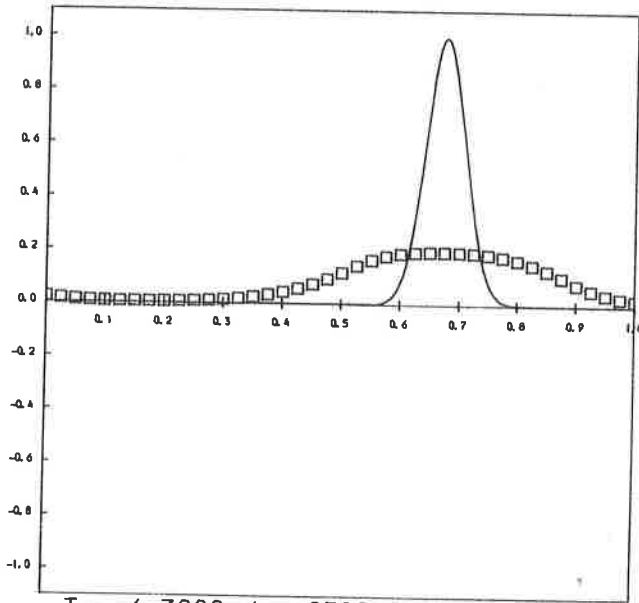
PERIODIC BOUNDARY CONDITIONS



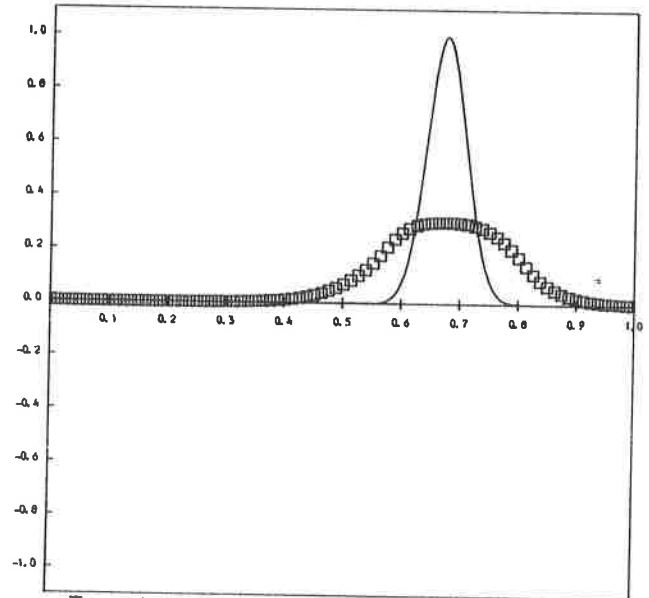
T= 6.3000 (630 STEPS)
 $\Delta x = 0.1000$
 GLOBAL ERROR = 1.49123



