Endogenous UK Housing Cycles and the Risk Premium: Understanding the Next Housing Crisis

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Abstract: Despite the lessons of the post-2007 housing crisis, it would be dangerous to suggest that there will be no similar future events. Here we define a ‘crisis’ as a period of sustained worsening in affordability followed by a collapse in house prices, both of which were features of the 1996-2009 period. Extending the standard life-cycle housing approach to a three-asset model which incorporates interactions with financial markets and uncertainty, it can be shown that endogenous housing cycles can explain volatility. Three parameters drive the system – the income and price elasticities of housing demand and the degree of risk aversion. Furthermore, a key feature of UK housing policy over the last ten years or more has been an attempt to increase housing supply in order to stabilise affordability. The model demonstrates that stabilisation is impossible for any plausible level of construction, if affordability is measured by the ratio of house prices to incomes. Nevertheless, the market has built-in stabilisers; this is demonstrated through the use of stochastic simulations, which illustrate the dynamics of house prices implied by our expected utility model. The model derives explicitly a housing risk premium as a key determinant of the user cost and, hence, house prices and affordability, a factor commonly ignored in many housing models. Moreover, we find that exogenous, persistent ‘ups and downs’ similar to the Great Moderation – Global Financial Crisis period complement the endogenous propagation mechanism of our model.

Keywords: house prices, life-cycle housing model, affordability, housing risk premium, endogenous housing cycles

JEL Classification: E32, E37, G11, R21, R31, R38

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1. Introduction

The rise in house prices during the Great Moderation (GM) from the mid-1990s and the subsequent collapse after 2007 with the Global Financial Crisis (GFC) was, with exceptions, a world-wide event. A strengthening of regulation has subsequently taken place including, explicitly for housing, controls on loan to value and loan to income ratios and stress tests, designed to limit lending to households who are considered at greatest risk of default. In the academic literature, the GFC has stimulated a substantial number of papers, both theoretical and empirical, on the interactions between housing and the wider economy; for example, vector autoregressive (VAR) models to examine the responses of housing to monetary policy shocks, which began emerging even before the GFC, have been particularly prolific after it: both within countries (e.g. Elbourne, 2008, on the United Kingdom (UK); Bjørnland and Jacobsen, 2013, on the United States (US)) and for international comparisons (e.g. Giuliodori, 2005, on nine European economies; Bjørnland and Jacobsen, 2010, on Norway, Sweden and the UK; Sá et al., 2014, on the OECD countries). It would, however, be optimistic to suggest that there will be no future housing crises; history is littered with recurrent housing booms and slumps.

For example, the first speculative boom and bust in the US took place in Florida in the 1920s (Simpson, 1933; Fisher, 1933); Australia – notably Melbourne – experienced a spectacular residential crash in the 1890s (Stapledon, 2010); Glasgow suffered a bank failure in 1878, with major housing consequences (Cairncross, 1934; Button, 2015). This was the last significant banking collapse in the UK until the GFC.

This paper adopts a new approach to understanding housing volatility; it begins by taking the conventional housing life-cycle model, which has been used in a number of UK empirical studies, most recently in Meen (2013). The implications of this model for housing policy – including housing supply measures, fiscal and monetary instruments – are examined. Conditions are derived to show that the stabilisation of some affordability indicators is almost impossible by supply policies alone (although these have been the focus of policy attention in the UK for at least the last ten years) for any plausible level of house building. Similarly, fiscal policy instruments are generally too weak to stabilise the housing market. Monetary policy can be more successful, but is, typically, directed towards wider objectives. Therefore, the standard model suggests that future volatility is unlikely to be avoided by traditional policies and, furthermore, affordability will continue to worsen.
But, in practice, affordability cannot worsen forever; if affordability is measured by the ratio of house prices to incomes, which is the most commonly-used indicator (although not the best), debt interest payments absorb an increasing share of income and the market becomes more vulnerable to unexpected shocks. In fact, in the UK, the ratio of house prices to incomes has shown only limited evidence of a trend over long periods. Therefore, there must be a missing element from the standard model; its omission is dangerous because the underlying probability of a housing collapse is understated.

