Informational efficiency and price reaction within in-play prediction markets

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Abstract

We propose a practical framework to detect mispricing, test informational efficiency and evaluate the behavioural biases within high-frequency prediction markets, especially in how prices react to news. We show this using betting exchange data for association football, exploiting the moment when the first goal is scored in a match as major news that breaks cleanly. There is mispricing in these markets and inefficiency, explained by reverse favourite-longshot bias. This is systematically absorbed or amplified after a goal, depending on the match conditions. We find that prices respond correctly when news is expected but overreact when it is a surprise.

Keywords: Market efficiency; Favourite-longshot bias; Mispricing; Sports forecasting; Probability forecasting; Behavioural bias; Betting strategy

JEL codes: G14, D01, L83, C58, Z2

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1 Introduction

In the past fifty years, many researchers have attempted to test Fama’s (1965; 1970) Efficient Market Hypothesis (EMH), using a variety of methods and contexts. Studies have not only looked at whether asset prices reflect all relevant historical information (weak-form efficiency) but also whether the arrival of new information is immediately and fully incorporated (semi-strong efficiency). The answers to these questions are important both practically and theoretically. Not least, if markets are inefficient, then it implies that better informed economic agents can gain at the expense of the less well informed. Moreover, if the pricing mechanism in a market is inefficient, then an asset’s price will not reflect its fundamental value, complicating any economic analysis. Investigations of financial market efficiency have proved somewhat burdensome in practice because, among other things, it is problematic to identify precisely the point at which news breaks and is known by some or all of the agents involved. It is typically hard to believe that efficiency tests on such markets are not affected by information leakages and asymmetries, which are unknown to the econometrician.

To overcome these challenges and gain insight on the validity of the EMH, a large literature has studied prediction or betting markets, typically those relating to sports events. Unlike conventional financial markets, sports betting provides ‘real world laboratories’ in which to test the EMH and study departures from it, as participants are generally regarded as being well-informed, motivated, experienced and, most importantly, breaking news is usually reported cleanly, in a way that is easy for the participants to share and process. The assets (bets) in these markets have defined end points upon which their values become certain, which is typically not the case when evaluating financial securities pricing (Thaler and Ziemba, 1988). The main findings from the literature studying betting markets, however, show mixed evidence both on the degree to which these markets are efficient and, in some cases, the potential behavioural biases which might account for why they are inefficient.

In this paper, we introduce a new approach to test the semi-strong efficiency of prediction markets when ‘in-play’ trading is allowed, i.e. after the event has begun (e.g. between kick-off and the final whistle in a game of football). This approach is based on the Mincer and Zarnowitz (1969) forecast evaluation framework, and extends a previous application by Angelini and De Angelis (2019), which only applied to testing the weak-form efficiency of traditional betting markets. First, we show how the approach applied to traditional

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1 See Malkiel, 2003; Williams, 2005; Lim and Brooks, 2011 for comprehensive reviews of this literature.
2 See among many others: Thaler and Ziemba, 1988; Pope and Peel, 1989; Golec and Tamarkin, 1991; Kuypers, 2000; Levitt, 2004; Snowberg and Wolfers, 2010; Page and Clemen, 2012; Franck et al., 2013; Brown, 2013; Croxson and Reade, 2014; Deutscher et al., 2018; Angelini and De Angelis, 2019.
3 Some studies have attempted to replicate these conditions in the laboratory, such as Plott and Sunder (1988); Plott et al. (2003); List (2004) and Koessler et al. (2012), though these naturally lack realism and are open to the standard critiques of the settings being artificial.
bookmakers in Angelini and De Angelis (2019) can be extended to the general case of prediction markets, to both detect mispricing and test whether prices are efficient just before an event begins. Second, we extend this to provide a way of similarly detecting bias and tests of market efficiency in the aftermath of in-play news which is major, relevant and plausibly arrives cleanly to all participants. Third, this framework of regression models and hypothesis testing provides a practical approach to describe and evaluate some of the possible behavioural biases present in these markets, such as the well-know favourite-longshot bias (e.g. Ottaviani and Sørensen, 2008; Snowberg and Wolfers, 2010), the home bias (e.g. Levitt, 2004), under or overreaction to major news (e.g. De Bondt and Thaler, 1985, 1990) and confirmation bias (e.g. Wason, 1960; Rabin, 1998).

We apply this methodological framework, using a high-frequency data set of pre-match and in-play odds (prices), to the final result markets of 1,004 English Premier League (EPL) association football matches. These data come from the Betfair Exchange, which is the world’s largest online betting exchange. As our primary focus is on the impact of news arriving on the market, betting exchange markets, where customers can bet against each other directly, are natural candidates to test for informational efficiency and to evaluate behavioural biases among the participants. Not least, betting exchange odds have more predictive power than corresponding bookmaker odds (Smith et al., 2009; Franck et al., 2010; Reade, 2014). The efficiency of these exchange markets has been studied before, most notably by Croxson and Reade (2014), who investigated the market reaction to goals scored just before half-time in a football match. This approach allowed the authors to separate the major news of a goal being scored from the continual flow of minor news. Croxson and Reade (2014) found that these markets were semi-strong efficient, as prices generally updated swiftly and fully following a goal. We add to this previous analysis in a number of ways.

First, we study the efficiency of the pre-match result market, finding significant evidence of mispricing, which can be explained by a reverse favourite-longshot bias. In other words, the team which the market did not expect to win was significantly underpriced. Studies of the football match result odds offered by bookmakers have tended to find the opposite, and are generally consistent with the majority of other examples from sports wagering analyses since the seminal study on horse-racing by Ali (1977). Explanations of the reverse bias that we find could include that the bettors on exchange markets, compared with those using bookmakers, are more risk averse (less risk loving, e.g. Ottaviani and Sørensen, 2015) and are better informed (fewer casual bettors, e.g. Smith et al., 2009). We use these results to carry out a simple strategy of systematically betting on the significantly

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4See among others Cain et al. (2000) and Deschamps and Gergaud (2007) for English football, as well as Angelini and De Angelis (2019) more generally for European professional football.

5Ours is by no means the only study to find a reverse favourite-longshot bias. For example, Woodland and Woodland (1994) found something similar in prediction markets for US baseball, though insufficient in that case to imply significant market inefficiency.
underpriced matches within our sample. Substantial returns on investment of around 50% could have been earned by market participants who exploited the presence of the reverse favourite-longshot bias.

Second, we evaluate whether the in-play prices of a football match result are efficient following the first goal, which is major news that implies a large change in the probability of one team winning. Specifically, we identify combinations of when the first goal was scored and the pre-match odds which implied that the price after was significantly mispriced. Consistent with Croxson and Reade's (2014) study of goals around the half-time break, we find that the win odds for the team playing at home (away) were semi-strong efficient when the first goal in the match was scored by the away (home) team. In other words, at whatever point in the game the first goal arrived and independent of the pre-match odds, the prices afterward fully incorporated the new information, responded immediately and did not drift. However, we find significant evidence that the home (away) odds were mispriced when the first goal was scored by the home (away) team, mostly explained by a reverse favourite-longshot bias afterward. This mispricing was strongest twenty seconds after the goal, but still remained significant as much as five minutes later. Again, applying a simple betting strategy based on the pre-match odds and the time when the goal was scored, we show that market participants could have systematically exploited these facts to make substantial returns on investment. Thus, there is evidence that these markets were inefficient in the aftermath of common instances of major news.

Third, we test for and evaluate the behavioural biases suggested by how the betting exchange markets reacted to major news. Depending both on whether the pre-match odds suggested favourite bias, longshot bias or no bias at all for the team playing at home or away, and depending on which team scored the first goal, we test whether this major news constituted a significant change in the degree and nature of mispricing. Prior expectations were not significantly updated when the pre-match odds implied favourite bias and the goal was scored early in a match. News which arrived early in the event and which reflected expectations did not cause significant revisions in the beliefs implied by market prices. However, when the first goal was scored later, in cases where the pre-match odds reflected favourite bias, the market significantly adjusted toward unbiasedness. Conversely, where the odds were generally priced correctly before the first goal, mispricing occurred after. If a longshot scored the first goal, particularly if this happened toward the end of the match, then prices had a significant tendency to overreact, amplifying the initial mispricing, i.e. the reverse favourite-longshot bias was increased. This behaviour has been observed before in football result betting exchange markets by Choi and Hui (2014). Overall, we find

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6There is mixed evidence from conventional financial markets on whether markets under or overreact to ‘surprise’ news, depending on the type of news. For example, Brooks et al. (2003) shows that markets overreact to industrial disasters or the death of a CEO, whereas Chan (2003) finds evidence that investors underreact to headline-making news about a company.
that how prices and expectations changed following major news on the Betfair Exchange was consistent with a pattern of markets responding efficiently to expected major news but inefficiently when this news came as a surprise.