T= 6.3000 (1260 STEPS)
 $\Delta x = 0.0500$
 GLOBAL ERROR = 1.40464

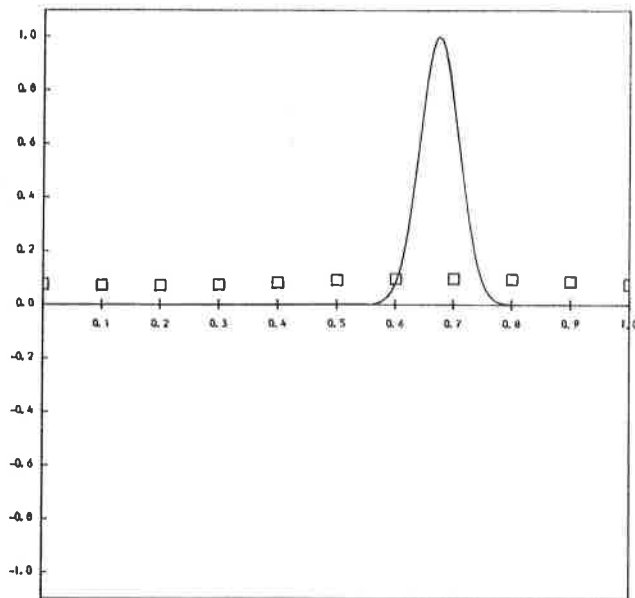


T= 6.3000 (2520 STEPS)
 $\Delta x = 0.0250$
 GLOBAL ERROR = 1.27053

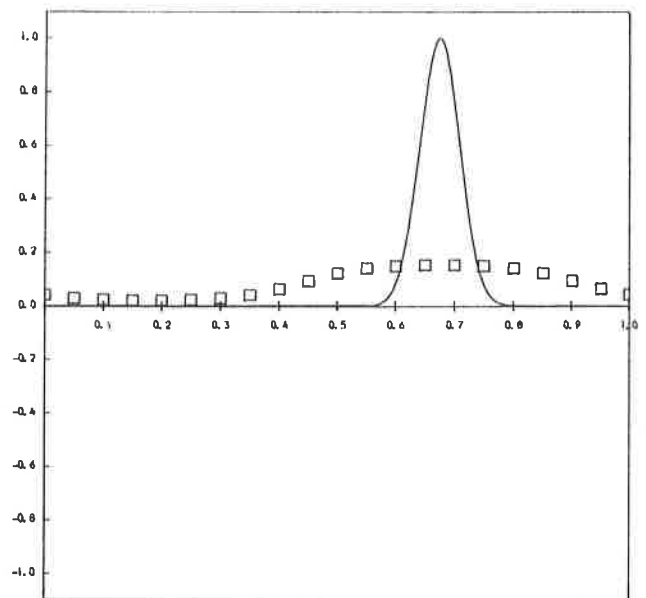


T= 6.3000 (5040 STEPS)
 $\Delta x = 0.0125$
 GLOBAL ERROR = 1.00019

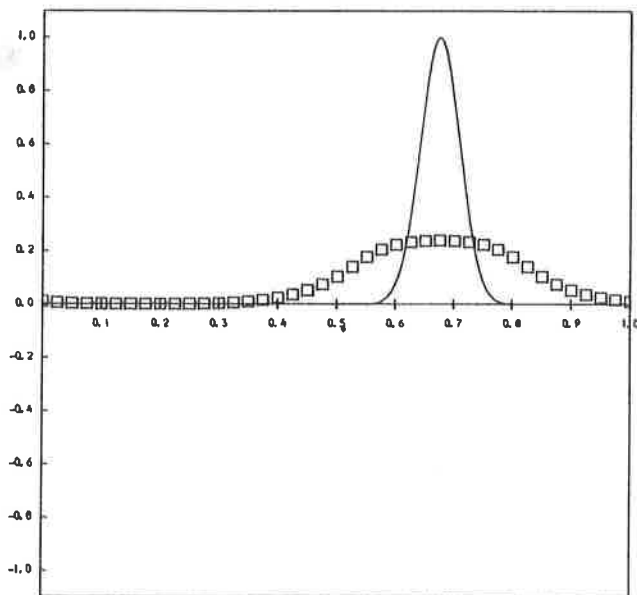
MESH RATIO= 0.10
 ENGQUIST-OSHER SCHEME
 MINMOD LIMITER
 LINEAR ADVECTION EQUATION
 GAUSSIAN DATA
 PERIODIC BOUNDARY CONDITIONS



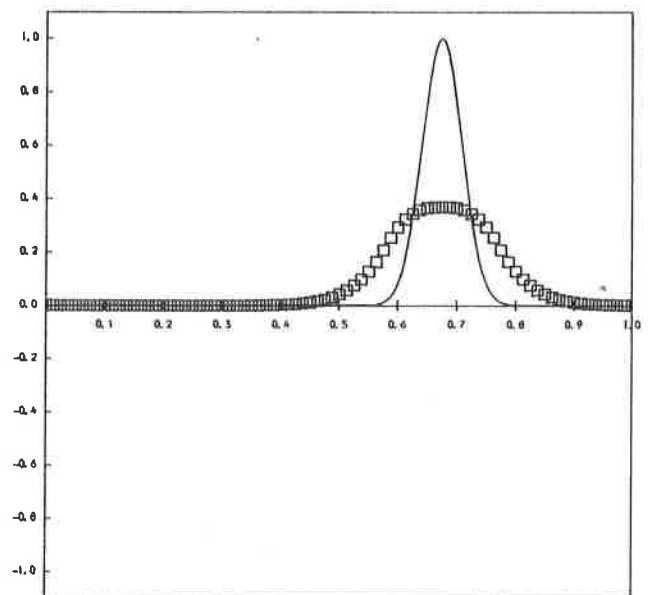
T= 6.3000 (126 STEPS)
 $\Delta x = 0.1000$
 GLOBAL ERROR = 1.44297