Consequently, the standard model is extended to incorporate uncertainty, which leads to the addition of a risk term in the user cost of capital. In empirical work, risk is often taken into account in an *ad hoc* manner by the inclusion of the variance of house prices. However, the theory shows that the appropriate measure is more complex and depends not only on the degree of risk aversion and the covariance of the returns in housing and financial assets, but also, more importantly, on the market value of the housing stock. Therefore, as house prices rise, risk and the probability of a market collapse increase. The market has a built-in stabiliser, which was not taken into account during the GFC by those who believed prices could increase forever. The risk premium only becomes quantitatively important when the market is undergoing a boom, so that the conventional model still works well under ‘normal’ conditions. Model properties are, then, assessed through the use of simulations, where the key exogenous stochastic variable is assumed to be the growth in income, evolving according to its empirical distribution, which is close to log-Normal.

Our paper builds on earlier work by Meen (1990, 2000) but also follows and extends two seminal benchmarks: it, first, generalises the Dougherty and Van Order (1982) life-cycle model by extending their deterministic user cost to the case of housing and financial risks, at the same time following their approach to embed persistence in expectations formation as evidenced in survey data (Case and Shiller, 1988; Nordvik, 1995; Case et al., 2005) and to include empirical testing of the proposed theory; and, second, following Dynan (1993), the paper adopts her ‘typical’ (in finance) constant absolute risk aversion (CARA) exponential utility and constant relative risk aversion (CRRA) isoelastic utility functional forms. In related work, Piazzesi et al. (2007) propose a model where housing services enter a nonseparable CRRA utility function, which allows them to derive what they term a ‘composition risk’. Their focus, however, is different from ours: they show that accounting for composition risk helps predict excess return on stocks. Another very recent paper – and perhaps closest to ours – is Pelletier and Tunc (2015). They stress three innovations relative to the literature:
(i) a comprehensive finite-horizon discrete time life-cycle model that includes features scattered in various studies (with a bequest function and probability of dying); (ii) estimation of two key parameters, the CRRA and the constant elasticity of intertemporal substitution (CEIS), which under the assumed Epstein-Zin (1989, 1991) recursive preferences can be disentangled; (iii) a better match to the US data. By contrast, our modelling strategy is to simplify, rather than to complicate. We are able to derive, empirically assess, and simulate to replicate the essential features of the most recent, GM-GFC housing boom-bust cycle in the UK with standard CARA – or CRRA – utility and using an estimated error-correction model (as in Meen, 1990, 2000). Our estimated and simulated model appears to fit the UK facts and data well.

In a preview of our results, the main theoretical contributions of our three-asset expected utility model can be summarised as follows: (i) because of the risk premium, and in contrast to the certainty case, the model has an inbuilt mechanism for endogenous boom-bust cycles as holdings of housing wealth rise; (ii) via the housing risk premium, the user cost is also positively related to the Pratt-Arrow parameter of risk aversion, which implies that house prices tend to decrease when households are more sensitive to housing risk; (iii) the housing risk premium is itself positively related to the (conditionally expected) correlation between housing and risky financial assets and to the (conditionally expected) variance of housing capital returns, but negatively related to the (conditionally expected) variance of financial returns.

Our simulations, based on the estimated parameters of the model, further reveal that the dynamics of house prices implied by our three-asset expected utility model are more consistent with the empirical regularities characterising our UK sample than the time path implied by a model ignoring the risk premium or uncertainty. However, we find that the persistent exogenous ‘ups and downs’ similar to the GM-GFC period are also important in generating the recent boom-bust housing cycle and complement the endogenous propagation mechanism of our model.