The remainder of the paper is organised as follows: Section 2 outlines a general approach to testing prediction market efficiency, both for the outcome of events before they have begun and in the aftermath of major in-play news; Section 3 describes a data set of football match prediction markets; Section 4 applies the testing approach to these data and analyses the degree of market efficiency on the Betfair exchange; and Section 5 concludes.

2 Testing the informational efficiency of in-play prediction markets

In this section, we outline a forecast-based approach to test the efficiency of prediction markets. This approach is directed toward addressing the following two main questions in Sections 2.1 and 2.2:

1. Are market prices efficient just before an event begins?

2. Are market prices efficient in the aftermath of relevant news, which occurs between the beginning of an event and its end?

The former question aims at discovering whether pre-event prediction market odds are mispriced, where the specific events we will apply this approach to later are the outcomes of association football matches. The latter question focuses on the reaction of market participants to the arrival of new and important news, which should almost certainly affect expectations about an event’s final outcome. In our application, the major news we will study are the instances of the first goal being scored by one of the teams playing in a match; football is a relatively low-scoring game, with the most common outcome of a match being 1-1, i.e. one goal scored by each of the two teams, and with typical goal-scoring rates for each team lying between one and two goals per match. These prediction markets for final event outcomes are active throughout an event’s duration, with in-play trading. Therefore, we study the new price equilibrium reached by the market after news arrives and the evolution of prices afterward. Moreover, in Section 2.3 we discuss the potential behavioural biases of prediction market participants, such as the well-known favourite-longshot bias (e.g. Ottaviani and Sørensen, 2008), and we attempt to interpret how deviations from no bias following major news could be related to any underreaction or overreaction by the participants. We also address the duration of any mispricing in prediction markets, studying whether any deviation from market efficiency persists or is absorbed quickly.

7Author calculations with thanks to J. James Reade, using the entire history of football matches listed on Soccerbase.com, i.e. from 511,759 recorded matches up to 8 January, 2019.
2.1 Are market prices efficient just before an event begins?

To evaluate whether prices in prediction markets (or betting exchanges) are set efficiently, or whether there is evidence of bias, we extend the analysis of betting markets by Angelini and De Angelis (2019). Let \( p_{i,0} \) be the (implied) probability of an outcome of event \( i \) observed pre-event (i.e. at time \( t = 0 \); or at kick-off in football terms) and let \( (p_{i,0})^{-1} \) be the corresponding pre-event prediction market price (decimal odds).\(^8\) For a given outcome (e.g. a win by the home side in a football match), we consider the market’s pre-event forecast error, computed as \( e_{i,0} = y_i - p_{i,0} \), where \( y_i = 1 \) if \( i \) ended with that specific outcome (e.g. a home win) and 0 otherwise (e.g. a draw or an away win). Then, following an approach akin to the Mincer and Zarnowitz (1969) forecast evaluation regression, consider the following model:

\[
e_{i,0} = \gamma_0 + \beta p_{i,0} + u_{i,0},
\]

where \( u_{i,0} \) is an i.i.d. error term. As Ioannidis and Peel (2005) show that forecast errors can exhibit heteroskedasticity under the null hypothesis of market efficiency, we estimate Equation (1) by Weighted Least Squares (WLS), where the \( n \times n \) weighting matrix is diagonal with elements \( \sigma^2_{1,0}, \ldots, \sigma^2_{n,0} \) and \( n \) denotes the total number of events studied. Since \( y_i \) is a Bernoulli random variable, its variance, \( \sigma^2_{i,0} \), can be approximated by \( p_{i,0}(1 - p_{i,0}) \).

The estimation results of (1) are then used to assess whether prediction markets are unbiased just before events began. In particular, a rejection of the null hypothesis,

\[
H_0: \gamma_0 = \beta = 0,
\]

implies that, conditional on all the information available regarding event \( i \), the expected value of the forecast error is not zero. Specifically, \( E(e_{i,0} | I_{i,0}) \neq 0 \), where \( I_{i,0} \) is the information set that also incorporates the implied probabilities, \( p_{i,0} \). If the null hypothesis (2) is not rejected, then the odds are set efficiently by the market participants and no bias is detected. If the null is rejected because of \( \gamma_0 \neq 0 \), then the odds imply significant forecast errors on average, perhaps because they are biased toward one outcome type over another. Similarly, we would anticipate rejecting the null if the markets were not competitive, such that on average one side of the market is earning significant profits in expectation, as is implied in the case of traditional bookmakers, for example, whereby \( \gamma_0 \) would then capture their expected profit margin (overround or vigourish in betting terms). A rejection of the null hypothesis (2) can also imply a significant relationship between the forecast error and the odds, \( \beta \neq 0 \), which in turn implies that the forecasts made by the market participants are biased for certain values of the pre-event implied probability. In other words, the odds before an event starts are mispriced, suggesting the presence of informational inefficiency.

\(^8\)See Wolfers and Zitzewitz (2006) and Manski (2006) for discussions on the interpretation of prediction market prices as probabilities.
In the spirit of Angelini and De Angelis (2019), we investigate whether any biases implied by the rejection of the null hypothesis (2) are large enough to generate market inefficiency, i.e. we test whether these deviations from no bias are significantly different from zero. In particular, consider the estimated parameter values of Equation (1), \( \hat{\theta}_0 = (\hat{\gamma}_0, \hat{\beta})' \), interpolate over all possible probability values, \( p_G \in (0, 1) \), and derive the ‘efficiency curve’ as:

\[
\hat{G}(p_G) = \hat{\gamma}_0 + \hat{\beta} p_G .
\] (3)

The related confidence bands are then computed as:

\[
CI_0 = [CI_{L0}, CI_{U0}] ,
= [\hat{G}(p_G) - z_{\alpha/2} \text{ s.e.} \left(\hat{G}(p_G)\right), \hat{G}(p_G) + z_{\alpha/2} \text{ s.e.} \left(\hat{G}(p_G)\right)] ,
\] (4)

where \( z_{\alpha/2} \) is the \( 100(1 - \alpha/2) \)-th percentile of the standard normal distribution, \( s.e. \left(\hat{G}(p_G)\right) = \left[\nabla \hat{G}(p_G)' V_{WLS} \nabla \hat{G}(p_G)\right]^{1/2} \), \( \nabla \hat{G}(p_G) = (1, p_G)' \) is the gradient, and \( V_{WLS} \) is the variance of the WLS estimator.

The confidence intervals in (4) are useful as a procedure to test market efficiency and to evaluate prediction market bias. We define the probability ranges where either the lower bound of the confidence interval is larger than zero or the upper bound of the confidence interval is smaller than zero:

\[
Q_0 = \{ p_{i,0} \in P : CI_{L0} > 0 \} ,
\]
(5)

\[
\bar{Q}_0 = \{ p_{i,0} \in P : CI_{U0} < 0 \} ,
\]
(6)

where \( P = \{ p : 0 < p < 1 \} \), whereas \( CI_{L0} \) and \( CI_{U0} \) denote the lower and upper confidence bounds reported in (4), respectively.

These ranges, if any, define the values of \( p_{i,0} \) to which a bias in the pre-event prices correspond. Specifically, \( Q_0 (\bar{Q}_0) \) defines the range of implied probabilities which corresponds to an underpricing (overpricing) in the prediction market. To test whether these biases are large enough that profitable opportunities exist for participants, which in turn would imply pre-event market inefficiency, we develop a simple betting strategy. We systematically wager on all events within the estimation sample in which odds are identified as being generally underpriced, i.e. in all cases where the implied probabilities belong to \( Q_0 \) in (5). According to Fama’s EMH, a positive return on investment (ROI) implies that the prediction market is (weak-form) inefficient.
2.2 Are market prices efficient in the aftermath of in-play news?

In the language of online in-play prediction and financial markets, we consider the time since the event began as a number of discrete ‘ticks’, which corresponds to discrete multiples of some amount of time, for example ten seconds. In terms of a football match, these ticks translate to the amount of time played, or ‘on the clock’, where the clock in football only stops for the half-time interval between the beginning (‘kick-off’) and the end of the match (‘final whistle’).