T= 6.3000 (252 STEPS)
 $\Delta x = 0.0500$
 GLOBAL ERROR = 1.26656



T= 6.3000 (504 STEPS)
 $\Delta x = 0.0250$
 GLOBAL ERROR = 1.14678



T= 6.3000 (1008 STEPS)
 $\Delta x = 0.0125$
 GLOBAL ERROR = 0.84632

MESH RATIO= 0.50

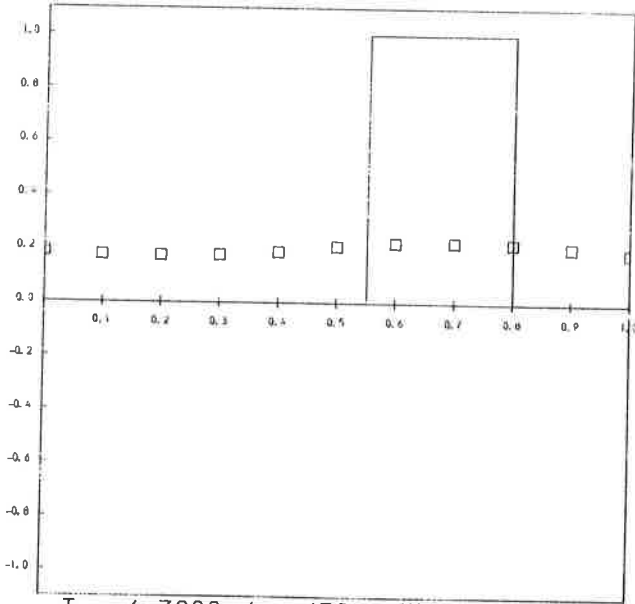
ENGQUIST-OSHER SCHEME

MINMOD LIMITER

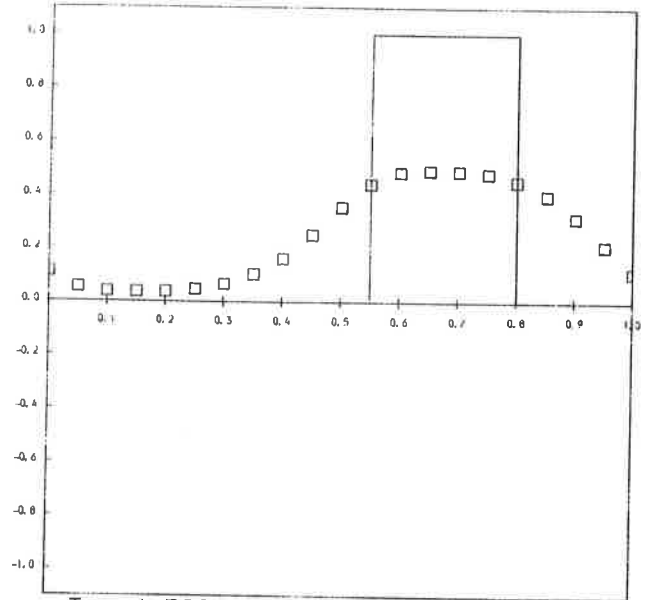
LINEAR ADVECTION EQUATION

GAUSSIAN DATA

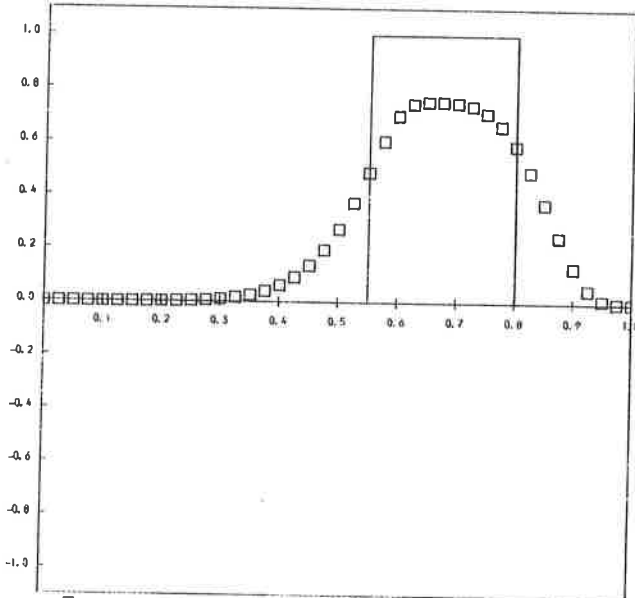
PERIODIC BOUNDARY CONDITIONS



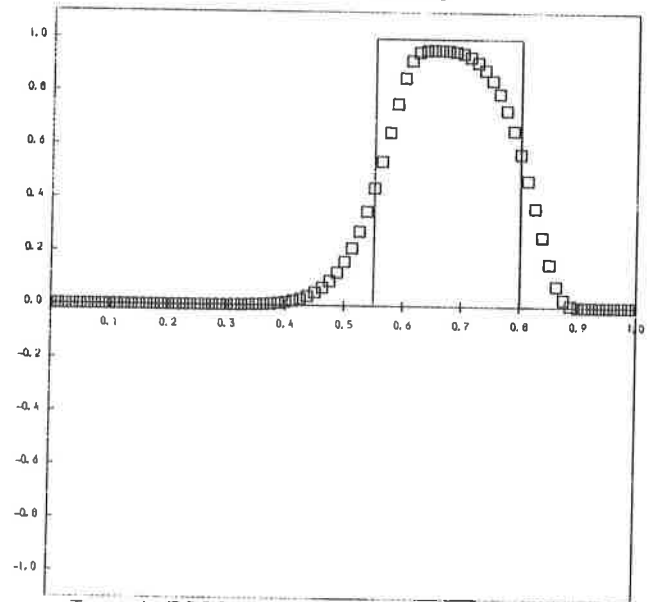
T= 6.3000 (630 STEPS)
 $\Delta x = 0.1000$
 GLOBAL ERROR = 1.32775



