Re-examining the importance of trade openness for aggregate instability

by

Stephen McKnight and Alexander Mihailov
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Stephen McKnight* Alexander Mihailov†
University of Reading University of Reading

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Abstract

This paper re-considers the importance of trade openness for equilibrium determinacy when monetary policy is characterized by interest-rate rules. We develop a two-country, sticky-price model where money enters the utility function in a non-separable manner. Forward- and current-looking policy rules that react to domestic or consumer price inflation are analyzed. It is shown that the introduction of real balance effects substantially limits the validity of the Taylor principle and challenges recent conclusions concerning the relative desirability of the inflation indicator targeted.

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*Correspondence address: Stephen McKnight, Department of Economics, University of Reading, Whiteknights, PO Box 219, Reading, RG6 6AW, UK. E-mail: Stephen.McKnight@reading.ac.uk.
†Correspondence address: Alexander Mihailov, Department of Economics, University of Reading, Whiteknights, PO Box 218, Reading, RG6 6AA, UK. E-mail: a.mihailov@reading.ac.uk.
1 Introduction

Recent papers have considered the real indeterminacy implications of designing interest-rate rules in sticky-price models allowing for trade openness.\(^1\) A general conclusion arising from these studies is that the degree of trade openness is only important for aggregate stability if monetary policy responds to expected future consumer price inflation. Reacting to expected future domestic price inflation or implementing a current-looking rule guarantees equilibrium determinacy if the Taylor principle is adhered to.\(^2\) However, the existing literature analyzes monetary policy in an environment where money demand plays no role for equilibrium determination.\(^3\) By contrast, this paper shows that money demand matters for the design of interest-rate rules. In the presence of real balance effects and for alternative interest-rate rules, the degree of trade openness can increase the prominence of aggregate instability.

Despite the intuitive appeal of money facilitating transaction services, the neglect of real balance effects in the equilibrium determinacy literature is common.\(^4\) Empirical estimates suggest, but are not conclusive, that such effects are probably small.\(^5\) Furthermore, closed-economy studies that have considered real balance effects have found that allowing for non-separability between consumption and real balances has no implications for real determinacy. For example, Benhabib \textit{et al.} (2001) using a continuous-time money-in-the-utility function (MIUF) framework and Kurozumi (2006) using the discrete-time counterpart both show that the Taylor principle is robust under a contemporaneous inflation feedback rule.\(^6\) In this paper we show that with a small degree of non-separability, there are important consequences for aggregate stability once international trade in goods is allowed. This arises because the effect of real money balances is magnified as an economy becomes increasingly open to trade.

\(^2\)That is, a policy that adjusts the nominal interest rate by proportionally more than the increase in inflation.
\(^3\)The vast majority of the literature either assumes a cashless economy or adopts a money-in-the-utility function model with separable preferences. A notable exception is De Fiore and Liu (2005) who employ a cash-in-advance constraint.
\(^4\)As stressed by Woodford (2003), if money is considered to provide transaction services then the benefits of this should be related to the individual’s volume of transactions.
\(^5\)See, for example, Woodford (2003), Ireland (2004) using US data and Andrés, López-Salido and Vallés (2006) using Euro-zone data. Kremer, Lombardo and Werner (2003), by contrast, find that such effects are of much greater importance using German data.
\(^6\)Interestingly, with non-separability Kurozumi (2006) finds that if the interest-rate rule also responds to current output indeterminacy is more likely to occur.
Using a discrete-time framework, this paper develops a two-country, sticky-price, monetary model. We adopt the widely used MIUF approach, but allow for non-separability between consumption and real money balances. The traditional end-of-period timing of money balances of the utility function is employed. The conditions for equilibrium determinacy are analyzed under forward- and current-looking versions of the interest-rate rule. In addition, two alternative price indexes, which can be chosen as the policy indicator, are considered: domestic-price inflation and consumer-price inflation. The main results from the analysis can be summarized as follows. If forward-looking rules are employed the range of indeterminacy increases in the presence of real balance effects. Indeed, with a small degree of non-separability and a sufficiently high degree of trade openness multiple equilibria is generated regardless of the index of inflation targeted. Furthermore, if monetary policy reacts to current-period inflation, a feedback rule responding to domestic-price inflation is also susceptible to indeterminacy. Consequently, we find that the Taylor principle only holds when monetary policy is characterized by a current-looking consumer price inflation rule.

Our results for consumer-price inflation feedback rules support the findings of the recent literature. For example, De Fiore and Liu (2005), Linnemann and Schabert (2006) and McKnight (2007) all show that the Taylor principle is satisfied if a current-looking consumer-price inflation rule is employed. However, under a forward-looking rule, indeterminacy is typically generated if the monetary authority pursues an active policy. By contrast the results from this analysis contradict with recent conclusions stipulated for feedback rules that respond to domestic-price inflation. For instance, Zanna (2003) and Linnemann and Schabert (2002, 2006) both find that the Taylor principle is satisfied under a current-looking domestic-price inflation rule. The latter authors also show that under a forward-looking rule the upper bound on the size of the inflation coefficient above which indeterminacy may appear is typically too large to have any practical relevance. Our analysis shows that these results are not robust once a small degree of non-separability is introduced.

\footnote{As discussed by Carlstrom and Fuerst (2001), alternative timing assumptions on money can have important consequences for equilibrium determinacy. Kurozumi (2006) considers the indeterminacy implications of these alternative timing-assumptions with non-separability in a closed-economy framework characterized by a current-looking interest-rate rule. This paper adopts the traditional convention that end-of-period money balances enter the utility function, to facilitate comparison with the vast majority of the literature which employs the same timing convention but imposes separability of the utility function. Assuming CIA-timing would make our model compatible with De Fiore and Liu (2005).}
Therefore the choice of a particular measure of inflation as the policy indicator does matter. This contradicts Carlstrom, Fuerst and Ghironi (2006) findings, where using a two-sector, closed-economy model they argue that the criteria for equilibrium determinacy does not imply a preference to any particular measure of inflation. Furthermore, our analysis does not lend support to Linnemann and Schabert (2002, 2006), Zanna (2003) and Batini et al. (2004) claim that domestic inflation targeting is superior to consumer inflation targeting, as it reduces the potential range of aggregate instability. As this paper shows, domestic-price inflation is superior to consumer-price inflation if a forward-looking rule is followed. However, under current-looking rules consumer-price inflation is superior to domestic-price inflation in minimizing the potential range of aggregate instability.

The remainder of the paper is organized as follows. Section 2 develops the two-country model. Section 3 examines the conditions for real equilibrium determinacy when forward-looking interest-rate rules are employed. Section 4 considers the implications of interest-rate rules that respond to contemporaneous inflation. Finally, Section 5 concludes.