Consider some type of major and relevant news about an event $i$’s outcome that arrives after it has begun (in-play) at tick $t$, and let $p_{i,t+h}$ be the implied probability of an outcome observed after the news arrives, i.e. at tick $t+h$, for $h = 1, 2, ..., H$, where $H$ in each case is constrained by the end of the event. Therefore, $(p_{i,t+h})^{-1}$ represents the new equilibrium price that the market sets $h$ ticks after the arrival of the specific piece of new information.

To evaluate whether the new equilibrium prices are set efficiently, or whether there is evidence of bias in how the market processes information, we extend the method described above to deal with high frequency data and in the style of an event study, where the event being studied in this case is the arrival of in-play news. Mimicking the approach described in Section 2.1 for the case of pre-event odds, the market forecast error $h$ ticks after the in-play news is given by $e_{i,t+h} = y_i - p_{i,t+h}$, and we consider the following model:

$$e_{i,t+h} = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \beta p_{i,\tau} + u_{i,t+h},$$

(7)

where $p_{i,\tau}$ is the probability of the outcome at tick $\tau$, for $\tau = 0, 1, ..., t - 1$. For example, $p_{i,0}$ denotes the pre-event probability of the outcome whereas $p_{i,t-1}$ is the probability of the outcome one tick before the news arrives and $u_{i,t+h}$ is an i.i.d. error term. As for the case of pre-event odds in Section 2.1, to account for the heteroskedasticity of the forecast errors, we estimate Equation (7) by Weighted Least Squares (WLS), where the $n_j \times n_j$ weighting matrix is diagonal with elements $\sigma^2_{1,\tau}, \ldots, \sigma^2_{n_j,\tau}$, where $n_j$ denotes the number of events where the in-play news of type $j$ occurs (e.g. a goal scored by the home team in a football match). Again, since $y_i$ is a Bernoulli random variable, we can approximate $\sigma^2_{i,\tau}$ with $p_{i,\tau}(1 - p_{i,\tau})$, thus yielding a weighting matrix $W_{\tau} = \text{diag}[\sigma^2_{i,\tau}] = \text{diag}[p_{i,\tau}(1 - p_{i,\tau})]$ in the WLS estimation.

The results from estimating Equation (7) are used to assess whether the prediction markets are generally unbiased after in-play news. In particular, a non-rejection of the null hypothesis,

$$H_0 : \gamma_0 = \gamma_1 = \gamma_2 = \beta = 0,$$

(8)

implies that the expected value of the forecast error is zero, conditional on all the information available on event $i$ until tick $t$, i.e. including all other in-play news before and related to the
particular news studied, which arrives at tick $t$. This would in turn imply that the prediction market is efficient $h$ ticks afterward. More specifically, we would have $E(e_{i,t+k}|I_{i,t}) = 0$, where $I_{i,t}$ is the information set which also incorporates regressors $t$ and $p_{i,\tau}$, for $\tau = 0, \ldots, t - 1$. Conversely, a rejection of the null hypothesis (8) implies a significant relationship between the forecast error and (at least) one of the regressors. This would in turn imply that the forecast of the market participants is biased for certain values of $t$ and $p$. In other words, the arrival of news creates informational inefficiency as mispricing is observed in the market. Similarly to the case of pre-event prices, we investigate whether these deviations from no bias are large enough to generate market inefficiency. Using the estimated parameter values of Equation (7), $\hat{\theta} = (\hat{\gamma}_0, \hat{\gamma}_1, \hat{\gamma}_2, \hat{\beta})'$, we interpolate over all possible values of $p_G \in (0, 1)$ and $t_G \in (0, 550)$ and derive the efficiency curve as:

$$\hat{G}(t_G, p_G) = \hat{\gamma}_0 + \hat{\gamma}_1 t_G + \hat{\gamma}_2 t_G^2 + \hat{\beta} p_G,$$

as well as the related confidence bands as:

$$CI = [\hat{G}(t_G, p_G) - z_{\alpha/2} \text{ s.e.} \left(\hat{G}(t_G, p_G)\right), \hat{G}(t_G, p_G) + z_{\alpha/2} \text{ s.e.} \left(\hat{G}(t_G, p_G)\right)],$$

where $s.e. \left(\hat{G}(t_G, p_G)\right) = \left[\nabla \hat{G}(t_G, p_G) V_{WLS} \nabla \hat{G}(t_G, p_G)\right]^{1/2}$ and $\nabla \hat{G}(t_G, p_G) = (1, t, t^2, p_G)'$ is the gradient and $V_{WLS}$ is the variance of the WLS estimator.

The purpose of this is to define regions where either the lower bound of the confidence interval is larger than zero or the upper bound of the confidence interval is smaller than zero. More specifically, for the case of post-news efficiency:

$$Q = \{(t, p_{i,\tau}), t \in T, p_{i,\tau} \in P : CI > 0\},$$

$$\overline{Q} = \{(t, p_{i,\tau}), t \in T, p_{i,\tau} \in P : CI < 0\},$$

where $T = \{t : 0 < t < 550\}$, $P = \{p : 0 < p < 1\}$, and $CI$ and $\overline{CI}$ denote the lower and upper confidence bounds reported in (10), respectively. These regions, if any, define the combinations of when the major in-play news arrives, $t$, and the event outcome probability prior to this, $p_{i,\tau}$, which correspond to bias in the prediction markets. $Q$ ($\overline{Q}$) defines the combinations of the news occurring at tick $t$ and the prior event outcome probability, $p_{i,\tau}$, which in general correspond to an underpricing (overpricing) in the in-play odds of the studied markets. To test whether these biases are large enough to imply in-play market inefficiency, we evaluate whether positive returns can be achieved. We do this by systematically betting on all the events within sample, when the combinations of implied probabilities of some final outcome and the arrival ticks of particular types of in-play news are in the region $Q$, i.e. in all cases where the in-play odds are underpriced. A positive
ROI would then imply that the set of prediction markets studied are generally not efficient in semi-strong form.

Moreover, the analysis of the prediction market forecast errors in (7) can be repeated for different values of $h$. This allows us to evaluate not only whether the market is inefficient but also how long any mispricing lasts, and how much time is required by the participants to accurately process the news, i.e. how much time is needed to absorb any biases and potentially adjust or re-adjust toward efficiency.

2.3 Detection of bias in prediction markets

The methods described above will potentially provide evidence of deviation from no bias within prediction markets. In the following, we provide an interpretation of possible biases and a test of whether the arrival of major news on the market provokes a significant change in participant’s beliefs about an event’s outcome.

The well-known favourite-longshot bias postulates that the odds on expected winners are underpriced while the odds on unlikely winners are overpriced, which typically implies that wagering on favourites is more profitable than wagering on longshots (e.g. Ali, 1977; Thaler and Ziemba, 1988; Ottaviani and Sørensen, 2008). In a prediction market containing bettors with heterogeneous beliefs, who are risk-neutral price takers, Manski (2006) showed formally that the overpricing (upward bias) of the longshot would arise in equilibrium due to the combination of budget constraints and skewed payoffs. Ottaviani and Sørensen (2015) and He and Treich (2017) generalised this result to broader sets of risk preferences, demonstrating sufficient conditions such that the favourite-longshot bias would emerge in prices. For example, the latter authors showed that this occurs when twice the degree of absolute risk aversion of participants is less than the degree of absolute prudence. In the case of constant relative risk aversion, this occurs when bettors are less risk averse than implied by logarithmic utility. If bettors are more risk averse, then the direction of the bias in prices could be reversed. Ottaviani and Sørensen (2015) also showed that the favourite-longshot bias would emerge among risk averse bettors with bounded wealth, or among bettors with unbounded wealth but decreasing risk aversion with wealth, as an underreaction to public information. However, a dynamic version of that model also predicts that this bias ought to be reversed over time. Besides these predictions from neoclassical theory, there is a competing set of behavioural explanations for the favourite-longshot bias, which emphasises the misperception of probabilities. Snowberg and Wolfers (2010) looked to distinguish the behavioural and neoclassical explanations using exotic bets on US horse racing. They found evidence suggesting that bettors’ inability to distinguish between different low probabilities, rather than risk-love, appears to explain why longshots are overbet.

