University of Reading
Mathematics Support Centre

Understanding the CHAIN RULE
Identifying different types of functions

1. Examples of simple functions:

x, $sin\, x$, $\sqrt{x}$, $3x^4$, $x+1$, x^2, $\ln x$, $6x^3$, $4cos\, x$, $5x$, $tan\, x$, x^{-2}, e^x, $3x + 2\ln x$, $4sin\, x$ - x^4

These simple functions can be differentiated straight away.

2. Examples of functions-of-functions:

$(x+1)^2$, $cos^2 x\ [= (cos\, x)^2]$, $\sqrt{(\ln x)}$, e^{2x+5}, $sin\, 3x$, $x^{\sin x}$

To differentiate these, we need the CHAIN RULE. (Although with practice, they can again be differentiated straight away.)

In each of these examples, there are TWO functions:

In $(x+1)^2$, the 1st function is $x+1$, the 2nd function is "square"
In $sin\, 3x$, the 1st function is $3x$, the 2nd function is "sin"

The 1st function is the one you'd work out first if you were going to evaluate the function for a given value of x:
e.g. to evaluate $(x+1)^2$, when $x = 3$, first work out $3 + 1$, then square, to get 16.
And to evaluate $sin\, 3x$ when $x = 30^\circ$, first work out $3x$, then take the $sine$ of 90°, which is 1.

3. Divide the following functions up into two groups, simple functions (ones that can be differentiated straight away) and functions-of-functions (ones that can't):

$cos\, x$, $sin\, 2x$, x^3, $\ln 4x$, $6x+5$, e^{sinx}, $3tan\, x$, $1+x^3$, $sin(4x+3)$, $2\sqrt{x}$, $7x^6$,
$1 - \ln x$, $2sin\, x + 3cos\, x$, $5e^{-2x}$, $cos\, 5x$, x^{-8}, $(6x+3)^5$

4. Differentiate all of the simple functions

5. To differentiate functions-of-functions, we need the CHAIN RULE:

Example: to differentiate $y = (x^3+4)^5$
We let u = 'the 1st function', which is …………
So now we have y = …………(i.e. a function of u)
Now work out $\frac{du}{dx}$ which is…………
And also work out $\frac{dy}{du}$, which is ……………………

Now the CHAIN RULE says: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
(think of multiplying fractions: the 'du's cancel out)

So we get $\frac{dy}{dx}$ = ……………… × …………………………..
= …………………………………..
= $15x^2(x^3+4)^4$ [Answer]

6. Now try to differentiate all of the functions-of-functions that you identified in Q3.

ANSWERS

Q3.
simple functions:
$cos\ x$, x^3, $6x + 5$, $3tan\ x$, $1 + x^3$, $2\sqrt{x}$, $7x^6$, $1 - \ln x$, $2sin\ x + 3cos\ x$, x^{-8}

functions-of-functions:
$sin\ 2x$, $\ln 4x$, e^{sinx}, $sin(4x + 3)$, $5e^{-2x}$, $cos\ 5x$, $(6x + 3)^5$

Q4. [in the same order as given above]
$-sin\ x$, $3x^2$, 6, $3sec^2x$, $3x^2$, $x^{-\frac{1}{2}}$, $42x^5$, $-\frac{1}{x}$, $2cos\ x - 3\sin x$, $-8x^{-9}$

Q5. i) $x^3 + 4$
ii) u^5
iii) $\frac{du}{dx} = 3x^2$
iv) $\frac{dy}{du} = 5u^4$
v) $5u^4 \times 3x^2 = 15x^2u^4 = 15x^2\ (x^3 + 4)^4$

Q6.
$2cos\ 2x$, $\frac{1}{x}$ or x^{-1}, $cos\ x\ e^{sinx}$, $4\cos(4x + 3)$, $2cos\ x - 3\sin x$, $-10e^{-2x}$, -5$sin\ 5x$, $30(6x + 3)^4$