2 Model

Consider a global economy that consists of two countries, home and foreign, where an asterisk denotes foreign variables. Within each country there exists a representative infinitely-lived agent, a representative final good producer, a continuum of intermediate good producing firms, and a monetary authority. The representative agent owns all domestic intermediate good producing firms and supplies labor to the production process. Intermediate firms operate under monopolistic competition and use domestic labor as inputs to produce tradeable goods which are sold to the home and foreign final good producers. The labor market is assumed to be competitive. Each representative final good producer is a competitive firm that bundles domestic and imported intermediate goods into non-tradeable final goods which are consumed by the domestic agent. Preferences and technologies are symmetric across the two countries. In this section, we present the features of the model for the home country on the understanding that the foreign case can be analogously derived.

\[\text{8} \text{ Indeed, Linnemann and Schabert (2006) conclude that the particular price index chosen as the policy indicator is irrelevant as long as the policy is not forward-looking. Our analysis suggests that this conclusion is a by-product of their cashless-economy assumption.}\]
2.1 Final-Goods Sector

The *home* final good \((Z)\) is produced by a competitive firm that uses \(Z_H\) and \(Z_F\) as inputs according to the aggregation technology index:

\[
Z_t = \left[ a^\frac{\theta}{\theta - 1} Z_H^{\theta - 1,t} + (1 - a)^\frac{\theta}{\theta - 1} Z_F^{\theta - 1,t} \right]^\frac{1}{\theta - 1},
\]  

(1)

where the relative share of domestic and imported intermediate inputs used in the production process is determined by \(0 < a < 1\) and the constant elasticity of substitution between aggregate *home* and *foreign* intermediate goods is \(\theta > 0\). The inputs \(Z_H\) and \(Z_F\) are defined as the quantity indices of domestic and imported intermediate goods, respectively:

\[
Z_{H,t} = \left[ \int_0^1 z_{H,t}(i) \frac{d}{\varphi - 1} di \right]^\frac{\varphi}{\varphi - 1}, \quad Z_{F,t} = \left[ \int_0^1 z_{F,t}(j) \frac{d}{\varphi - 1} dj \right]^\frac{\varphi}{\varphi - 1},
\]  

(2)

where the elasticity of substitution across individual *home* (*foreign*) intermediate goods is \(\varphi > 1\), and \(z_H(i)\) and \(z_f(j)\) are the respective quantities of the domestic and imported type \(i\) and \(j\) intermediate goods. Let \(p_{H}(i)\) and \(p_{F}(j)\) represent the respective prices of these goods in *home* currency. Cost minimization in final good production yields the aggregate demand conditions for *home* and *foreign* goods:

\[
Z_{H,t} = a \left( \frac{p_{H,t}}{P_t} \right)^{-\theta} Z_t, \quad Z_{F,t} = (1 - a) \left( \frac{p_{F,t}}{P_t} \right)^{-\theta} Z_t,
\]  

(3)

where the demand for individual goods is given by

\[
z_{H,t}(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varphi} Z_{H,t}, \quad z_{F,t}(j) = \left( \frac{p_{F,t}(j)}{P_{F,t}} \right)^{-\varphi} Z_{F,t}.
\]  

(4)

Since the final good producer is competitive, it sets its price equal to marginal cost:

\[
P_t = \left[ a P_{H,t}^{1-\theta} + (1 - a) P_{F,t}^{1-\theta} \right]^\frac{1}{1-\varphi},
\]  

(5)
where $P$ is the consumer price index and $P_H$ and $P_F$ are the respective price indices of home and foreign intermediate goods, all denominated in home currency

$$P_{H,t} = \left[ \int_0^1 p_{H,t}(i)^{1-\varphi} \, di \right]^{1-\varphi}, \quad P_{F,t} = \left[ \int_0^1 p_{F,t}(j)^{1-\varphi} \, dj \right]^{1-\varphi}. \quad (6)$$

We assume that there are no costs to trade between the two countries and the law of one price holds, which implies that

$$P_{H,t} = S_t P^*_{H,t}, \quad P^*_{F,t} = P_{F,t} S_t, \quad (7)$$

where $S$ is the nominal exchange rate. Letting $Q \equiv \frac{S^*_p}{p}$ denote the real exchange rate, under the law of one price the CPI index (5) and its foreign equivalent imply:

$$\left( \frac{1}{Q_t} \right)^{1-\theta} \left( \frac{P_t}{S_t P^*_t} \right)^{1-\theta} = \frac{a P^{1-\theta}_{H,t} + (1-a) S_t P_{F,t}^{1-\theta}}{a (S_t P^*_{F,t})^{1-\theta} + (1-a) P^{1-\theta}_{H,t}} \quad (8)$$

and hence the purchasing power parity (PPP) condition is satisfied only in the absence of any bias between home and foreign intermediate goods (i.e. $a = 0.5$). The relative price of foreign goods in terms of home goods, or the terms of trade $T$, is defined as $T \equiv \frac{S^*_p}{p_H}$.

2.2 Intermediate-Goods Sector

Intermediate-sector firms hire labor $h$ to produce output given a real wage rate $w_t$. A firm of type $i$ has a linear production technology

$$y_t(i) = h_t(i). \quad (9)$$

Given competitive prices of labor, cost minimization yields

$$mc_t = w_t \frac{P_t}{P_{H,t}} \quad (10)$$

where $mc_t \equiv \frac{MC_t}{P_{H,t}}$ is real marginal cost. Firms set prices according to Calvo (1983), where in each period there is a constant probability $1 - \psi$ that a firm will be randomly selected to adjust its price, which is drawn independently of past history. A domestic firm $i$, faced
with changing its price at time $t$, has to choose $p_{H,t}(i)$ to maximize its expected discounted value of profits, taking as given the indexes $P$, $P_H$, $P_F$, $Z$ and $Z^*$:\(^9\)

$$\max_{p_{H,t}(i)} E_t \sum_{s=0}^{\infty} (\beta \psi)^s X_{t,t+s} \left\{ [p_{H,t}(i) - MC_{t+s}(i)] \left[ z_{H,t+s}(i) + z^*_{H,t+s}(i) \right] \right\}, \quad (11)$$

where

$$z_{H,t+s}(i) + z^*_{H,t+s}(i) \equiv \left( \frac{p_{H,t}(i)}{P_{H,t+s}} \right)^{-\psi} \left[ Z_{H,t+s} + Z^*_{H,t+s} \right]$$

and the firm’s stochastic discount factor used to value random date $t+s$ payoffs is $\beta^s X_{t,t+s} = [U_C(C_{t+s}, m_{t+s})/U_C(C_t, m_t)](P_t/P_{t+s})$.\(^{10}\) Firms that are given the opportunity to change their price, at a particular time, all behave in an identical manner. The first-order condition to the firm’s maximization problem yields

$$\bar{P}_{H,t} = \frac{\psi}{\varphi - 1} E_t \sum_{s=0}^{\infty} q_{t,t+s} MC_{t+s}, \quad (12)$$