The rest of the paper is organised as follows. Section 2 illustrates some of the key data; Section 3 highlights issues from the literature relevant to the study; Section 4 considers the conventional model and its policy implications; Section 5 sets out the extended model, whereas Section 6 conducts dynamic stochastic simulations. Conclusions are drawn in Section 7. Appendixes A and B contain detailed derivations.
2. Key Data

Figure 1a-1d sets out four key indicators: the first defines the annual growth in UK real house prices, measured by the Office for National Statistics (ONS) mix-adjusted house price index for all dwellings relative to the consumers’ expenditure deflator; the second divides the same house price index by household disposable income; the third shows net mortgage advances deflated by house prices; and the fourth plots the cost of capital. Real house price growth indicates four cycles since the early 1970s, but each has been different in character and amplitude; the boom in 1972/3 was strong, with real growth peaking at 40%, but relatively short-lived; whereas the post-1996 boom during the GM, leading up to the GFC, was much more prolonged. The two peaks in the late 1970s and late 1980s, although certainly important, were slightly less dramatic.

[Insert Figure 1a – 1d about here]

Figure 1b is sometimes taken as an indicator that there is a long-run price to income ratio to which the market must tend; indeed price to income or price to earnings ratios are the most commonly adopted indicator of affordability. In addition, there is a significant literature that considers cointegration between house prices and income, although the evidence for cointegration is, by no means, clear cut (see, for example, Gallin 2006 for the US). But, as the theory in Section 4 shows, there is no necessary cointegrating relationship between the two variables alone; demographics, housing supply or the user cost of capital may shift the relationship. Visually, the figure indicates that the ratio has risen in recent years, consistent with the fall in nominal interest rates. Nevertheless, at least on the basis of Augmented Dickey-Fuller (ADF) tests for 1969Q3 to 2012Q4, affordability is nearly stationary: an ADF(4) yields a test statistic of -2.72 which is close to the 5% critical value of -2.88. For comparison, the real house price (in levels rather than the growth rate) yields ADF(4) = -1.0 and is non-stationary (see Table 1).

[Insert Table 1 about here]

Figure 1c turns to the mortgage market and considers real net mortgage advances. The collapse in advances in the GFC is clearly evident, but the volume of advances had still not recovered by the end of the sample; in real terms, advances remain below the level of the 1970s when mortgage rationing was the norm. Empirical models have to take this into account; formally, the existence of credit constraints raises the user cost of capital. This is discussed further below, but the theoretical conditions are shown in Meen (1990). Including credit constraints, Figure 1d graphs the cost; importantly for the later discussion, the cost of capital (defined fully below) is stationary, as might be
expected since it is a form of real interest rate and yields an ADF(4) test statistic of -4.31 over the full sample of available data.

3. Issues from the Literature

Housing has typically received limited attention in portfolio models in finance and macroeconomics; a few models that explicitly analyse risky housing in addition to risky and/or safe financial assets have recently been proposed, but their focus is different from ours. For example, Cocco (2005) shows that the presence of housing in a CRRA utility function crowds out younger and poorer households from the stock market; generalising the early work of Grossman and Laroque (1990), where infinitely-lived households derive utility from a single illiquid and durable good interpreted as housing, Flavin and Nakagawa (2008) argue that habit persistence in consumption has similar implications to housing with adjustment costs in the utility function; Attanasio et al. (2012) study the implications of individual housing demand, in particular the response to changes in institutional features of the mortgage market and of exogenous shock processes, for aggregate demand; Case et al. (2010) investigate the risk-return relationship in the determination of housing asset pricing by specifying and testing a multi-factor housing asset pricing model based on behavioural hypotheses advanced in Case and Shiller (1988).

Nevertheless, theoretical and empirical models of volatility and cycles in North America and the UK have a long history in the housing literature dating back to at least the 1950s, although the work of Cairncross (1934) on the collapse of the Glasgow City bank provides an even earlier forerunner. There are a number of possible explanations for cycles, which are not mutually exclusive; first, construction lags and transactions costs are inherent to housing. Both the time required to build new properties (Marino and Özbaş, 2014), and search costs (Anenberg and Bayer, 2014), exacerbate housing volatility, because of the failure of markets to adjust quickly and smoothly to changes in economic conditions.