The presence of the favourite-longshot bias in prediction markets can be evaluated by
testing whether the slope of the \( p_{i,0} \) or \( p_{i,\tau} \) regressors in Equations (1) and (7), respectively, are zero against the following two alternatives:

\[
\begin{align*}
H_0 : \beta &= 0 \quad \text{no bias} \\
H_{1A} : \beta &> 0 \quad \text{favourite-longshot bias} \\
H_{1B} : \beta &< 0 \quad \text{reverse favourite-longshot bias} .
\end{align*}
\] (13)

Further, we can compare the degree of bias in the market pre-event with the aftermath of major types of in-play news. Consider the initial or pre-event forecast errors, \( e_{i,0} \), and the post-news forecast errors, \( e_{i,t+h} \), to investigate whether the same biases apply before an event begins and after in-play news changes participants’ prior expectations. In particular, define the variable:

\[
\xi_{i,t} = \hat{\theta}' x_{i,t},
\] (14)

which measures the mispricing at tick \( t \) for \( x_{i,t} = (1, p_{i,0}, t, t^2)' \), and where \( \hat{\theta} = (\hat{\gamma}_0, \hat{\beta}, \hat{\gamma}_1, \hat{\gamma}_2)' \) are the estimated parameters from (3) or (9). In the former case for \( t = 0 \): \( (\hat{\gamma}_1, \hat{\gamma}_2) = 0 \).

The difference between the post-news and pre-event bias is defined as:

\[
\Xi_{i,t} = \xi_{i,t} - \xi_{i,0} ,
\] (15)

and we test the following null hypothesis against the alternatives:

\[
\begin{align*}
H_0 : E(\Xi_{i,t}) &= 0 \\
H_{1A} : E(\Xi_{i,t}) &> 0 \\
H_{1B} : E(\Xi_{i,t}) &< 0 .
\end{align*}
\] (16)

Not rejecting the null in (16) implies that market participants do not significantly react to the arrival of new information and stick to their prior beliefs about an event’s outcome. A rejection of the null in favour of either alternative in (16) suggests that the news significantly changes the participants’ expectations of an event’s outcome.

In Table 1, we summarise possible combinations of the pre-event degree of bias, \( \xi_{i,0} \), and the post-news market reaction, measured by \( \Xi_{i,t} \). If we observe no pre-event deviation from no bias, \( \xi_{0,t} \approx 0 \), (top panel of Table 1) and we reject the null in (16), then the arrival of new information on the market creates mispricing, as participants adjust their expectations and deviate from no bias. The middle panel of Table 1 shows that, starting from a situation of pre-event positive mispricing, \( \xi_{i,0} > 0 \), the in-play news at tick \( t \) may lead to: (i) the same degree of bias on the market as before, i.e. we do not reject the null in (16); (ii) an amplification of the positive mispricing when \( H_0 \) is rejected in favour of \( H_{1A} \); (iii) a significant reduction of the mispricing when \( H_0 \) is rejected in favour of \( H_{1B} \). The
latter may result in the market completely absorbing or even reversing the previous bias in the aftermath of the new information. The interpretation of the market reaction to new information in the case of pre-event negative mispricing, $\xi_{i,0} < 0$, is reported in the lower panel of Table 1 and is opposite to the case of $\xi_{i,0} > 0$.

Table 1: Combinations of pre-event market mispricing, results of the hypotheses testing in (16) and interpretations of the market reaction to in-play news at tick $t$.

<table>
<thead>
<tr>
<th>Pre-event</th>
<th>Result of the test</th>
<th>Interpretation of the market reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>No bias ($\xi_{i,0} \approx 0$)</td>
<td>Accept $H_0$</td>
<td>No change in beliefs: still no bias</td>
</tr>
<tr>
<td></td>
<td>Reject $H_0$ for $H_{1A}$</td>
<td>Creating positive mispricing</td>
</tr>
<tr>
<td></td>
<td>Reject $H_0$ for $H_{1B}$</td>
<td>Creating negative mispricing</td>
</tr>
<tr>
<td>Positive mispricing ($\xi_{i,0} &gt; 0$)</td>
<td>Accept $H_0$</td>
<td>No change in beliefs: still positive mispricing</td>
</tr>
<tr>
<td></td>
<td>Reject $H_0$ for $H_{1A}$</td>
<td>Amplifying mispricing</td>
</tr>
<tr>
<td></td>
<td>Reject $H_0$ for $H_{1B}$</td>
<td>Absorbing mispricing</td>
</tr>
<tr>
<td>Negative mispricing ($\xi_{i,0} &lt; 0$)</td>
<td>Accept $H_0$</td>
<td>No change in beliefs: still negative mispricing</td>
</tr>
<tr>
<td></td>
<td>Reject $H_0$ for $H_{1A}$</td>
<td>Absorbing mispricing</td>
</tr>
<tr>
<td></td>
<td>Reject $H_0$ for $H_{1B}$</td>
<td>Amplifying mispricing</td>
</tr>
</tbody>
</table>

The idea that market participants overreact to salient new information, suggested by Kahneman and Tversky (1973) has been extensively studied and, for example, has been demonstrated in practice by participants in the stock market (e.g. De Bondt and Thaler, 1985, 1990). For betting markets on professional football matches, Choi and Hui (2014) found that market reaction to particularly surprising in-play outcomes overcompensates the typical underreaction in these markets to news, and, therefore, creates an opposite mispricing on the market after the surprise. We also investigate whether unexpected news during an event, i.e. in-play outcomes that are characterised by a low probability, lead to overreaction. In our particular football betting application, we look for evidence of this by comparing the reaction and evolution of in-play odds after goals are scored by teams which are either less or more fancied to win the match.

Then, we empirically investigate under which conditions a confirmation bias (Wason, 1960; Rabin, 1998) can be suggestively found in prediction markets. We interpret any cases in which we do not reject the null in (16) and mispricing was detected before the event began, i.e. $\xi_{i,0} \neq 0$, as evidence of a confirmation bias. Such evidence would suggest that participants stick to their prior beliefs after new information breaks on the market, even though these beliefs were biased. In that sense, the new information is perceived by the market as not informative enough to provoke a reaction to compensate for the previous mispricing.

Finally, we analyse a further well-documented bias in sports betting markets. Among others, Levitt (2004) and Vlastakis et al. (2009) show that bettors tend to overestimate the probability of the home team winning, i.e. home bias. Notwithstanding the fact that playing at home significantly increases the probability of winning the match (e.g. Nevill
and Holder, 1999), bettors tend to overvalue the actual chance of the home team winning, and this bias is amplified when the home team is the favourite to win.

3 Data & estimation

We use a sample of $n = 1,004$ matches played in the English Premier League (EPL) from 15th August, 2009 to 11th May, 2015. For each match we observe the in-play odds (prices) collected every 10 seconds on the Betfair Exchange betting market for the final result outcome, i.e. whether the game finishes in a draw (tie) or a win for either the team playing at home or away.

A betting exchange operates as a limit order-driven market, which matches the ‘back’ and ‘lay’ orders, that is the bets on and against an outcome, respectively. Essentially, this allows individuals to bet against each other directly. The back and lay odds are equivalent to the bid and ask prices in financial markets. In betting exchange markets, the prices (odds) are not dictated by market makers (bookmakers), but the bettors can buy (back) or sell (lay) bets both pre-match and during the game. Moreover, as Betfair charges a commission of up to 5% on net winnings ex post, falling to 2% for heavy bettors, this is not reflected in the data. In a nutshell, the matched bets on each outcome are zero-sum games between the back and lay bettors and, given that there is no bookmaker, any biases we observe from the betting exchange odds should derive from the behaviour of market participants.