T= 6.3000 (1260 STEPS)
 $\Delta x = 0.0500$
 GLOBAL ERROR = 0.86858

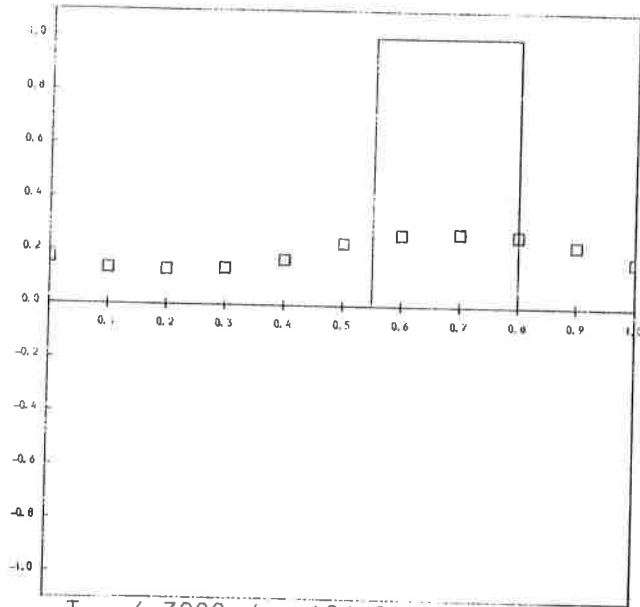


T= 6.3000 (2520 STEPS)
 $\Delta x = 0.0250$
 GLOBAL ERROR = 0.52584

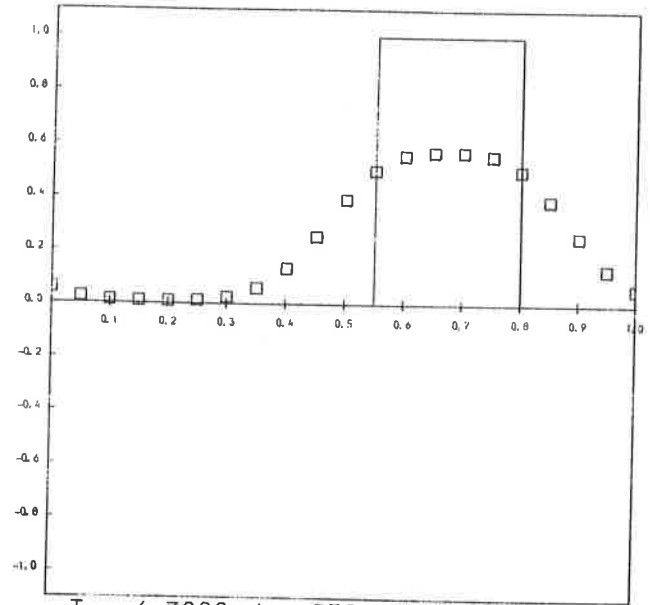


T= 6.3000 (5040 STEPS)
 $\Delta x = 0.0125$
 GLOBAL ERROR = 0.28893

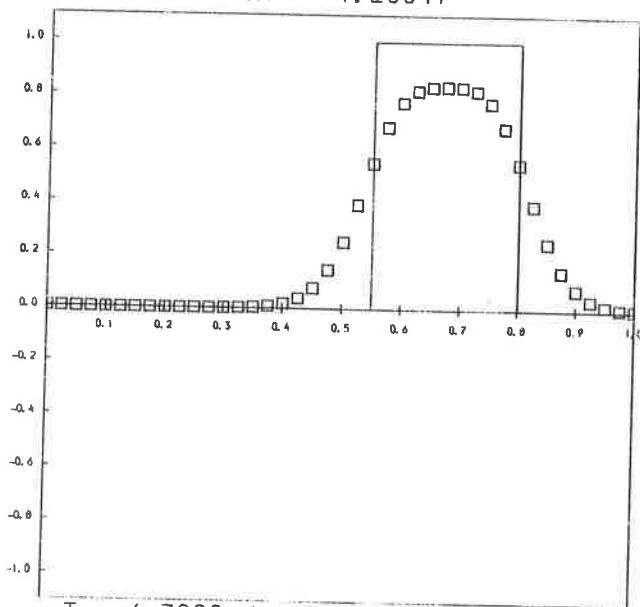
MESH RATIO= 0.10
 ENGQUIST-OSHER SCHEME
 VAN LEER LIMITER
 LINEAR ADVECTION EQUATION
 SQUARE WAVE DATA
 PERIODIC BOUNDARY CONDITIONS



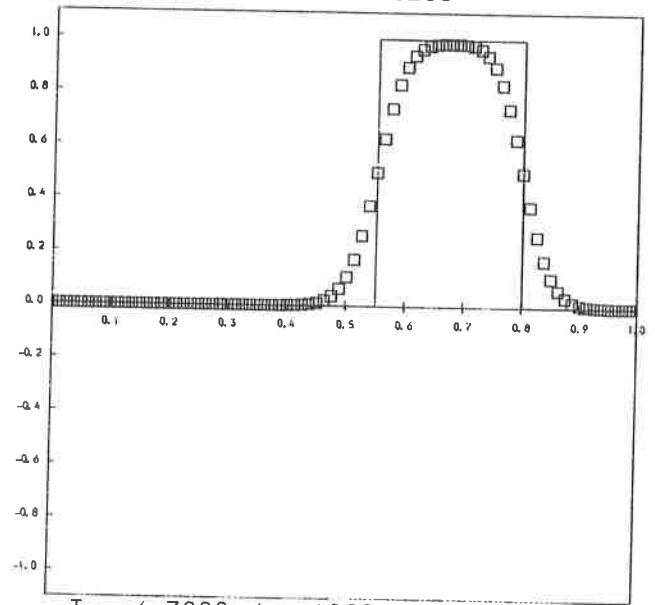
T= 6.3000 (126 STEPS)
 $\Delta x = 0.1000$
 GLOBAL ERROR = 1.20041



T= 6.3000 (252 STEPS)
 $\Delta x = 0.0500$
 GLOBAL ERROR = 0.70283

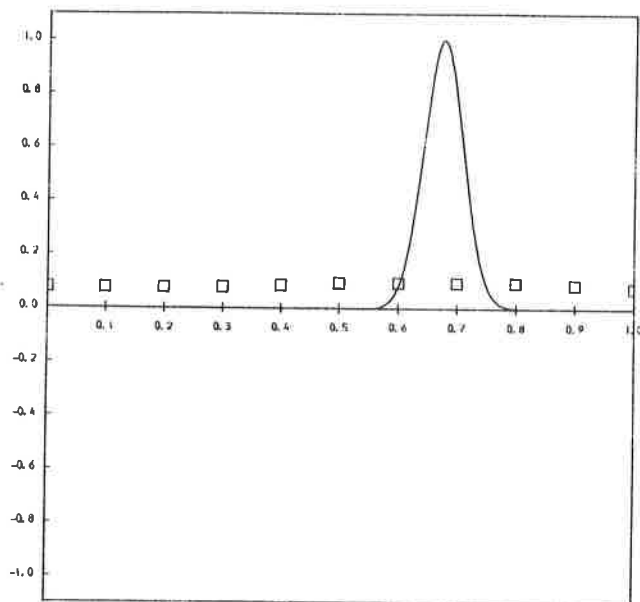


T= 6.3000 (504 STEPS)
 $\Delta x = 0.0250$
 GLOBAL ERROR = 0.39262

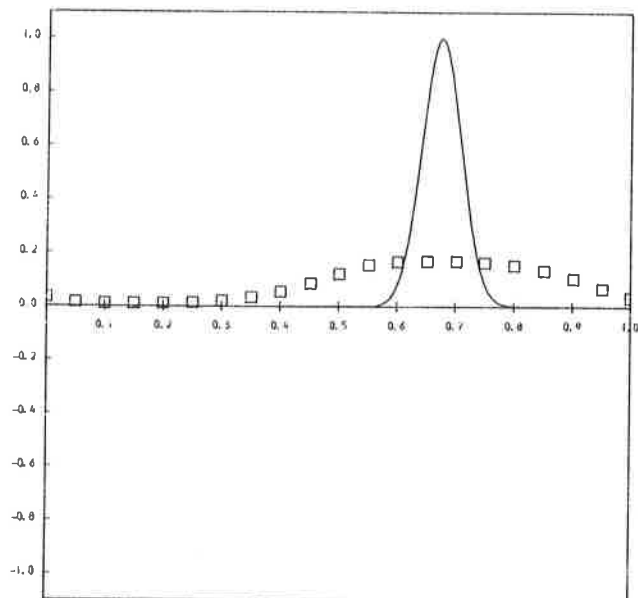


T= 6.3000 (1008 STEPS)
 $\Delta x = 0.0125$
 GLOBAL ERROR = 0.21280

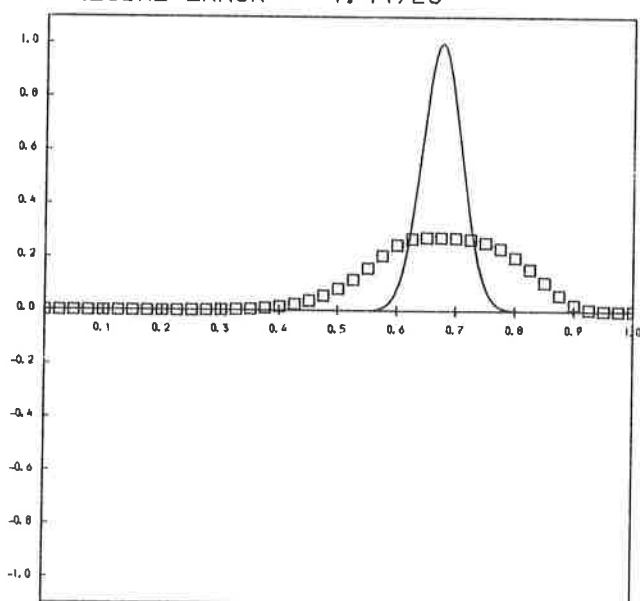
MESH RATIO= 0.50
 ENGQUIST-OSHER SCHEME
 VAN LEER LIMITER
 LINEAR ADVECTION EQUATION
 SQUARE WAVE DATA
 PERIODIC BOUNDARY CONDITIONS