The optimal price set by a home firm $\bar{P}_{H,t}$ is a mark-up $\frac{\psi}{\varphi - 1}$ over a weighted average of future nominal marginal costs, where the weight $q_{t,t+s}$ is given by

$$q_{t,t+s} = \frac{(\beta \psi)^s X_{t,t+s} P_{H,t+s}^\varphi \left( Z_{H,t+s} + Z^*_{H,t+s} \right)}{E_t \sum_{s=0}^{\infty} (\beta \psi)^s X_{t,t+s} P_{H,t+s}^\varphi \left( Z_{H,t+s} + Z^*_{H,t+s} \right)}. \quad (13)$$

As all prices have the same probability of being changed, with a large number of firms the evolution of the price subindexes is given by

$$P_{H,t}^{1-\varphi} = \psi P_{H,t-1}^{1-\varphi} + (1 - \psi) \bar{P}_{H,t}^{1-\varphi} \quad (14)$$

since the law of large numbers implies that $1 - \psi$ is also the proportion of firms that adjust their price each period.

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\(^9\)While the demand for a firm’s good is affected by its pricing decision $p_{H,t}(i)$, each producer is small with respect to the overall market.

\(^{10}\)The assumption that all firms are owned by the representative household implies that the firm’s stochastic discount factor is equivalent to the household’s intertemporal marginal rate of substitution.
2.3 Representative Agent

The representative agent chooses real consumption $C$, domestic real money balances $m = M/P$, and labor $h$ to maximize expected discounted utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left( C_t, \frac{M_t}{P_t}, h_t \right),$$

where the discount factor is $0 < \beta < 1$, subject to the period budget constraint

$$E_t \Gamma_{t,t+1} B_{t+1} + M_t + P_t C_t \leq B_t + M_{t-1} + P_t w_t h_t + \int_0^1 \Pi_t d(h) - \Upsilon_t.$$

The agent carries $M_{t-1}$ units of money and $B_t$ units of nominal bonds into period $t$. Before proceeding to the goods market, the agent visits the financial market where a state-contingent nominal bond $B_{t+1}$ can be purchased that pays one unit of domestic currency in period $t+1$ if a specific state is realized at a period $t$ price $\Gamma_{t,t+1}$. During period $t$ the agent supplies labor to the intermediate-sector firms receiving real income from wages $w_t$, nominal profits from the ownership of domestic intermediate firms $\Pi_t$ and a lump-sum nominal transfer $\Upsilon_t$ from the monetary authority. The agent then uses these resources to purchase the final good.

The period utility function is assumed to be non-separable between consumption and real money balances but additively separable with respect to labor: \(^{11}\)

$$U\left( C_t, \frac{M_t}{P_t}, h \right) \equiv u\left( C_t, \frac{M_t}{P_t} \right) - v\left( h_t \right).$$

The first-order conditions from the home agent’s maximization problem yield:

$$\beta E_t \frac{u_C(C_{t+1}, m_{t+1})}{P_{t+1}} = \frac{u_C(C_t, m_t)}{P_t} \frac{1}{R_t}$$

$$\frac{u_m(C_t, m_t)}{u_C(C_t, m_t)} = \frac{R_t - 1}{R_t}$$

$$\frac{v_h(h_t)}{u_C(C_t, m_t)} = w_t$$

\(^{11}\)As is standard, we assume that $u(C, m)$ is concave and strictly increasing in each argument and both consumption and real money balances are normal goods. It is further assumed that $v(h)$, the disutility of labor supply, is an increasing, convex function.
where $R_t$ denotes the gross nominal yield on a one-period discount bond defined as $R_t^{-1} = E_t\{\Gamma_{t,t+1}\}$. (18) is the consumption Euler equation, (19) defines the money demand function, and (20) determines labor supply. Optimizing behavior further implies that the budget constraint (16) holds with equality in each period and the appropriate transversality condition is satisfied. Analogous conditions apply to the foreign agent.

From the first-order conditions for the home and foreign agent, the following risk-sharing conditions can be derived:

$$R_t = R_t^* E_t \left[ \frac{S_{t+1}}{S_t} \right], \quad (21)$$

$$Q_t = q_0 \frac{u_C(C_t^*, m_t^*)}{u_C(C_t, m_t)}, \quad (22)$$

where the constant $q_0 = Q_0 \left[ \frac{u_C(C_0, m_0)}{u_C(C^*_0, m^*_0)} \right]$. Equation (21) is the standard uncovered interest rate parity (UIP) condition, whereas (22) follows from the assumption of complete asset markets.

### 2.4 Monetary Authority

The monetary authority can adjust the nominal interest rate $R_t$ in response to changes in domestic price inflation (PPI) $\pi_{t+v}^H$ or to changes in consumer price inflation (CPI) $\pi_{t+v}$ according to the corresponding rules:

$$R_t = \mu (\pi_{t+v}^H) = R \left( \frac{\pi_{t+v}^H}{\pi^H} \right)^\mu, \quad (23)$$

$$R_t = \mu (\pi_{t+v}) = \overline{R} \left( \frac{\pi_{t+v}}{\overline{\pi}} \right)^\mu, \quad (24)$$

where $\overline{R} > 1$ and the timing index $v$ represents the behavior of the monetary authority. If $v = 0$, the monetary authority reacts to current-period inflation, whereas $v = 1$ corresponds to a forward-looking rule. The inflation coefficient $\mu$ determines whether monetary policy is active or passive. An active monetary policy corresponds to $\mu > 1$, i.e. the so-called Taylor principle, where the real interest rate rises in response to higher inflation, as the monetary authority increases the nominal interest rate by more than the increase in inflation. A passive monetary policy, on the other hand, corresponds to $0 \leq \mu < 1$, where the real interest rate falls in response to higher inflation.
2.5 Market Clearing and Equilibrium

Market clearing for the home goods market requires

\[ Z_{H,t} + Z^*_{H,t} = Y_t. \]  

(25)

Total home demand must equal the supply of the final good,

\[ Z_t = C_t, \]  

(26)

and the labor, money and bond markets all clear:

\[ Y_t = M_t - M_{t-1}, \quad B_t + B^*_t = 0. \]  

(27)

**Definition 1** (Rational-Expectations Equilibrium): Given an initial allocation of \( B_{t_0}, B^*_{t_0}, M_{t_0} - 1, M^*_{t_0} - 1, \) a rational-expectations equilibrium is a set of sequences \( \{C_t, C^*_t, M_t, M^*_t, h_t, h^*_t, B_t, B^*_t, R_t, R^*_t, MC_t, MC^*_t, w_t, w^*_t, Y_t, Y^*_t, S_t, Q_t, P_t, P^*_t, P_{H,t}, P^*_{H,t}, \) \( \tilde{P}_{H,t}, \tilde{P}^*_{F,t}, P_{F,t}, P^*_{F,t}, Z_t, Z^*_t, Z_{H,t}, Z^*_{H,t}, Z_{F,t}, Z^*_{F,t}\} \) for all \( t \geq t_0 \) characterized by: (i) the optimality conditions of the representative agent, (18) to (20), the budget constraint (16) and the transversality condition; (ii) cost-minimization and price-setting behavior of intermediate firms, (10) and (12), and the aggregate version of the production function (9); (iii) the final good producer's optimality conditions, (2) and (5); (iv) all markets clear, (25) to (27); (v) the monetary policy rule is satisfied, (23) or (24); along with the foreign counterparts for (i)-(v) and conditions (7), (8), (21) and (22).