Betfair operates the world’s biggest exchange by volume traded and claims to have millions of customers. The arrival of the online betting exchanges in 2000 in the UK is credited with revolutionising the betting industry, driving down bookmaker profit margins (overrounds) and increasing competition (Forrest et al., 2005). As outlined by Croxson and Reade (2014), the number of daily trades on the Betfair Exchange has historically been greater than all the European Stock Exchanges combined. There is no liquidity issue in the prediction markets we study, as also described by Croxson and Reade (2014). This is the case both pre-match and during the match itself. It is generally a feature of the most popular betting exchange markets that the volume of trading multiplies after the event has begun. To illustrate this, we present in Figure 1 scraped data from a recent EPL match, showing the cumulative amount of money (pounds sterling) matched on the market for the final result (i.e. home, away win or draw), from 90 minutes before kick-off up to the point the market closed after the final whistle. The vertical dashed line shows when the match began. At this point, £4.7 million of bets had been matched, but by market close this figure was £12.3 million. Also shown in Figure 1 is what the exchange terms the ‘book percentage’ (right axis) throughout the duration of the market. From the perspective of the backer, this

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9The authors were unable to collect data for all EPL matches in this period due to technical and practical difficulties, though the selection is random.
is the sum over all possible event outcomes of the odds-implied probabilities, or in other words one plus the exchange market equivalent of a bookmaker’s overround, and thus it gives a measure of competitiveness. For the vast majority of the event this measure was less than 101%, implying that the prices being offered were competitive.\textsuperscript{10}

The data we analyse concern the actual prices at which trades were made, rather than the back or lay prices being offered at any point in time. Figure 2 shows an example of these in-play betting exchange data, using the Southampton vs. Manchester United match played on 11th May, 2014. The time series of the odds-implied probabilities are depicted for each of the possible final result outcomes and for all ticks from 1 to 550, where each additional tick corresponds to 10 seconds of the match. We can observe two distinct jumps in the patterns of the implied probability series in this match, caused by two separate instances of major news. The first of these is a goal scored at tick 163 by Southampton, which provoked an abrupt change in the three outcome probabilities and, subsequently, a new market equilibrium was reached. Specifically, the implied probability of a Southampton win increased from 0.35 to 0.62 after the goal, while the draw and Manchester United win probabilities dropped to 0.23 and 0.14 from 0.31 and 0.33, respectively. The second major news is a goal by Manchester United at tick 322, which promptly increased the implied probabilities of the draw and the away win, and dramatically decreased the home win probability. Thereafter, since no other major news arrived on the market, such as more goals or a player being dismissed, the draw probability tended to 1 toward the end of the match, while both the home and away win probabilities shrank toward 0.

Importantly for the application of our methodology in what follows, Betfair briefly suspends the final result markets at kick-off and whenever a ‘Material Event’ is deemed to have occurred. The latter includes the awarding of red cards or penalty kicks, but the most significant material events in terms of price reactions are goals being scored. In these cases, the market is suspended only so long as it takes for a goal to be awarded with certainty to one team or the other, which is typically just a few seconds. This delay is put in place in case the referee or another official rules the goal out, for example because of a player being deemed offside or a foul being spotted in the build-up to the goal. When the market is suspended, all unfilled orders are cancelled, clearing out the market.\textsuperscript{11} As soon as the market re-opens we observe an immediate jump in prices, as shown in Figure 2 and previously demonstrated by Croxson and Reade (2014). We observe the tick and prices just before and just after a goal is scored.

\textsuperscript{10}Unfortunately we do not have data on the market volumes traded or competitiveness of the matches in our analysis sample. However, in addition to the example shown, the data employed by Croxson and Reade (2014) for an earlier period demonstrate that these markets are heavily trade, liquid and competitive.

\textsuperscript{11}In fact, Betfair actively voids any bets that were ‘unfairly’ matched after a material event if the market was not suspended on time. See Betfair Rules and Regulations, Part B, 1.3 (November 2019): https://www.betfair.com/aboutUs/Rules.and.Regulations/.
3.1 Estimation

We apply the methodology described in Section 2 to the data set of \( n = 1,004 \) matches (events), to evaluate the efficiency of the exchange betting market. We use the prices backing either a home or away win throughout the analysis. The odds on ties in football are generally bounded, both pre-match and after the first goal, in which latter case the draw outcome becomes even more unlikely. As a consequence, the efficiency curves in Equations (3) and (9) cannot be computed over all values of \( p_G \in (0,1) \). Therefore, we do not consider the draw outcome in our analysis.

First, we concentrate on the pre-event market, as per Section 2.1. Second, we study the markets after the arrival of major in-play news, as per Section 2.2. In particular, we focus on the ‘first goal’ of a match as major news.\(^{12}\) Therefore, for this part of the analysis, we exclude all matches which ended with no goals scored, such that \( \tilde{n} = 882 \), of which \( \tilde{n}_H = 513 \) are ‘home team goal’ matches and \( \tilde{n}_A = 369 \) are ‘away team goal’ matches. In estimating Equation (7), we consider \( p_{i,\tau} \) with \( \tau = 0 \), i.e. we consider the pre-match probability as the regressor.\(^{13}\) Focusing on the first goal of a match and \( p_{i,0} \) as the regressor in (7) allows us to consider an exhaustive range of possible combinations of tick and implied probabilities, which enables us to study the evolution of prices after events and investigate the market participants’ reaction in four different scenarios: (i) home odds after the first goal is scored by the home team \((HH)\); (ii) home odds after the first goal is scored by the away team \((HA)\), (iii) away odds after the first goal is scored by the home team \((AH)\), (iv) away odds after the first goal is scored by the away team \((AA)\).

4 Empirical analysis of Betfair Exchange prediction markets

In this section, we show the results from applying the bias and efficiency testing approach described above to our sample of Betfair Exchange prediction markets. By doing so, we look to answer the questions posed in Section 2 within this particular context. First, in Section 4.1, we investigate the market efficiency for the prices of final result outcomes set just before the beginning of a football match (event). Second, in Section 4.2, we address the semi-strong form of market efficiency, focusing on how these prediction markets react

\(^{12}\)We consider the ‘first goal’ as major news in the sense that it significantly affects the in-play odds (see Figure 2).

\(^{13}\)We also estimate the model using the probability of the match outcome prior to the goal, i.e. \( p_{i,t-1} \), in (7), and these results are available upon request. However, despite the informative content provided by this regressor, the issue is that considering the probability before the goal, in practice, rules out several combinations of \((t,p)\), especially cases of large \( t \) and large \( p \). As a matter of fact, it is unlikely to observe cases in which no goals have yet to be scored as a match nears its end but where one of the teams has a high win probability. In these cases the draw is the most likely outcome and, in general, odds are set accordingly by the market.
after the first goal is scored (major news). Finally, in Section 4.3 we analyse and interpret the possible presence of the behavioural biases described in Section 2.3.

4.1 Are market prices efficient just before a match kicks off?

We evaluate whether the exchange market participants set prices (odds) efficiently before the beginning of a match. This analysis replicates what was carried out by Angelini and De Angelis (2019) for online bookmaker markets. The results are reported in Table 2. The top panel of the table reports the estimates of Equation (1), the middle panel depicts the derived efficiency curves \( \hat{G}(p_G) \) as per Equation (3) and over all possible values of \( p_G \in (0, 1) \), and the bottom panel shows the returns on investment from applying a betting strategy based on Equation (5), i.e. betting the same amount in all cases where the implied probabilities at the start of a match are significantly underpriced.

The results of the \( F \)-tests reported in the top panel of Table 2 show a rejection of the null hypothesis in (2) for both the home and away win odds. Therefore, the pre-match odds for the winning outcomes on the Betfair Exchange markets are generally not set efficiently, and thus there is evidence of mispricing (bias). The estimated slopes, \( \hat{\beta} \), are significantly negative. This result suggests a reverse favourite-longshot bias, as we reject the null hypothesis, \( H_0: \beta = 0 \), in favour of the alternative, \( H_{1B} \) in (13). Therefore, bettors operating in the exchange betting markets back too strongly the teams which are expected to win, such that wagering on longshots would tend to offer greater expected returns. This evidence is opposite to the result typically found for fixed-odds bookmaker markets on football matches (e.g. Angelini and De Angelis, 2019). In line with Smith et al. (2009) and Snowberg and Wolfers (2010), in addition to the theoretical predictions in Ottaviani and Sørensen (2015) and He and Treich (2017), this evidence of a reverse favourite-longshot bias in exchange betting markets could be explained by a higher degree of risk aversion among the bettors operating in these markets compared with those using fixed-odds bookmakers.

The results also show that the estimated intercept of Equation (1) is significantly positive, though not enough on average to offset the favourite-longshot bias, for both the home and away win outcomes, implying on average negative forecast errors. This would suggest that the market participants are biased toward a win outcome and away from the draw, which would be consistent with a ‘splitting’ bias, or ‘black and white thinking’. The significant tendency of individuals to under-predict draws in football matches has been documented before by Na et al. (2019) in an experimental setting.

The estimated efficiency curves, \( \hat{G}(p_G) \), and the related confidence bands computed as per (4) define the probability ranges \( Q_0 \) in (5) for home and away odds (middle panel of Table 2). Due to the estimated negative slope (and positive \( \hat{\gamma}_0 \)), these probability ranges are defined by the lowest probabilities (i.e. the highest odds), as \( Q_0 \) is given by \( 0 < p_G \leq 0.24 \) and \( 0 < p_G \leq 0.14 \) for home and away odds, respectively. Therefore, consistent with the reverse
favourite-longshot bias, higher probabilities (smaller odds) are more likely overpriced, while smaller probabilities (higher odds) are more likely underpriced.