Second, the observation that changes in house prices and mortgage advances are highly correlated has provided a stimulus to work on the role of credit markets, although it cannot be concluded from the correlation alone that variation in the availability of credit causes fluctuations in house prices. Tomura (2009) and Sommervoll et al. (2010) provide recent examples of housing models emphasising credit markets. A third strand of the literature concentrates on expectations (in some
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cases in conjunction with credit conditions); among the earlier papers, DiPasquale and Wheaton (1994) show that the housing dynamics differ according to whether agents use backward or forward-looking expectations formation processes. Furthermore, in contrast to the standard representative household approach, agent-based models can be used to examine aggregate housing market fluctuations arising from heterogeneous expectations. The starting point is the Artificial Stock Market Model first developed by LeBaron et al. (1999), arising from the research programme in complex systems at the Santa Fe Institute, but recent experiments for UK housing markets have been conducted by Li and Meen (2016). Overall, the importance of transaction costs, credit conditions and expectations as explanations for housing market volatility should certainly not be dismissed, but the focus of this paper is different, concentrating on endogenous dynamics of the housing market arising from the nature of risk.

A second relevant theme for later sections concerns the impact of housing on the wider economy. Although it has always been accepted that housing is strongly affected by macro events, the potential reverse causation has been controversial, at least until the GFC. In the UK, the topic first became important in the late 1980s, when consumers’ expenditure expanded at a faster rate than traditional consumption functions had predicted. Since house prices were growing rapidly at the same time, a ‘housing wealth effect’ on consumption provided one explanation (Maclennan et al., 1998). However, as discussed by Benito et al. (2006), the correlation might also have been caused by a common third factor, for example an underlying improvement in productivity reflected in higher permanent incomes or, alternatively, the easing of collateral constraints as prices rose. A significant number of empirical studies of the relationship between consumption and house prices have also been conducted for other countries (see Iacoviello, 2000; Giuliodori, 2005; Case et al., 2005; Chen, 2006). The studies generally find a significant relationship between consumption and the housing market, although of varying strengths; Case et al. (2005) find stronger propensities for consumption from housing wealth than from stock market wealth. In addition to time-series studies, micro household studies have been conducted by Attanasio and Weber (1994), Campbell and Cocco (2005), and Attanasio et al. (2009). As noted above, housing wealth may also act as collateral for consumer loans. Using a financial accelerator approach, Aoki et al. (2004) indicate that an increase in housing wealth raises credit worthiness and, therefore, reduces the external finance premiums that households face under asymmetric information.
The GFC led to an explosion in empirical studies of the relationship between housing and the economy. Although early time-series tests of consumption and housing wealth were, typically, conducted using single equation methods, more recent work is dominated by models using VAR or VECM methods particularly in order to capture the joint effects of monetary shocks on the different markets (see, for example, Goodhart and Hoffman, 2008; Iacoviello and Minetti, 2008; Jarocinski and Smets, 2008; Vargas-Silva, 2008a, 2008b; Gabriel and Lutz, 2014; and Rahal, 2015). However, in some cases, empirical VAR models bear only a superficial resemblance to theoretical models of housing markets, based on a life-cycle approach, which is the work horse of the housing economics literature. This is particularly the case where international comparisons are conducted – for example, differential responses to monetary shocks – and international data availability limits the structure of the models.

Beyond VAR and VECM models, dynamic stochastic general equilibrium (DSGE) models that incorporate housing have also started to appear in response to the GFC (Iacoviello, 2005; Iacoviello and Neri, 2010; Adam and Woodford, 2013). However, in most cases such DSGE systems oversimplify the institutional features of housing markets as well as the specificity of housing assets, in the sense we have been discussing thus far. Surveying the emerging DSGE literature which incorporates housing, Iacoviello (2010) concludes that it misses details on banking sector and labour market institutional features (various frictions and adjustment costs, in particular). Since these DSGE models with housing generate, as an optimality condition, an asset price equation which is purely forward-looking, he further suggests that house and consumer price inflation share the implausible property of being ‘too forward-looking relative to the data’.

A question for the next section is the conditions under which house prices can validly be estimated as a single equation rather than a system, since it enables the structure of the model to be simplified.