2.6 Local Equilibrium Dynamics

In order to analyze the equilibrium dynamics of the model, a first-order Taylor approximation is taken around a steady state to replace the nonlinear equilibrium system with an approximation which is linear.12 In what follows, a variable \( \tilde{X}_t \) denotes the percentage deviation of \( X_t \) with respect to its steady state value \( \overline{X} \). To be precise, the model is

12The Aoki (1981) decomposition approach is then employed, which splits the linearized model into two decoupled dynamic systems: the aggregate system that captures the properties of the closed world economy and the difference system that portrays the open-economy dimension. Consequently, for the equilibrium to be determinate it must be the case that there is a unique solution for both cross-country differences and world aggregates.
Cross-Country Differences Equations

$$\hat{m}_t^R = \eta_c \hat{Z}_t^R - \eta_R \hat{R}_t^R \quad \text{LM}^R$$

$$\hat{Z}_t^R = E_t \hat{Z}_{t+1}^R - \sigma \left[ ( \hat{R}_t^R - E_t \hat{\pi}_{t+1}^R ) + \chi ( E_t \hat{m}_{t+1}^R - \hat{m}_t^R ) \right] \quad \text{IS}^R$$

$$\hat{\pi}_t^R (H-F^*) = \beta E_t \hat{\pi}_{t+1}^R + \lambda \left[ \sigma^{-1} + (2a-1) \omega \right] \hat{Z}_t^R - \lambda \chi \hat{m}_t^R$$

$$+ 2\lambda (1 + 2a\theta\omega) (1 - a) \hat{T} \quad \text{AS}^R$$

$$\hat{R}_t^R = \mu E_t \hat{\pi}_{t+1}^R \quad \text{MPR}^R$$

$$\hat{\pi}_t^R = (2a-1) \hat{\pi}_t^R (H-F^*) + 2 (1 - a) \Delta \hat{S}_t$$

$$\hat{Q} = \sigma^{-1} \hat{Z}_t^R - \chi \hat{m}_t^R = (2a-1) \hat{T} \quad \text{RER}$$

World Aggregates

$$\hat{m}_t^W = \eta_c \hat{Z}_t^W - \eta_R \hat{R}_t^W \quad \text{LM}^W$$

$$\hat{Z}_t^W = E_t \hat{Z}_{t+1}^W - \sigma \left[ ( \hat{R}_t^W - E_t \hat{\pi}_{t+1}^W ) + \chi ( E_t \hat{m}_{t+1}^W - \hat{m}_t^W ) \right] \quad \text{IS}^W$$

$$\hat{\pi}_t^W = \beta E_t \hat{\pi}_{t+1}^W + \lambda \left[ \sigma^{-1} + \omega \right] \hat{Z}_t^W - \chi \hat{m}_t^W \quad \text{AS}^W$$

$$\hat{R}_t^W = \mu E_t \hat{\pi}_{t+1}^W \quad \text{MPR}^W$$

| Table 1: Aoki Decomposition of the Linearized Model |

linearized around a symmetric steady state in which prices in the two countries are equal and constant \((P_H = P_F = P = P^* = P_H^* = P_F^*)\). Then, by definition, inflation is zero \((\pi = \pi^* = 0)\), and the steady state terms of trade and nominal and real exchange rates are all unity: \(T = S = Q = 1\). An important consequence of assuming non-separability of the utility function is that the money demand equation affects the local dynamics of the model.\(^{13}\) Following Woodford (2003), let \(\Delta_t \equiv \frac{R_t}{R_t}^{-1}\), where in a steady state \(\Delta = 1 - \beta > 0\). Then from the money demand equation (19), one can solve for real money balances

$$\frac{M_t}{P_t} = L(C_t, \Delta_t) \quad (28)$$

where the right-hand term is the liquidity preference function \(L\), which is increasing in \(C_t\) and decreasing in \(\Delta_t\). Linearizing equation (28) around the steady state yields the LM equation

$$\hat{m}_t = \eta_c \hat{c}_t - \eta_R \hat{R}_t$$

where the constant coefficients are: \(\eta_c \equiv \overline{\alpha} \frac{\partial L}{\partial C} > 0\) is the income elasticity of money

\(^{13}\)If money balances are separable, the money demand equation is irrelevant for equilibrium determinacy.
### Table 2: Benchmark parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Intertemporal elasticity of substitution in consumption</td>
<td>6.4</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Output elasticity of (real) marginal cost</td>
<td>0.47</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution between aggregate <em>home</em> and <em>foreign</em> goods</td>
<td>1.5</td>
</tr>
<tr>
<td>$a$</td>
<td>Degree of trade openness</td>
<td>$0 &lt; a &lt; 1$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Degree of price stickiness</td>
<td>0.835</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Degree of non-separability of utility function</td>
<td>$0 \leq \chi \leq 0.1$</td>
</tr>
<tr>
<td>$\eta_Z$</td>
<td>Output elasticity of money demand</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_R$</td>
<td>Interest-rate semi-elasticity of money demand</td>
<td>28</td>
</tr>
</tbody>
</table>

Demand and $\eta_R \equiv -\left(\frac{1-\chi}{m}\right) \frac{\partial L}{\partial \lambda} > 0$ is the interest semi-elasticity of money demand. For convenience, the complete linearized system of equations is summarized in Table 1.

In order to minimize unnecessary complications for the analytical determinacy derivations, following Kurozumi (2006) the ensuing analysis imposes an assumption on $\chi$, which according to Woodford (2003) is of most empirical relevance:

**Assumption 1** $0 \leq \chi < (\eta_Z \sigma)^{-1}$.

It will also be convenient to illustrate the determinacy conditions using the benchmark values for the parameters specified in Table 2. The majority of the benchmark parameter values chosen are based on Kurozumi (2006). However, empirical studies offer no clear conclusion on the size of $\theta$ and $\sigma$. For instance, the literature suggests that $\sigma$ be between 1 and 10, e.g. Gali *et al.* (2007). For $\theta$, evidence suggests that it can take a value anywhere between 1 and 7, e.g. Trefler and Lai (1999). Therefore, in the benchmark analysis we set $\theta = 1.5$ and $\sigma = 6.4$. For robustness we additionally use values of $\theta = 5$ and $\sigma = 1.5$. Since our focus is on the interaction between the degree of trade openness $a$ and the degree of non-separability $\chi$, we compute the numerical eigenvalues of the model for alternative values of $0 \leq \chi \leq 0.1$ (which is consistent with Assumption 1) and $a \in (0, 0.5) \cup (0.5, 1)$.