To evaluate the forecasting performance of these markets and their efficiency, the simple betting strategy described in Section 2.1 is applied in-sample (ex post) to all \( n \) matches in our data set. The results reported in the bottom panel of Table 2 show that by systematically wagering the same amount on all the home (away) odds of matches whose pre-match probability was in the range \( Q_0 = (0, 0.24) \) (\( Q_0 = (0, 0.14) \)), i.e. on each of the 183 home and 195 away matches where the odds were larger than 4.16 and 7.14, respectively, we would have earned a substantial ROI of 40% in the case of home wins and 56% for away wins, before paying any commission. From this evidence, we can conclude that the reverse favourite-longshot bias detected within pre-match odds, for final results on the Betfair Exchange, is large enough to create profitable opportunities for bettors in expectation, using a relatively simple betting strategy. Therefore, these markets are (weak-form) inefficient. This result is in spite of the apparent overpricing of the win outcomes relative to the draw outcome in these football matches.

### 4.2 Are market prices efficient after the first goal is scored?

We now consider whether the odds set on the Betfair Exchange in the aftermath of the first goal scored are unbiased or not. As a preliminary analysis of the in-play data, Figure 3 reports the mean of the jumps in the implied probabilities \( h = 2 \) ticks after the first goal in each of the four cases considered, namely \{HH, HA, AH, AA\}. The behaviour of the implied win probabilities, \( p_{i,t+2} \), in the HH and AA cases is similar. The magnitude of the jump increases along with the tick when the first goal was scored. The higher jumps in the probability of a win are concentrated around cases where the pre-match probability was 0.50. However, for the cases of HA and AH, it is less easy to observe regularity in the pattern of probability jumps following the arrival of the first goal, and overall the mean changes are lower in absolute value.

We estimate the model in Equation (7) with the pre-match probability \( p_{i,0} \) as a regressor (i.e. \( \tau = 0 \)) and then consider different horizons after the major news arrives, namely \( h = \{2, 5, 30\} \) ticks after the first goal.\(^{14}\) Table 3 reports the estimated coefficients and the efficiency tests for the null hypothesis in (8). The F-tests reject the null of market efficiency for the cases of HH and AA in the top-left and bottom-right panels, respectively, for all horizons \( h \) considered, expect for \( h = 5 \) in the AA case. Conversely, for the two cases of HA and AH (top-right and bottom-left of Table 3, respectively) the exchange market prices evolve efficiently after the arrival of major news for all horizons considered, as the F-tests

\(^{14}\)We exclude from the analysis all matches in which further major news arrives after the first goal and before tick \( t + h \), e.g. a second goal.
for the null in (8) do not reject market efficiency.\footnote{We control for misspecification in the functional form of (7) with a set of Ramsey RESET tests. The results indicate that the model in (7) is correctly specified.}

When the home team scores first, the estimated coefficients reported in Table 3 show that the average market bias is mainly explained by the time when the first goal was scored, i.e. $\hat{\gamma}_1$ is significantly greater than zero at the 5\% level for all horizons considered, while $\hat{\gamma}_2$ is only so for $h = 2$. Conversely, the pre-match implied probability is (negatively) significant at the 5\% level only twenty seconds after the first goal is scored, i.e. $h = 2$, but not for $h = 5$ (one minute) or $h = 30$ (five minutes). Therefore, we find evidence of reverse favourite-longshot bias, i.e. we reject $H_0$ in (13) in favour of $H_1$, though only for the case of $h = 2$. As such, this bias tends to be absorbed by the market as $h$ increases. However, the markets generally still do not achieve efficiency as long as five minutes after the first goal was scored, mainly because of the significance of $\hat{\gamma}_1$. This is also the case when the away team scores the first goal. However, the reverse favourite-longshot bias is more evident in this case. The results in the bottom-right panel of Table 3 show that the reaction of away odds is only significantly explained by the pre-match probability of an away team win when they have scored the first goal, as $\hat{\beta}$ is significantly negative for $h = 2$ and $h = 30$, at least at the 5\% level, and for $h = 5$ at the 10\% level.

Figure 4 plots in blue the efficiency curves $\hat{G}(t_G,p_G)$ according to (9), and in red the related 90\% confidence bands as per (10), for $h = 2$ ticks after the first goal is scored. From these plots we can identify the combinations of tick, $t$, and pre-match probability, $p_{i,0}$, where the conditions $C_I > 0$ and $\bar{C}_I < 0$ in (11) and (12), respectively, are satisfied. For instance, we observe that the combination of large (small) $t$ and small (large) $p_{i,0}$ satisfies condition $C_I > 0$ ($\bar{C}_I < 0$) for the cases of $HH$ and $AA$. In line with the results in Table 3, we cannot distinguish such combinations in the cases of $HA$ and $AH$, especially for the condition $C_I > 0$.

To better identify the combinations ($t, p_{i,0}$) which imply inefficiency, Figure 5 extrapolates from Figure 4 the underpriced areas, $Q$, (in green) and the overpriced areas, $\bar{Q}$, (in yellow) for $h = \{2, 5, 30\}$ ticks after the first goal. It shows that the evolution of the in-play odds afterward is similar in the cases of $HH$ and $AA$. The underpriced odds are concentrated in the area where the pre-match probability is roughly below 0.7 and 0.5 for $HH$ and $AA$, respectively. One implication suggested by these results is that a goal scored by a pre-match longshot team leads to mispricing of the team’s win probability. In particular, this “surprise” news is unexpected by the market participants, and thus the underpriced odds set after the goal is scored could be explained by a lack of confidence in the now greater possibility that the longshot will eventually win the match. Conversely, the overpriced odds are concentrated in the area where the pre-match probability is larger and the first goal is scored in the early stages of a match, namely $t$ approximately smaller than
150 ticks (the first twenty-five minutes). This suggests that a match’s first goal scored early by the favourite team is generally expected by the market participants, and thus the price after the goal comes is set with excess confidence, since it reaffirms beliefs that the favourite team will win.

To evaluate the efficiency of the market, the betting strategy outlined in Section 2.2 is applied in-sample (ex post) to the sub-sample of $\tilde{n} = 882$ matches. The results for the underpriced odds after the first goal is scored, i.e. those satisfying the condition in $Q$ for the $HH$ and $AA$ cases, are reported in the left panel of Table 4. These results show that, by systematically wagering the same amount on all matches in the region $Q$ for the $HH$ ($AA$) case, a ROI is earned of 35.27%, 10.74% and 11.97% (70.42%, 67.16% and 24.58%) for $h = 2, 5, \text{ and } 30$ ticks after the goal, respectively. As a robustness check, we present in the right panel of Table 4 the results form similarly and systematically betting in-sample on the region of overpriced odds, $\overline{Q}$. From these results, it is evident that betting in this way would generally lead to negative returns.\footnote{We also performed an out-of-sample betting strategy exercise for the last 100 matches in the sample, for both pre-match and in-play odds; i.e. we did not use these matches to identify the regions of mispricing. The results from this are qualitatively similar to the in-sample ones reported in Tables 2-4, and are available on request.}

In summary, we find significant and substantial evidence that the in-play odds in football result Betfair Exchange markets are (semi-strong form) inefficient in the aftermath of the first goal, if we consider the reaction of home (away) win odds to a goal scored by the home (away) team. However, this is not the case for other combinations of outcome and identity of the team scoring the first goal, i.e. the $HA$ and $AH$ cases.

4.3 Detection of bias on the Betfair Exchange

In this section, we evaluate whether and how new information impacts on bettors’ previous expectations. Table 5 shows the results from testing whether these expectations (biases) changed for all three of the different pre-match cases reported in Table 1, for $h = 2$ ticks after the first goal is scored. In particular, we consider the odds of the overpriced favourite teams, with an implied pre-match win probability larger than or equal to 0.75 ($p_{i,0} \geq 0.75$), and the underpriced longshot teams, $p_{i,0} \leq 0.20$. We also consider the odds when the pre-match market was unbiased, as previously described in Table 2: $0.25 \geq p_{i,0} \geq 0.45$ and $0.15 \geq p_{i,0} \geq 0.32$ for the home and away odds, respectively.