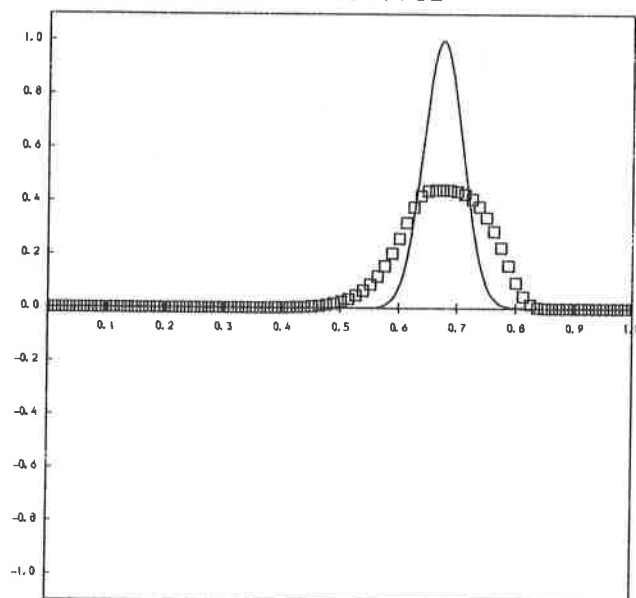
T= 6.3000 (630 STEPS)
 $\Delta x = 0.1000$
 GLOBAL ERROR = 1.44923



T= 6.3000 (1260 STEPS)
 $\Delta x = 0.0500$
 GLOBAL ERROR = 1.19732



T= 6.3000 (2520 STEPS)
 $\Delta x = 0.0250$
 GLOBAL ERROR = 1.04443



T= 6.3000 (5040 STEPS)
 $\Delta x = 0.0125$
 GLOBAL ERROR = 0.70007

MESH RATIO= 0.10

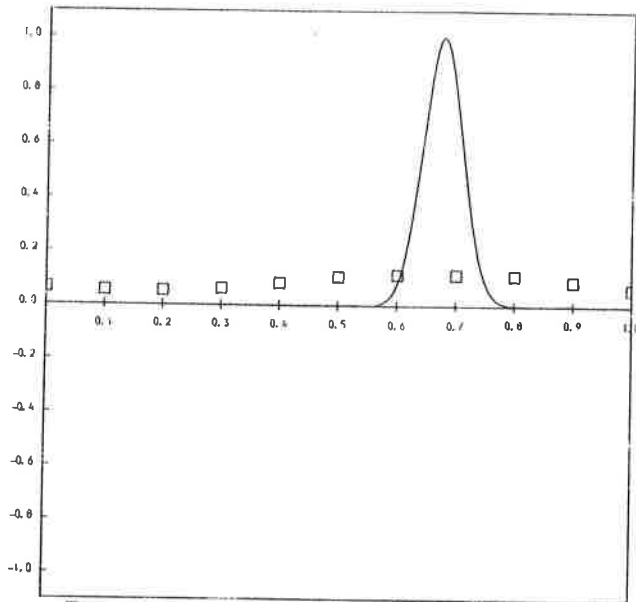
ENGQUIST-OSHER SCHEME

VAN LEER LIMITER

LINEAR ADVECTION EQUATION

GAUSSIAN DATA

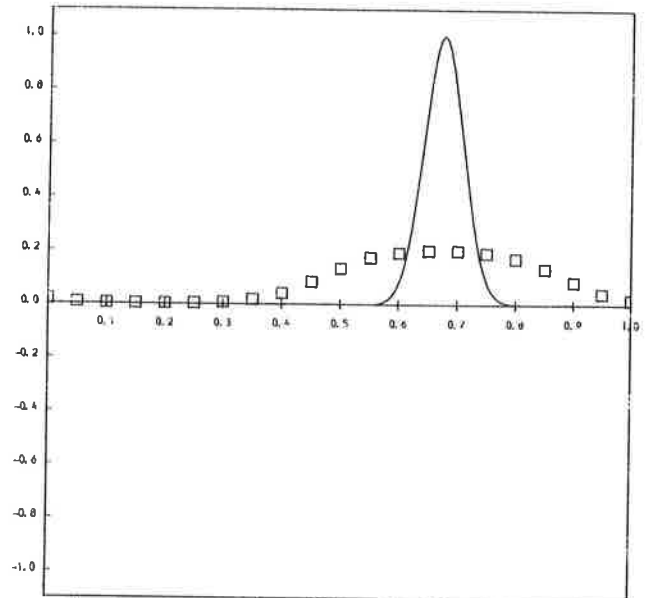
PERIODIC BOUNDARY CONDITIONS



T= 6.3000 (126 STEPS)