---

4 As also pointed out by Iacoviello (2010), housing (residential fixed) investment leads non-housing (non-residential fixed) investment, and is more volatile, in the US data, which led to the now-famous quote of Leamer (2007) that ‘housing is the business cycle’.
4. The Standard Model and the Implications for Policy

4.1 The Life-Cycle Housing Model

The standard inter-temporal life-cycle model extended to housing assumes two goods, housing services and a composite consumption good (C). The model has a long history, for example Dougherty and Van Order (1982), and can therefore be described briefly. If, for simplicity, the flow of housing services is proportional to the demand for the housing stock (HD) and, given an assumed constant real discount rate (r), lifetime utility is described in an infinite horizon continuous time\(^5\) by equation (1):

\[
\int_0^\infty e^{-rt} \mu(H^D(t), C(t)) dt
\]

\(\mu(H^D, C)\) denotes the instantaneous utility of the representative household. (1) is maximised with respect to the budget constraint (2) and technical constraints (3) and (4) which describe changes to real asset stocks (housing and financial, respectively) over time.

\[
g(t)X(t) + S(t) + C(t) = (1 - \theta(t))RY(t) + (1 - \theta(t))i(t)A(t)
\]

\[
\dot{H}^D(t) = X(t) - \delta(t)H^D(t)
\]

\[
\dot{A}(t) = S(t) - \pi(t)A(t)
\]

where:

- \(H^D(t)\) stock demand for housing;
- \(g(t)\) real purchase price of dwellings;
- \(X(t)\) new purchases of dwellings;
- \(S(t)\) real savings net of real new loans;

\(^5\) Differences in results between CARA and CRRA utility functional forms are discussed in Section 5. Furthermore, variations between finite and infinite horizon models or continuous versus discrete time models are not essential to our findings. Proofs of the latter are available upon request.
\( \theta(t) \) household marginal tax rate;
\( RY(t) \) real household income;
\( i(t) \) market interest rate;
\( A(t) \) real net non-housing assets;
\( \delta(t) \) depreciation rate on housing;
\( \pi(t) \) general inflation rate;
\( \ddot{x}(t) \equiv \frac{dx(t)}{dt} \) denotes the time derivative for any variable \( x(t) \).

From the first-order conditions (see Appendix A for the expected utility case), the marginal rate of substitution between housing and the consumption good, \( \mu_H / \mu_C \), is given by (5);

\[
\mu_H(t)/\mu_C(t) = g(t) \left[ (1 - \theta(t))i(t) - \pi(t) + \delta(t) - \dot{g}^e(t)/g(t) \right]
\]

(5)

The right-hand side in (5) is the widely-used standard definition of the real housing user cost of capital (UCC) and represents the real price of housing services, where \( (\dot{g}^e(t)/g(t)) \) is the expected real capital gain. Below the term in […] is referred to as the cost of capital (CC).

If, for illustration, instantaneous utility takes the popular additive CRRA isoelastic form, \( \mu(C(t),H^D(t)) = [C(t)^{1-\gamma} + H^D(t)^{1-\gamma}]/(1 - \gamma) \), where \( \gamma \) is the CRRA and, equivalently, \( 1/\gamma \) is the CEIS, then, \( H^D(t)/C(t) = UCC(t)^{-1/(1-\gamma)} \), where UCC is equal to the right-hand side of (5) and forms the basis for Figure 1d.

Aggregating, total demand for housing is given by the log-linear housing demand function (6), which assumes that aggregate non-housing consumption is related to permanent income, \( RPY \), (see Pain and Westaway, 1997).

\[
\ln(H^D(t)) = \alpha_0 + \alpha_1 \ln[RPY(t)/HH(t)] - \alpha_2 \ln[g(t) * CC(t)] + \alpha_3 \ln[HH(t)] + \epsilon_1(t)
\]

(6)

where:
\( HH(t) \) total number of households;
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\[ RPY(t)/HH(t) \] per household permanent income;
\[ \varepsilon_1(t) \] error term.

\[ CC(t) = (1 - \theta(t))i(t) - \pi(t) - \delta(t) - g^e(t)/g(t) \]  \hspace{1cm} (7)

\[ \alpha_1 = 1.0; \quad \alpha_2 = \frac{1}{\gamma}; \quad \alpha_3 = 1.0. \]

Conditional on the supply of the housing stock, the equilibrium (log) real house price is given by (8).

\[ \ln(g(t)) = (\alpha_0/\alpha_2) + (\alpha_1/\alpha_2) \ln[RPY(t)/HH(t)] + (\alpha_3/\alpha_2) \ln[HH(t)] - (1/\alpha_2) \ln[H^s(t)] - \ln[CC(t)] + \varepsilon_2(t) \]  \hspace{1cm} (8)

where:
\[ H^s(t) \] stock supply of housing.