### 3 Equilibrium Determinacy

We start by examining the conditions for equilibrium determinacy under a forward-looking interest-rate rule.
3.1 Aggregate System

The set of linearized equations for the world aggregates, given in Table 1, can be reduced to the following two-dimensional system:

\[ E_t x_{t+1}^W = A x_t^W, \quad x_t = \begin{bmatrix} \hat{m}^W_t \ 2^W_t \end{bmatrix}, \]

where \( A \equiv \begin{bmatrix} -\Lambda_1 + \frac{1}{\eta_r \mu} \left( [1 + \sigma(\omega + 1) + \beta \Lambda_1] \sigma(\mu - 1) \Lambda_1 + \sigma \right) - \Lambda_2 + \frac{1}{\eta_r} \lambda \left( [1 + \sigma(\omega + 1) + \beta \Lambda_2] \sigma(\mu - 1) \Lambda_2 \right) 
\end{bmatrix} \]

\( \Lambda_1 \equiv \frac{1}{\eta_r \mu} > 0 \) and \( \Lambda_2 \equiv \frac{\eta_r}{\eta_r \mu} > 0 \). Since \( x \) is a column vector of non-predetermined variables, equilibrium determinacy requires that the two eigenvalues of \( A \) are outside the unit circle. Then by Proposition C.1 of Woodford (2003) the following result is obtained:

**Proposition 1** Suppose that monetary policy is characterized by a forward-looking interest-rate rule. Then the necessary and sufficient conditions for determinacy of the aggregate system are:

\[(I) \quad 1 < \mu < \min \{ \Gamma^A_1, \Gamma^A_2 \} \quad \text{or} \quad (II) \quad \max \{1, \Gamma^A_1\} < \mu < \Gamma^A_2 \quad (29)\]

where \( \Gamma^A_1 \equiv \frac{\beta (1 - \sigma \chi \eta_z)}{\eta_r \lambda \omega \sigma \chi} \) and \( \Gamma^A_2 \equiv \frac{2(1 - \sigma \chi \eta_z)(1 + \beta) + \lambda [1 - \sigma \chi \eta_z + \sigma \omega]}{\lambda [1 - \sigma \chi \eta_z + \sigma \omega(1 + 2 \eta_r \chi)]} \).

Under a forward-looking rule, non-separability results in more restrictive conditions for determinacy than when separability of the utility function is assumed (\( \chi = 0 \)). With separability it is straightforward to show that the necessary and sufficient condition for determinacy is given by

\[ 1 < \mu < 1 + \frac{2(1 + \beta)}{\lambda (\sigma \omega + 1)} \equiv \Gamma^A_{\chi=0} \quad (30) \]

and hence \( \Gamma^A_2 < \Gamma^A_{\chi=0} \). The impact of the magnitude of \( \chi \) on the bounds on \( \mu \) given in (29) is negative:

\[ \frac{\partial \Gamma^A_1}{\partial \chi} = \frac{-\beta}{\lambda \sigma \omega \chi^2 \eta_r} < 0, \]

\[ \frac{\partial \Gamma^A_2}{\partial \chi} = \frac{-2 \sigma \omega \left( (1 + \beta)(\sigma \eta_z + 2 \eta_r) + \lambda \eta_r \left( 1 - \sigma \chi \eta_z + \sigma (\omega + \chi) \right) \right)}{\lambda \left( 1 - \sigma \chi \eta_z + \sigma \omega(1 + 2 \chi \eta_r) \right)^2} < 0, \]

and thus indeterminacy increases as \( \chi \) increases. Using the benchmark calibration to compute the upper bounds on the inflation coefficient confirms that (second-order) indetermi-
nacy is more likely the higher is \( \chi \) and the larger is \( \sigma \). For instance, setting \( \sigma = 6.4 \), then with \( \chi = 0 \) determinacy requires \( \mu < 29.99 \). If \( \chi = 0.02 \), then the upper bound is significantly reduced to \( \mu < 14.51 \) and if \( \chi = 0.04 \), then \( \mu < 8.61 \). However when a lower value of \( \sigma \) is chosen, the upper bound on \( \mu \) can be high enough so as to be of little empirical relevance. For example, if \( \sigma = 1.5 \), then with \( \chi = 0 \), \( \mu < 69.15 \) is required for determinacy, whereas if \( \chi = 0.02 \), then \( \mu < 46.41 \) and even if \( \chi = 0.1 \), then \( \mu < 18.23 \).

### 3.2 Difference System

#### 3.2.1 Reacting to Domestic-Price Inflation

The set of linearized conditions for cross-country differences yields a system of the form:

\[
E_t x_{t+1}^R = B x_t^R, \quad x_t = \left[ \hat m_t^R \hat Z_t^R \right]'
\]

where \( \Lambda_1 = \frac{1}{\eta_R \mu} \), \( \Lambda_2 = \frac{\eta}{\eta_R \mu} \), \( \Lambda_3 = \frac{\lambda(2-a)+2\theta \omega a}{\sigma(2a-1)} + \lambda \left[ \omega(2a-1) + \sigma^{-1} \right] + \beta \Lambda_2 \), and \( \Lambda_4 = \frac{\lambda(2-a)+2\theta \omega a}{\sigma(2a-1)} + \lambda \chi + \beta \Lambda_1 \). As before, determinacy requires that both eigenvalues of the coefficient matrix are outside the unit circle.