$\Delta x = 0.1000$

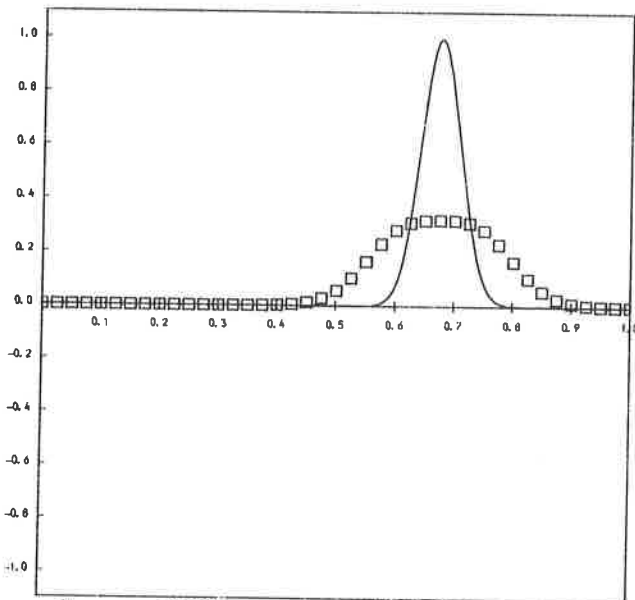
GLOBAL ERROR = 1.35277



T= 6.3000 (252 STEPS)

$\Delta x = 0.0500$

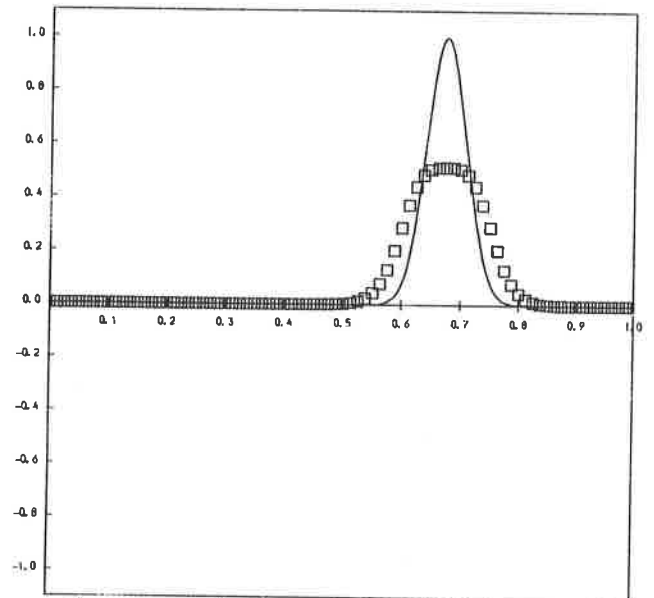
GLOBAL ERROR = 1.11166



T= 6.3000 (504 STEPS)

$\Delta x = 0.0250$

GLOBAL ERROR = 0.90850



T= 6.3000 (1008 STEPS)

$\Delta x = 0.0125$

GLOBAL ERROR = 0.55678

MESH RATIO= 0.50

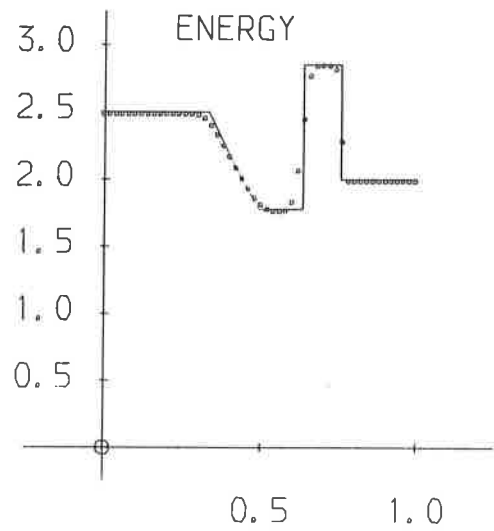
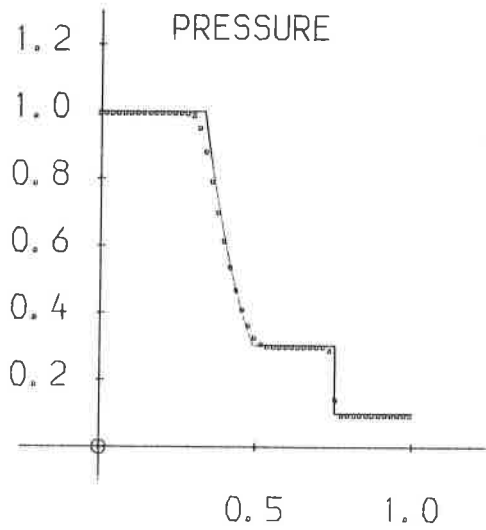
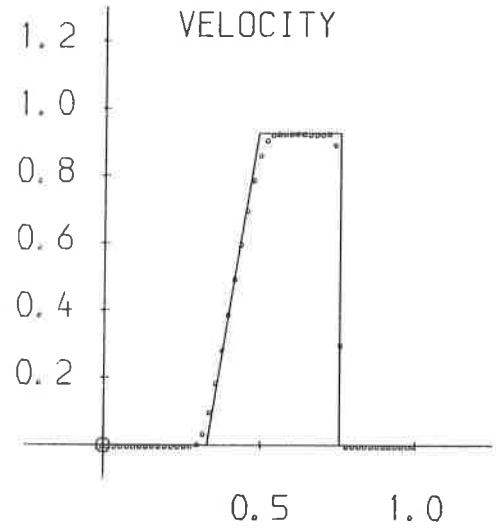
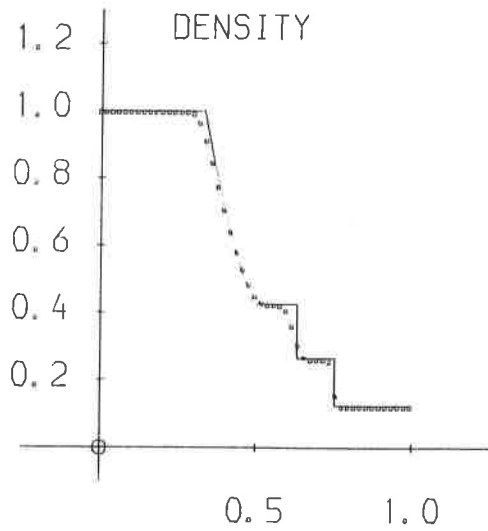
ENGQUIST-OSHER SCHEME