Since \( \alpha_3 = 1 \) (i.e. the demand for housing rises proportionately to the number of households), then a proportionate rise in both the housing stock and the number of households leaves real house prices unchanged. But if homogeneity holds, then affordability as measured by the (log) ratio of house prices to per household income\(^6\), \( \ln(g(t)/RPY(t)/HH(t)) \), is constant only if \( \alpha_1/\alpha_2 = 1 \), for given values of the other variables. If \( \alpha_1/\alpha_2 > 1 \), then affordability worsens over time (assuming growing incomes). \( \alpha_1/\alpha_2 > 1 \) implies that the income elasticity of housing demand, \( \alpha_1 \), is higher than the price elasticity, \( \alpha_2 \), which is consistent with the empirical evidence in Muellbauer and Murphy (1997), but further evidence is presented below. Note that it is not the absolute size of the two elasticities that matters, but the relative sizes. Therefore, the first important condition is that affordability will worsen over time unless the supply of housing rises faster than the number of households or some other market stabiliser operates.

A fuller stock-flow specification endogenises the housing stock by the inclusion of an equation for housing construction (see, for example, Poterba 1984), where stock-flow consistency is obtained from the identity (9).

\(^6\) Although permanent income is used here, whereas most empirical measures use current earnings.
\[ H^S(t) \equiv (1 - d(t)) H^S(t - 1) + N(t) \]  
(9)

where:
\[ d(t) \] demolition rate;
\[ N(t) \] new construction of dwellings.

Ball et al. (2010) survey the international literature on empirical housing construction models and find that most studies still adopt simple specifications, related to house prices, construction costs and borrowing costs, summarised in (10).

\[ \ln(N(t)) = b_0 + b_1 \ln(g(t)) + b_2 \ln(CCost(t)) + b_3 rc(t) + \varepsilon_3(t) \]  
(10)

where:
\[ CCost(t) \] index of construction costs;
\[ rc(t) \] interest rate on borrowing by construction companies.

The joint model of prices and construction yields a vector \( X' = [g, RPY, HH, CC, H^S, N, CCost, rc] \) which, in principle, could be estimated jointly as a VAR or VECM testing for possible cointegrating relationships; the addition of dynamics is important because of both the transactions costs associated with housing demand and time-to-build for housing supply (Topel and Rosen, 1988). This general specification allows the testing of a range of hypotheses; first, the possible feedback of housing markets to the aggregate economy through consumption and income; second, the model could be used to estimate the effects of housing markets on household formation; third, a sub-set of the variables could be used to investigate housing market interactions between prices, building, construction costs and interest rates. In each case, the appropriate structure depends on weak exogeneity restrictions. For example, in the third case, Meen (2000) shows that there are two cointegrating vectors amongst the four variables and both house prices and interest rates can be treated as weakly exogenous. At first sight, this result may appear surprising; but it arises from the fact that house prices are determined in (8) as a stock equilibrium condition and the flow, \( N(t) \), is a small percentage of the stock in any period. The weak exogeneity of interest rates is perhaps less surprising since short-term rates are set with regard to wider inflation objectives and house prices.
are only one amongst a range of influences. The restrictions imply that house prices can be estimated as a single equation conditional on the remaining variables, with the proviso that house prices and incomes could still be jointly determined. As noted above, a single equation approach contrasts with VAR or DSGE models that examine the responses of house prices to monetary and other shocks in the context of a system of equations.