**Proposition 2** Suppose that monetary policy reacts to forward-looking domestic-price inflation. Then the necessary and sufficient conditions for determinacy of the difference system are:

\( (I) \) \( a > 0.5 \) and either \( (i) 1 < \mu < \min\{\Gamma_1^B, \Gamma_2^B\} \) or \( (ii) \max\{1, \Gamma_1^B\} < \mu < \Gamma_2^B \)

\( (II) \) \( a < 0.5 \) and either \( (i) 1 < \mu < \Gamma_3^B \) if \( \eta_R > X^B \) or \( (ii) 1 < \mu < \min\{\Gamma_3^B, \Gamma_2^B\} \) if \( \eta_R < X^B \),

where

\[
\Gamma_1^B = \frac{\beta (1-\sigma \chi \eta)}{\eta_R \lambda \sigma \chi \omega(2a-1)}.
\]
\[
\Gamma^B_2 \equiv \frac{2(1 - \sigma \chi \eta_z)(1 + \beta) \lambda^{-1} + (1 - \sigma \chi \eta_z)(1 + 4(1 - a) \omega \theta a) + \omega \sigma(2a - 1)^2}{(1 - \sigma \chi \eta_z)[1 + 4(1 - a) \omega \theta a] + \omega \sigma(2a - 1)^2 + 2\sigma \chi \omega \eta_R(2a - 1)},
\]
\[
\Gamma^B_3 \equiv \frac{(1 - \beta)(1 - \sigma \chi \eta_z)}{\eta_R \lambda \sigma \chi \omega(1 - 2a)} \text{ and } X^B \equiv \frac{\omega \sigma(2a - 1)^2 + (1 - \sigma \chi \eta_z)[1 + 4(1 - a) \omega \theta a]}{2\sigma \chi \lambda \omega(1 - 2a)}.
\]

First consider the case when \( \chi = 0 \). Then the determinacy conditions above collapse to

\[
1 < \mu < 1 + \frac{2(1 + \beta)}{\lambda[1 + \sigma \omega + 4\omega a(1 - a)(\theta - \sigma)]} \equiv \Gamma^B_{\chi=0}.
\] (31)

Consequently, if \( \theta = \sigma \) then the determinacy conditions of the difference system (31) and the closed-economy (30) are analogous. For any \( \theta \leq \sigma \), the open-economy introduces no additional requirements for determinacy, such that if (30) is satisfied, then both the aggregate and difference systems are determinate. However, if \( \theta > \sigma \) then the upper bound on the inflation coefficient for the difference system (31) is reduced relative to (30). Therefore, the potential range of indeterminacy is greater in the open-economy and increases the larger the difference between \( \theta \) and \( \sigma \). The impact that the degree of trade openness has on this upper bound is given by:

\[
\frac{\partial \Gamma^B_{\chi=0}}{\partial a} = \frac{8(1 + \beta) \Lambda_1 \omega(\theta - \sigma)(2a - 1)}{\lambda^2[1 + \sigma \omega + 4\omega a(1 - a)(\theta - \sigma)]^2} \geq 0,
\] (32)

where for any \( \theta > \sigma \), (32) > 0 if \( a > 0.5 \) and (32) < 0 if \( a < 0.5 \). Using the baseline calibration to compute the upper bound on \( \mu \) given by (31) for \( \theta = 5 \) and \( \sigma = 1.5 \), the impact for determinacy is as follows. If \( a = 0.8 \) or \( a = 0.2 \) then \( \mu < 43.13 \) is required to prevent (second-order) indeterminacy. If \( a = 0.6 \) or \( a = 0.4 \) then \( \mu < 36.38 \). While these bounds are much lower than the closed-economy case (i.e. \( \mu < 69.15 \)), they are very unlikely to bind. Hence, the impact of trade openness for equilibrium determinacy under a forward-looking domestic-price inflation does not seem to matter at a practical level.

Now consider the case when the utility function is assumed to be non-separable. Here the upper bounds on \( \mu \) are much more likely to bind if there is a sufficient degree of trade openness.\(^\text{14}\) Figure 1 depicts the regions in the parameter space \((a, \mu)\) that are associated with determinacy (D) and (second-order) indeterminacy \((I^2)\) given the baseline calibration

\(^{14}\)The numerical analysis suggests that if \( a > 0.5 \) then the upper bound on \( \mu \) is larger for the difference system relative to the aggregate system when \( \sigma > \theta \). Thus, the open-economy dimension places no additional restrictions for determinacy. If \( \theta > \sigma \) then the upper bound on \( \mu \) is relatively lower for the difference system. However, even in this case the bounds are such that they are extremely unlikely to bind.
Figure 1: Regions of determinacy under a forward-looking domestic-price inflation rule values. Even with a small degree of non-separability indeterminacy can exist. Its range expands as both $\chi$ and the degree of trade openness increase. Furthermore note that this result is qualitatively robust regardless of the relative size between $\theta$ and $\sigma$.

### 3.2.2 Reacting to Consumer-Price Inflation

If the policy rule reacts to expected consumer-price inflation, the set of linearized conditions for cross-country differences yields a system of the form:

$$E_t x_{t+1}^R = C x_t^R,$$

$$x_t = \left[ \tilde{m}_t^R \right]^\prime,$$

$$C = \begin{bmatrix}
\frac{\lambda_1 \sigma (2a-1)(\chi + (\mu - 1) \Lambda_1) - \Lambda_1 [1 - 2(1-a)] \mu}{\lambda \sigma \chi (2a-1)^2 + \beta (\sigma \chi \Lambda_1 - \Lambda_1) [1 - 2(1-a)] \mu} & \frac{\lambda_2 [1 - 2(1-a)] \mu - \Lambda_3 [2a-1] [1 + \sigma (\mu - 1) \Lambda_2]}{\lambda \sigma \chi (2a-1)^2 + \beta (\sigma \chi \Lambda_2 - \Lambda_1) [1 - 2(1-a)] \mu} \\
\frac{\lambda_0 \sigma (2a-1)(\chi + (\mu - 1) \Lambda_1) - \Lambda_1 [1 - 2(1-a)] \mu}{\lambda \sigma \chi (2a-1)^2 + \beta (\sigma \chi \Lambda_1 - \Lambda_1) [1 - 2(1-a)] \mu} & \frac{\lambda_3 [1 - 2(1-a)] \mu - \Lambda_6 [2a-1] [1 + \sigma (\mu - 1) \Lambda_3]}{\lambda \sigma \chi (2a-1)^2 + \beta (\sigma \chi \Lambda_2 - \Lambda_1) [1 - 2(1-a)] \mu}
\end{bmatrix},$$

where $\Lambda_1 \equiv \frac{1}{\eta \Lambda_1}$, $A_2 \equiv \frac{\eta}{\eta \Lambda_2}$, $\Lambda_5 \equiv \frac{\lambda_2 (1-a) [1 + 2\theta \omega a]}{\sigma (2a-1)} + \lambda \left[ \omega (2a - 1) + \sigma^{-1} \right] + \frac{\beta \lambda_2 [1 - 2(1-a)] \mu}{(2a-1)}$, and $\Lambda_6 \equiv \frac{\lambda_2 (1-a) [1 + 2\theta \omega a]}{(2a-1)} + \lambda \chi + \frac{\beta \Lambda_1 [1 - 2(1-a)] \mu}{(2a-1)}$. As before, determinacy requires that both eigenvalues of the coefficient matrix are outside the unit circle.