VAN LEER LIMITER

LINEAR ADVECTION EQUATION

GAUSSIAN DATA

PERIODIC BOUNDARY CONDITIONS



SODS SHOCKTUBE PROBLEM

IMPOSED INITIAL VELOCITY : 0.000

T= 0.144 (16 STEPS)

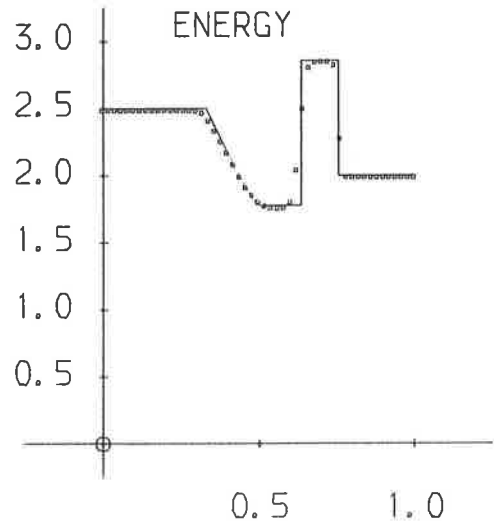
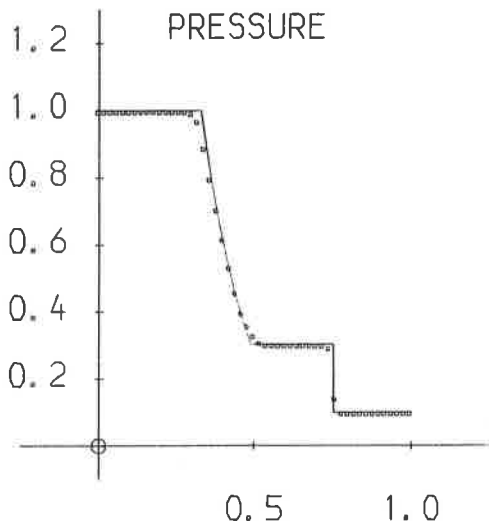
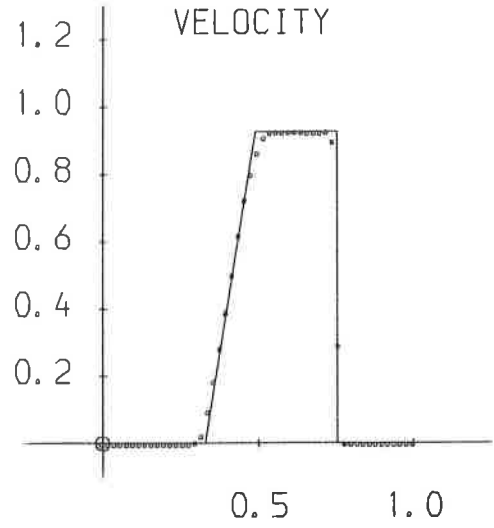
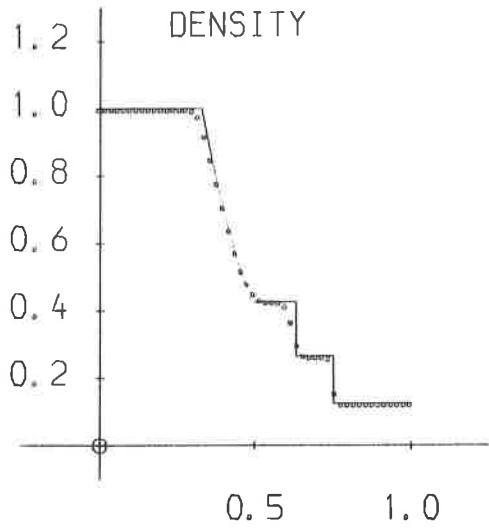
MESH RATIO = 0.450

$\Delta x = 0.020$

U-A FIELD : - FROMM BASED LIMITER

U FIELD : - FROMM BASED LIMITER

U+A FIELD : - FROMM BASED LIMITER



SODS SHOCKTUBE PROBLEM

IMPOSED INITIAL VELOCITY : 0.000

T= 0.144 (16 STEPS)

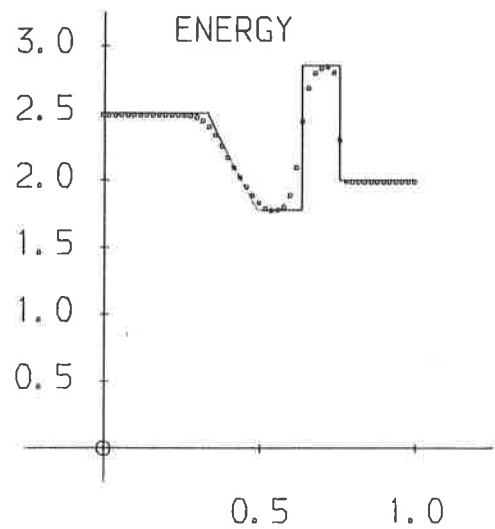
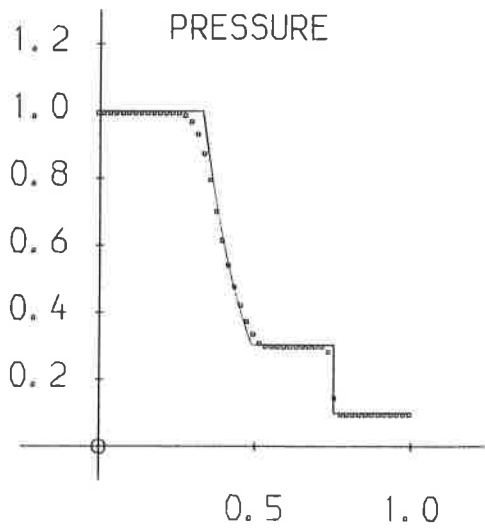
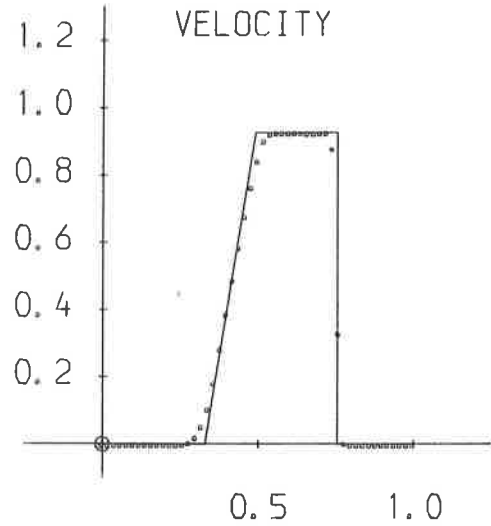
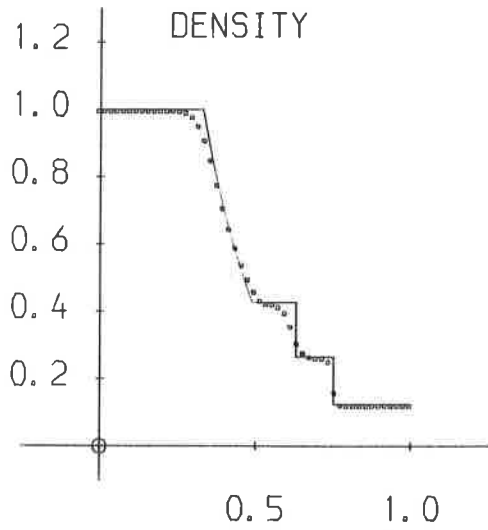
MESH RATIO = 0.450