There are two cases where it appears as though previous expectations are not updated after the first goal arrives in a match. Both relate to the favourite team scoring in the first fifteen minutes ($t \leq 90$), i.e. cases of pre-match upward biased prices for either the home or away win. In these two cases we do not reject the null hypothesis in (16). This emphasises that an early goal scored by a favourite team is somehow expected by market participants,
and does not alter the pre-match mispricing. In other words, the probability that the bettors aspire to the favourite team remains higher than it should be. This is evidence that prices are affected by confirmation bias, i.e. individuals process the implications of the major news in a way which confirms their prior expectations (Wason, 1960; Rabin, 1998). However, from the results reported in the first row of each panel of Table 5, we observe an absorption of the pre-match mispricing, rejecting $H_0$ in favour of $H_{1A}$ in (16), as a reaction when the first goal is either scored later in a match by a favourite or scored at any time by a longshot. These latter results suggest that in some circumstances the market participants do correctly process the arrival of new information and the exchange market then tends to adjust toward unbiasedness.\footnote{Page and Clemen (2012) also found that the favourite long-shot bias in prediction market prices of political events tends to be absorbed by markets over long periods of time and as the expiration date approaches. However, this study did not address whether the absorption was driven by major news arriving on the market, but rather instead modelled it as a mechanical effect of time discounting by participants, given that the length of time studied was considerably longer.}

Another interesting case concerns the effect of the first goal arriving when the market was generally unbiased at kick-off. The second row of each panel in Table 5 shows that we reject $H_0$ in (16) in all these cases, thus losing market unbiasedness after the major news arrives. The first goal being scored by the away team generally produces a downward (upward) bias on the home (away) odds, suggesting that the market participants do not believe sufficiently that the away team will eventually win. This result is in support of a home bias (e.g. Levitt, 2004). In the case of the first goal being scored by the home team, the results are mixed. We observe a downward (upward) bias for home (away) odds if the first goal occurs in the first fifteen minutes of a match ($t \leq 90$), while the opposite market reaction is observed when the first goal occurs late in a match ($t \geq 275$). These results are difficult to interpret and this could be due to the large range of $p_{i,0}$ considered.

The last row of each panel in Table 5 reports results relating to the pre-match upward bias of longshot odds. In this case, the mispricing tends to be absorbed after the favourite scores the first goal, both for home and away win odds. Conversely, this mispricing is amplified when the first goal is scored by the longshot, expect for the case of an early goal scored when the longshot is playing at home. Therefore, we find evidence that the market participants react correctly to expected news (i.e. the first goal scored by the favourite team), while the reverse favourite-longshot bias is amplified when there is surprise news (i.e. the first goal scored by the longshot), exaggerating the pre-existing reverse favourite-longshot bias and mispricing in the market. These results are consistent with the findings in Choi and Hui (2014), also from the Betfair Exchange football markets, which show that unanticipated major news typically leads to a market overreaction.
5 Concluding remarks

In this paper, we proposed a practical framework which could be used to generally investigate the behaviour of participants in prediction markets. We showed this using a high-frequency data set of sports betting exchange prices (odds) on the final result markets of football matches. The methodology could be readily applied to other prediction markets with high-frequency data and the clean arrival of major news, beyond sports, such as those run within major companies among employees (e.g. Cowgill and Zitzewitz, 2015) and public markets on political or financial events.\textsuperscript{18}

In our application, we tested for weak-form market efficiency, by analysing pre-match exchange odds, and semi-strong form efficiency, by focusing on the in-play odds after the arrival of the major news that the first goal of a match had been scored. The results suggested a reverse favourite-longshot bias for both pre-match and in-play odds. This is opposite to findings from fixed-odds bookmaker markets, where the evidence in favour of the favourite-longshot bias has been widely documented (e.g. Kuypers, 2000; Direr, 2013; Angelini and De Angelis, 2019). The reverse bias on the betting exchange created profitable opportunities that could have been exploited by simple betting strategies. By wagering on longshots, we showed that substantial positive returns were possible both from betting before and during matches.

From our analysis of in-play pricing, we also tested for the presence of behavioural biases, focusing on how market participants reacted to major news. We found evidence that prior beliefs were not significantly updated only in the case when a favourite team scores at the beginning of a match. Conversely, when the first goal was scored by either a longshot or a favourite team later in a match, the response tended to either amplify or absorb the initial mispricing, depending on conditions. In particular, if the news was somehow expected by the market, then prices reacted correctly, i.e. when a favourite team scored first. On the other hand, when the news was unexpected, then the market tended to overreact to the surprise, i.e. when the first goal was scored by a longshot. Moreover, in the case that no pre-match bias was detected in prices, the arrival of the first goal created mispricing. Empirical evidence of home bias and confirmation bias in these markets was also found.

\textsuperscript{18}For example: The Iowa Electronic Markets (IEM), PredictIt and the now defunct Intrade.com.
References


Table 2: Pre-match analysis of prediction market mispricing and efficiency

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<thead>
<tr>
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<th>Home Odds Estimates</th>
<th>Away Odds Estimates</th>
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<tr>
<td></td>
<td>$\hat{\gamma}_0 = 0.0942^{<em><strong>}$, $\hat{\beta} = -0.2599^{</strong></em>}$ $(0.0031)$</td>
<td>$\hat{\gamma}_0 = 0.0606^{<em><strong>}$, $\hat{\beta} = -0.2530^{</strong></em>}$ $(0.0006)$</td>
</tr>
<tr>
<td></td>
<td>$F$-test $= 7.94^{***}$ $(0.0003)$</td>
<td>$F$-test $= 5.85^{***}$ $(0.0030)$</td>
</tr>
</tbody>
</table>

**Efficiency curves**

<table>
<thead>
<tr>
<th></th>
<th>Home Odds</th>
<th>Away Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_0 = (0, 0.24]$, $\overline{Q}_0 = [0.46, 1)$</td>
<td>$Q_0 = (0, 0.14]$, $\overline{Q}_0 = [0.33, 1)$</td>
</tr>
</tbody>
</table>

**Betting strategy**

<table>
<thead>
<tr>
<th></th>
<th>Home Odds</th>
<th>Away Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Odds $&gt; 4.16$</td>
<td>Odds $&gt; 7.14$</td>
</tr>
<tr>
<td></td>
<td>Matches $= 183$</td>
<td>Matches $= 195$</td>
</tr>
<tr>
<td></td>
<td>Correct bets $= 20.77%$</td>
<td>Correct bets $= 14.36%$</td>
</tr>
<tr>
<td></td>
<td>ROI $= 40.05%$</td>
<td>ROI $= 56.41%$</td>
</tr>
</tbody>
</table>

Notes: author calculations using pre-match Betfair result odds from $n = 1,004$ matches. Top panel: WLS estimates of Equation (1), $p$-values in parentheses. $^{***}$ indicates significance at the 1% level, two-sided tests. $F$-test displays the test statistic for the test of the null hypothesis in (2). Middle panel: efficiency curves as per Equation (3). Bottom panel: application of a simple betting strategy described in the text.
Table 3: In-play analysis of market mispricing when the first goal is scored

<table>
<thead>
<tr>
<th></th>
<th>Home Goal ((n_H = 513))</th>
<th>Away Goal ((n_A = 369))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(h = 2)</td>
<td>(h = 5)</td>
</tr>
<tr>
<td>Home Odds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\tilde{\gamma}_0)</td>
<td>-0.0170</td>
<td>-0.0638</td>
</tr>
<tr>
<td></td>
<td>(0.7746)</td>
<td>(0.2895)</td>
</tr>
<tr>
<td>(\tilde{\gamma}_1)</td>
<td>0.0011***</td>
<td>0.0010***</td>
</tr>
<tr>
<td></td>
<td>(0.0055)</td>
<td>(0.0098)</td>
</tr>
<tr>
<td>1000(\tilde{\gamma}_2)</td>
<td>-0.0015**</td>
<td>-0.0013*</td>
</tr>
<tr>
<td></td>
<td>(0.0238)</td>
<td>(0.0509)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-0.1784**</td>
<td>-0.1209</td>
</tr>
<tr>
<td></td>
<td>(0.0244)</td>
<td>(0.1391)</td>
</tr>
<tr>
<td>(F)-test</td>
<td>4.1158***</td>
<td>3.4686***</td>
</tr>
<tr>
<td></td>
<td>(0.0027)</td>
<td>(0.0083)</td>
</tr>
<tr>
<td>Away Odds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\tilde{\gamma}_0)</td>
<td>0.0289</td>
<td>0.0442*</td>
</tr>
<tr>
<td></td>
<td>(0.2442)</td>
<td>(0.0851)</td>
</tr>
<tr>
<td>(\tilde{\gamma}_1)</td>
<td>-0.0002</td>
<td>-0.0003</td>
</tr>
<tr>
<td></td>
<td>(0.2658)</td>
<td>(0.1324)</td>
</tr>
<tr>
<td>1000(\tilde{\gamma}_2)</td>
<td>0.0003</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>(0.2664)</td>
<td>(0.1870)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-0.0423</td>
<td>-0.0185</td>
</tr>
<tr>
<td></td>
<td>(0.3425)</td>
<td>(0.6497)</td>
</tr>
<tr>
<td>(F)-test</td>
<td>0.5164</td>
<td>0.7769</td>
</tr>
<tr>
<td></td>
<td>(0.7238)</td>
<td>(0.5405)</td>
</tr>
</tbody>
</table>