**Proposition 3** Suppose that monetary policy reacts to forward-looking consumer-price inflation. Then the necessary and sufficient conditions for determinacy of the difference
system are:

(I) \( (i) \) \( 1 < \mu < \min \{ \Gamma_C^1, \Gamma_C^2, \Gamma_C^3 \} \) or \( (ii) \) \( \max \{ 1, \Gamma_C^2 \} < \mu < \min \{ \Gamma_C^1, \Gamma_C^3 \} \)

(II) \( \max \{ \Gamma_C^1, \Gamma_C^3 \} < \mu < \min \{ 1, \Gamma_C^2 \} \) and \( 2(1 - \beta)(1 - a)(1 - \sigma \eta_z) > \lambda \sigma \omega \eta_R (2a - 1)^2 \),

where

\[
\Gamma_C^1 = \frac{1}{2(1-a)}, \quad \Gamma_C^2 = \frac{\beta(1 - \sigma \eta_z)}{\beta(1 - \sigma \eta_z - 2(1-a) + (2a-1)^2 \lambda \sigma \omega \eta_R)} \quad \text{and} \quad \Gamma_C^3 = \frac{(1 - \sigma \eta_z)[2(1 + \beta) + \lambda[1 + 4(1-a)\theta \omega a]] + \lambda \sigma \omega (2a - 1)^2}{(1 - \sigma \eta_z)(1 + 2(1-a)(1 + \beta) + \lambda[1 + 4(1-a)\theta \omega a]) + \lambda \sigma \omega (2a - 1)^2(1 + 2\chi \eta_R)}. 
\]

First note that part (II) of Proposition 3 is ignored since under a passive monetary policy the aggregate system is always indeterminate. Under separability the inflation coefficient is constrained by two upper bounds

\[
1 < \mu < \min \left\{ \frac{1}{2(1-a)}, \frac{2(1 + \beta) + \lambda[1 + \sigma \omega + 4\omega a(1-a)(\theta - \sigma)]}{\lambda[1 + \sigma \omega + 4\omega a(1-a)(\theta - \sigma)] + 4(1 + \beta)(1-a)} \right\}. 
\]

However with non-separability, the inflation coefficient is now constrained by three upper bounds. In both the separability and non-separability cases the range of indeterminacy

Figure 2: Regions of determinacy under a forward-looking consumer-price inflation rule \( \theta = 1.5 \) and \( \sigma = 6.4 \).
is potentially greater the higher the degree of trade openness (i.e. the lower is \( a \)). This is most evident from the upper bound \( \Gamma^C_1 \), where only an \( a > 0.5 \) can be consistent with \( \mu > 1 \). Figure 2 depicts the regions in the parameter space \((a, \mu)\) that are associated with determinacy (D) and (first-order) indeterminacy (I) using the baseline calibration. Here it is apparent that the higher the degree of non-separability the higher the potential range of indeterminacy.  

Therefore we can conclude that even a small degree of non-separability between consumption and real balances increases the range of indeterminacy under forward-looking rules in the open economy. This occurs regardless of whether monetary policy reacts to domestic-price or consumer-price inflation. Thus, unlike studies that assume separability, indeterminacy can arise when targeting domestic inflation. What is the economic intuition behind these results? Suppose that in response to a non-fundamental shock agents in the home country believe inflation will increase. Under an active monetary policy \((\mu > 1)\) the real interest rate increases and from the aggregate demand channel of monetary policy this reduces (real) marginal cost. The higher the degree of trade openness the smaller the change in \( \hat{mc}_t \). The increase in the real interest rate also results in an improvement in the terms of trade \((\hat{T}_t \downarrow)\), which puts additional downward pressure on domestic inflation from the Phillips curve (33):

\[
\hat{\pi}^H_t = \beta E_t \hat{\pi}^H_{t+1} + \lambda \left[ a^{-1} + (2a - 1) \omega - \chi \eta Z \right] \hat{Z}_t + \lambda \chi \eta R \hat{R}_t + 2\lambda (1 + 2a \theta \omega) (1 - a) \hat{T}_t. \tag{33}
\]

The higher the degree of trade openness the greater is the terms of trade effect. However with non-separability, real balance effects put upward pressure on domestic inflation: a direct effect, where the increase in \( \hat{R}_t \) puts upward pressure on domestic inflation; and an indirect effect, which reduces the change in \( \hat{mc}_t \). Consequently, if \( \hat{R}_t \) dominates the downward pressure on domestic inflation implied by \( \hat{T}_t \) and \( \hat{mc}_t \), then domestic inflation can actually rise validating the initial inflationary belief. If consumer-price inflation is the price indicator, then multiple equilibria is more likely since the CPI inflation rate depends

\[15\]While the values of \( \theta \) and \( \sigma \) do influence the upper bounds \( \Gamma^C_2 \) and \( \Gamma^C_3 \), the sensitivity analysis suggests that the quantitative impact on the threshold levels for determinacy is small.
on both the domestic inflation rate and the terms of trade:

\[ E_t \hat{\pi}_{t+1} = E_t \hat{\pi}_{t+1}^h + (1-a) \left( E_t \hat{T}_{t+1} - \hat{T}_t \right). \] (34)

Even if future domestic inflation is expected to be lower, the expected deterioration in the terms of trade \((E_t \hat{T}_{t+1} \text{ increases relative to } T_t)\) puts upward pressure on CPI inflation, the effect of which is greater the higher the degree of trade openness.

4 Current-looking Rules

So far the analysis has focused on forward-looking interest-rate rules. In this section we consider interest-rate rules that react to contemporaneous inflation.

4.1 Aggregate System

Under a current-looking rule, the set of linearized equations for the world aggregates, given in Table 1, can be reduced to the following two-dimensional system:

\[ E_t x_{t+1}^W = D x_t^W, \quad x_t = \left[ \hat{Z}_t^W \hat{\pi}_t^W \right]'. \]

Since \(x_t\) is a column vector of non-predetermined variables, equilibrium determinacy requires that the two eigenvalues of \(D\) are outside the unit circle. Then by Proposition C.1 of Woodford (2003) the following result is obtained:

**Proposition 4** Suppose monetary policy is characterized by a current-looking interest-rate rule. Then the Taylor principle (i.e. \(\mu > 1\)) is the necessary and sufficient condition for determinacy of the aggregate system.

Here non-separability has no impact on the determinacy conditions for the aggregate system. This corresponds to Kurozumi (2006) and Benhabib et al. (2001) findings that the Taylor principle is robust in guaranteeing equilibrium determinacy regardless of non-separability.
of the utility function between consumption and real balances.\footnote{Kurozumi (2006) however shows that if the interest-rate rule also responds to contemporaneous output, then a small degree of non-separability is more likely to induce indeterminacy.}

4.2 Difference System

4.2.1 Reacting to Domestic-Price Inflation

The set of linearized equations for cross-country differences, given in Table 1, can be reduced to the following two-dimensional system:

$$E_t x_{t+1} = E x_t,$$

where $E = \left[ \begin{array}{c} \lambda \beta \\frac{1}{1 - \sigma \chi \eta} \\frac{\Lambda_7}{\beta} \\ \frac{\lambda \beta}{\beta} \\frac{\Lambda_8}{\beta} \end{array} \right]$, and $\Lambda_7 \equiv 1 + \omega \sigma (2a - 1) + \frac{\varphi(1-a)(1-z)}{2a-1}$ and $\Lambda_8 \equiv 1 - \frac{\lambda \eta R \mu (1+4(1-a)\omega\theta a)}{2a-1}$.