$\Delta x = 0.020$

U-A FIELD : - SUPERBEE LIMITER

U FIELD : - SUPERBEE LIMITER

U+A FIELD : - SUPERBEE LIMITER



SODS SHOCKTUBE PROBLEM

IMPOSED INITIAL VELOCITY : 0.000

T= 0.144 (16 STEPS)

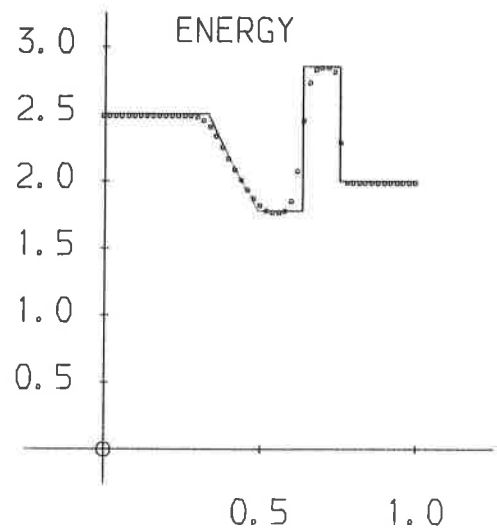
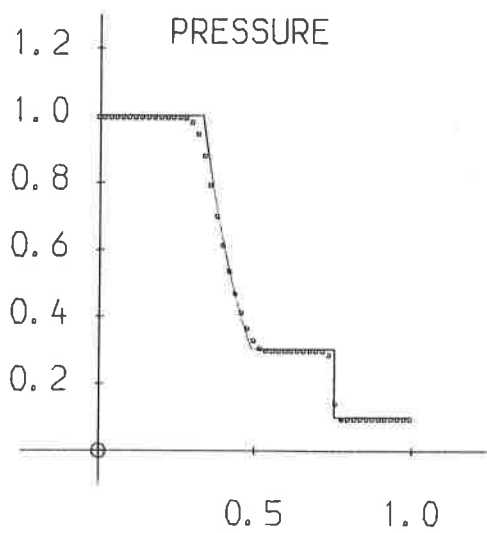
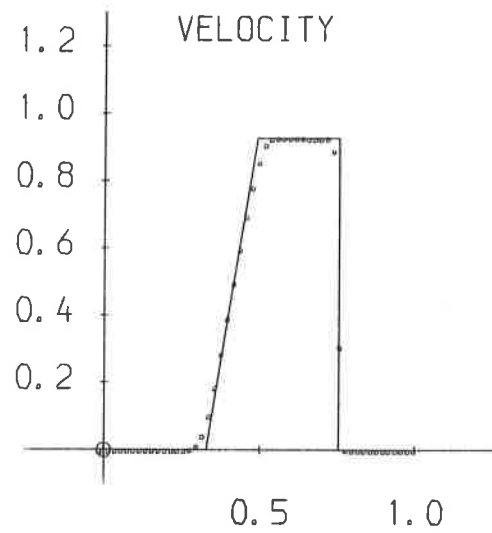
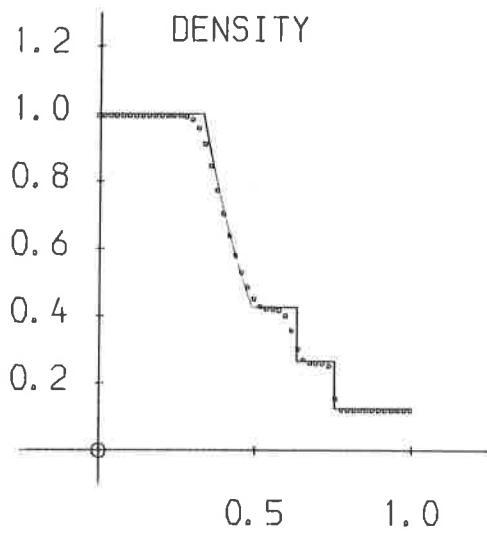
MESH RATIO = 0.450

$\Delta x = 0.020$

U-A FIELD : - MINMOD LIMITER

U FIELD : - MINMOD LIMITER

U+A FIELD : - MINMOD LIMITER



SODS SHOCKTUBE PROBLEM

IMPOSED INITIAL VELOCITY : 0.000

T= 0.144 (16 STEPS)

MESH RATIO = 0.450

$\Delta x = 0.020$

U-A FIELD : - VAN LEER LIMITER

U FIELD : - VAN LEER LIMITER

U+A FIELD : - VAN LEER LIMITER