Notes: author calculations using within match Betfair result odds from \(\tilde{n}_H = 513\) ‘home team goal’ matches and \(\tilde{n}_A = 369\) ‘away team goal’ matches. Presents WLS estimates of Equation (7). \(p\)-values in parenthesis. ***, **, * indicate significance at the 1%, 5% and 10% levels, respectively, two-sided tests. \(F\)-test displays the test statistic for the test of the null hypothesis in (8).
Table 4: In-play betting strategy results on final result outcomes after the first goal is scored

<table>
<thead>
<tr>
<th></th>
<th>Betting Strategy: underpriced odds</th>
<th>Robustness check: overpriced odds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Home Odds/Home Goal</td>
<td>Away Odds/Away Goal</td>
</tr>
<tr>
<td></td>
<td>$h = 2$</td>
<td>$h = 5$</td>
</tr>
<tr>
<td>ROI (%)</td>
<td>35.27</td>
<td>10.74</td>
</tr>
<tr>
<td>Correct bets (%)</td>
<td>67.95</td>
<td>74.14</td>
</tr>
<tr>
<td>Mean winning odds</td>
<td>1.99</td>
<td>1.49</td>
</tr>
<tr>
<td>Matches</td>
<td>78</td>
<td>58</td>
</tr>
</tbody>
</table>

Notes: see Figure 5. Shows results from a simple betting strategy, systematically wagering the same amount on all cases identified as underpriced or overpriced (for robustness), following the first goal scored during an in-sample match.
Table 5: Change in market participants expectations and bias, $\Xi_{i,t}$, following the first goal scored in a match, depending on pre-match conditions

<table>
<thead>
<tr>
<th></th>
<th>Home Odds</th>
<th>Away Odds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_H = 513$</td>
<td>$n_A = 369$</td>
</tr>
<tr>
<td></td>
<td>$t \leq 90$</td>
<td>$t \geq 275$</td>
</tr>
<tr>
<td></td>
<td>$t \geq 275$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0 \leq t \leq 550$</td>
<td></td>
</tr>
<tr>
<td><strong>Favourite bias, $p_{i,0} \geq 0.75$</strong></td>
<td>-0.0069 (0.2223)</td>
<td>0.0517 (0.0000)</td>
</tr>
<tr>
<td><strong>No bias 0.25 \leq p_{i,0} \leq 0.45</strong></td>
<td>-0.0342*** (0.0000)</td>
<td>-0.0061*** (0.0001)</td>
</tr>
<tr>
<td><strong>Longshot bias $p_{i,0} \leq 0.20$</strong></td>
<td>-0.0511*** (0.0000)</td>
<td>-0.0360*** (0.0000)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th><strong>Favourite bias, $p_{i,0} \geq 0.75$</strong></th>
<th><strong>No bias 0.15 \leq p_{i,0} \leq 0.32</strong></th>
<th><strong>Longshot bias $p_{i,0} \leq 0.20$</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1067*** (0.0000)</td>
<td>0.0101*** (0.0000)</td>
<td>-0.0085*** (0.0000)</td>
</tr>
<tr>
<td></td>
<td>0.0812*** (0.0000)</td>
<td>-0.0118*** (0.0000)</td>
<td>-0.0340*** (0.0000)</td>
</tr>
<tr>
<td></td>
<td>0.0963*** (0.0000)</td>
<td>-0.0028*** (0.0034)</td>
<td>-0.0237*** (0.0000)</td>
</tr>
<tr>
<td></td>
<td>-0.0045 (0.4716)</td>
<td>0.0482*** (0.0000)</td>
<td>0.0627*** (0.0000)</td>
</tr>
<tr>
<td></td>
<td>0.1004*** (0.0000)</td>
<td>0.1561*** (0.0000)</td>
<td>0.1676*** (0.0000)</td>
</tr>
<tr>
<td></td>
<td>0.578*** (0.0000)</td>
<td>0.1043*** (0.0000)</td>
<td>0.1188*** (0.0000)</td>
</tr>
</tbody>
</table>

Notes: see Tables 2 & 3. Shows test results of the null hypothesis (16), i.e. whether the nature of the pre-match _reverse_ favourite-longshot bias changes after the first goal is scored. $p$-values in parentheses. *** indicates significance at the 1% level, two-sided tests. See Table 1 for interpretation. Results shown in _green_ are the cases in which the pre-match bias tends to be absorbed following the goal. Results shown in _red_ are the cases in which the pre-match bias is amplified; favourites were negatively mispriced and longshots were positively mispriced.
Figure 1: An example of the in-play liquidity and competitiveness of Betfair Exchange English Premier League match result markets

Notes: author calculations from Betfair Exchange: time series from 90 minutes before kick-off to the market close for the final result outcome of Liverpool vs. Manchester City, 10th November, 2019. Dashed line shows the time of kick-off.
Figure 2: In-play match result probabilities from Betfair Exchange: Manchester United vs. Southampton, 11th May 2014

Notes: author calculations from Betfair Exchange, time series from tick 0 to tick 550 of the probabilities of a home win (red), a draw (blue) and an away win (green) implied by in-play odds. The match ended 1-1.
Figure 3: Mean of the odds-implied outcome probability jump after the first goal is scored at tick $t$ and for pre-match probability $p_i$.

<table>
<thead>
<tr>
<th>Home Goal</th>
<th>Away Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Home Odds</strong></td>
<td><strong>Away Odds</strong></td>
</tr>
<tr>
<td><strong>500-500</strong></td>
<td><strong>500-500</strong></td>
</tr>
<tr>
<td>0.73</td>
<td>0.03</td>
</tr>
<tr>
<td>0.73</td>
<td>0.55</td>
</tr>
<tr>
<td>0.73</td>
<td>0.40</td>
</tr>
<tr>
<td>0.71</td>
<td>0.34</td>
</tr>
<tr>
<td>0.70</td>
<td>0.44</td>
</tr>
<tr>
<td>0.57</td>
<td>0.45</td>
</tr>
<tr>
<td>0.62</td>
<td>0.40</td>
</tr>
<tr>
<td>0.57</td>
<td>0.33</td>
</tr>
<tr>
<td>0.41</td>
<td>0.33</td>
</tr>
<tr>
<td>0.78</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Notes: author calculations using $\tilde{n}_H = 513$ ‘home team goal’ matches and $\tilde{n}_A = 369$ ‘away team goal’ matches. Black cells indicate that there are no events in the data set for the corresponding combination of tick and pre-match outcome probability.
Figure 4: Estimated efficiency curves for the first goal scored at tick $t$ and for pre-match probabilities $p_{i,0}$

Notes: author calculations using the $\tilde{n}_H = 513$ ‘home team goal’ matches and $\tilde{n}_A = 369$ ‘away team goal’ matches. The blue plane shows the estimated efficiency curves as per Equation (9) and in red the related 90% confidence intervals as per (10). In black the zero plane is depicted, and in yellow the pre-match efficiency curve is also shown (see also Table 2).
Figure 5: Estimated regions of under and overpricing following the first goal scored

Notes: shows the estimated regions \( Q \) and \( \overline{Q} \) in (11) and (12), respectively, for \( h = \{2, 5, 30\} \) ticks after the first goal event. The green areas depict combinations of \( t \) and \( p_{i,0} \) when the Betfair Exchange in-play odds are in general underpriced. The yellow areas depict the combinations of \( t \) and \( p_{i,0} \) when the Betfair Exchange in-play odds are in general overpriced.