Proposition 5 Suppose monetary policy is characterized by an interest-rate rule responding to current-period domestic-price inflation. Then the necessary and sufficient conditions for determinacy of the difference system are:

(I) $a > 0.5$ and $\mu > 1$

(II) $a < 0.5$ and

(i) $\mu > 1$ if $\chi < X^E$

(ii) $1 < \mu < \Gamma^E$ if $\chi > X^E$

where $X^E \equiv \frac{1 + 4(1-a)\omega\theta a + \sigma \omega(1-2a)^2}{\sigma \omega \eta R (1-2a) + \sigma \eta z [1 + 4(1-a)\omega\theta a]}$ and

$$\Gamma^E \equiv \frac{(1-\beta)(1-\sigma \chi \eta z)}{\lambda [\sigma \omega \chi \eta R (1-2a) - (1 - \sigma \chi \eta z) [1 + 4(1-a)\omega\theta a] - \sigma \omega (2a-1)^2]}.$$

First note that the range of indeterminacy is highly sensitive to the values of $\theta$ and $\sigma$. The greater $\sigma - \theta > 0$ the greater the range of indeterminacy (see Figure 3). Why? From the above proposition it is clear that determinacy can be achieved for all $\mu > 1$ provided $\chi < X^E$. Rearranging $X^E$ yields:

$$\chi \eta_R (1-2a) < 1 + \frac{1 - \sigma \chi \eta z}{\sigma \omega} + \frac{4(1-a)\omega a}{\sigma} [\theta (1 - \sigma \chi \eta z) - \sigma].$$
If $\sigma$ is sufficiently larger than $\theta$, this condition is less likely to bind, and thus indeterminacy is more likely. Moreover, for the Taylor principle to generate indeterminacy, a sufficiently large $\chi$ is required. Figure 4 shows the range of determinacy ($D$), (first-order) indeterminacy ($I^1$) and (second-order) indeterminacy ($I^2$) when $\sigma = 6.4$ and $\theta = 1.5$. As can be seen, the range of second-order indeterminacy increases as $\chi$ increases and the impact of changes in the inflation coefficient $\mu$ to variations in $\chi$ are quantitatively very minor.\footnote{Note that generating indeterminacy requires at least a $\chi > 0.04$ with the baseline parameter values chosen. This may be of too high a magnitude given current estimates of this parameter, to have any practical importance.} Interestingly, for relatively high values of $\chi$ the range of indeterminacy switches from second-order to first-order indeterminacy, the latter becoming larger the higher the inflation coefficient.

\subsection*{4.2.2 Reacting to Consumer-Price Inflation}

If the policy rule reacts to current-looking consumer-price inflation, the set of linearized conditions for cross-country differences yields a system of the form:
Figure 4: Regions of determinacy under a current-looking domestic-price inflation rule $\theta = 1.5$ and $\sigma = 6.4$

\[
E_t x_{t+1} = F x_t \quad x_t = \begin{bmatrix} \hat{Z}_t^R & \hat{Z}_t^R & \hat{Z}_t^R \end{bmatrix},
\]

where $\Lambda_9 \equiv (1 - \sigma \chi \eta \mu)[1 + 4(1 - a) \alpha \theta \omega] + (2a - 1)^2 \omega \sigma$ and $\Lambda_{10} \equiv 1 + 2(1 - a) \mu - \lambda \chi \mu \eta R \mu [1 + 4(1 - a) \alpha \theta \omega]$.

**Proposition 6** Suppose monetary policy is characterized by an interest-rate rule responding to current-period consumer-price inflation. Then the necessary and sufficient conditions for determinacy of the difference system are $\mu > 1$ and either:

\[
\frac{2(1-a)\mu}{\beta} \left[ 2(1-a)\mu(1-\beta) - (1+\beta) - \lambda[1 + 4(1-a)\alpha \theta \omega] - \frac{\lambda \sigma \omega(2a - 1)^2[1 + \lambda \eta R \mu]}{1 - \sigma \chi \eta \mu} \right] + (1-\beta) + 2(1-a)\mu(1+\beta) + \lambda \mu[1 + 4(1-a)\alpha \theta \omega] + \frac{\lambda \sigma \omega(2a - 1)^2[1 + \lambda \eta R \mu]}{1 - \sigma \chi \eta \mu} > 0 \quad (35)
\]
or
\[
\mu > \frac{2\beta - 1 - \lambda[1 + 4(1 - a)a\theta\omega] - \frac{\lambda\sigma\omega(2a-1)^2}{1-\sigma\chi\eta\xi}}{2\beta(1-a) + \frac{\lambda\sigma\chi\eta\xi(2a-1)^2}{1-\sigma\chi\eta\xi}}.
\]  

(36)

Here determinacy is always achieved for standard parameter values. The numerical analysis suggests that explosiveness can occur if \( \chi > 0.16 \), but this violates Assumption 1. Recall that the CPI inflation rate depends on both the domestic inflation rate and the terms of trade:
\[
\hat{\pi}_t = \hat{\pi}_t^h + (1-a) (\hat{T}_t - \hat{T}_{t-1}),
\]
where \( \hat{T}_{t-1} \) is predetermined. Thus, even if domestic inflation was to increase (i.e. due to the real balance effect outweighing the aggregate demand and terms of trade effects), the increase in the real interest rate results in an improvement in the terms of trade (\( \hat{T}_t \downarrow \)). From (37), this trade channel of monetary policy generates additional downward pressure on consumer-price inflation. Consequently, multiple equilibria is less likely to occur.

5 Conclusion

This paper has re-examined the importance of trade openness for equilibrium determinacy for a number of alternative interest-rate rules in the presence of real balance effects. Contrary to the previous literature, the appropriateness of the Taylor principle, as a policy guideline in preventing aggregate instability, is not as robust as initially believed. Under forward-looking rules the range of indeterminacy increases once real balance effects are introduced; given a sufficiently high degree of trade openness, even a small degree of non-separability generates multiple equilibria. If monetary policy is current-looking, a feedback rule responding to domestic-price inflation also leads to indeterminacy. Significantly, we find that the Taylor principle is only satisfied under a current-looking consumer-price inflation rule. A key conclusion that emerges is that the policy indicator used in the feedback rule does matter, which contradicts earlier findings based on cashless models. Overall, domestic-price inflation is only superior if a forward-looking rule is followed, whereas under current-looking rules, consumer-price inflation is superior in minimizing the potential range of aggregate instability.
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