To stress the point, given the focus of our attention, there are few statistical advantages to estimating a system rather than a single equation, under the validity of the identification and weak exogeneity restrictions found in Meen (2000). There are, in fact, very large numbers of single-equation time-series studies of house prices in the UK including Nellis and Longbottom (1981), Hendry (1984), Giussani and Hadjimatheou (1990), Meen (1990, 2013), Drake (1993), Ashworth and Parker (1997), Holly and Jones (1997), Muellbauer and Murphy (1997), Pain and Westaway (1997), although these have, perhaps, given way to VAR and DSGE approaches in recent years. For future reference, across these studies, the mean income elasticity of house prices is 2.6 and the minimum value is 1.7.

As a single equation, the error-correction model (ECM) to be estimated is given by (11):

$$
\Delta \ln[g(t)] = \gamma_1 \Delta \ln[g(t-1)] + \gamma_2 \Delta \ln[X(t)] + \gamma_3 \ln[g(t-1)] - \gamma_4 \ln[X(t-1)] + \epsilon(t) \quad (11)
$$

where:

$$X' = \left[ \frac{RY}{HH}, RW, \frac{H^s}{HH} \frac{\lambda}{CC}, WSH \right]$$

is a vector of weakly exogenous regressors and $\epsilon$ is an error term;

$g$       real house prices (index, 2002=100);
$RY/HH$   real per household disposable income (£m);
$RW$      real household wealth (£m);
$HH$      number of households (000s);
$H^s$     housing stock (000s);
$\lambda$ measure of mortgage rationing (%);
$CC$      cost of capital (%);
$WSH$     ratio of post-tax wage income to personal disposable income (see Meen and Andrew, 1998);
$\Delta$  (quarterly) first difference operator.
This operational version of \( X' \) includes real household income, household wealth, the number of households, the housing stock, the share of wages in household incomes, the cost of capital and a measure of mortgage rationing to be discussed in the next sub-section.

### 4.2 The Cost of Capital and Initial Empirical Results

The basic cost of capital definition is given by equation (7), but there are both conceptual and operational issues. Conceptually, the measure includes no risk premium – an issue considered in the next section. Furthermore, the variable does not allow for the credit restrictions that are implied by Figure 1c. Formally, as shown by Meen (1990), credit restrictions require an amendment to (7), given by (7')

\[
CC(t) = (1 - \theta(t))i(t) - \pi(t) + \delta(t) - \frac{\hat{g}^c(t)}{g(t)} + \lambda(t)/\mu_c(t)
\]  

(7')

where, \( \lambda(t) \) is the shadow price of the rationing constraint. Since \( \lambda(t) \) is not directly observable, we assume that it is related to the difference between the growth in the demand and supply for the mortgage stock \( (M^D \text{ and } M^S) \); this is a logarithmic approximation for net advances, since net advances are identical to the change in the stock.

\[
\lambda(t) = \lambda_1 [\Delta \ln(M^D(t)) - \Delta \ln(M^S(t))]
\]  

(12)

The measure of rationing prior to the early 1980s is taken from Meen (1990); however, from this period until the GFC, the rapid liberalisation of mortgage markets meant that advances were primarily demand-determined; this implies that in unconstrained periods \( \lambda(t) \) takes a value of zero.

A post-2007 rationing variable is, then, constructed by calculating what mortgage demand would have been in the absence of the constraints imposed by the GFC\(^7\). Furthermore, it also implies a form

\(^7\) This is calculated from the constructed value of mortgage demand, derived from an equation estimated between 1983 and 2007, i.e. the unconstrained period. Details of the equation are available from the authors.
of regime-switching model, so that in the vector $X'$ above, it is not sufficient simply to include the mortgage stock or mortgage advances as one of the variables.

Equation (7') includes the expected capital gain on housing and two issues arise. First, the literature indicates strongly (Dougherty and Van Order, 1982; Case and Shiller, 1988; Nordvik, 1995; Case et al., 2005) that expectations are not fully rational and, as shown below, in this case are, empirically, best measured adaptively by the annual growth of real house prices lagged one period. Second, if the front-end loading of mortgage payments is important, then nominal as well as real interest rates should affect the user cost. This can be incorporated by adding a coefficient, $\beta \leq 1.0$, to the capital gains term.

The final definition of the cost of capital is (7''); it includes, for completeness, property taxes $p_t(t)$ and maintenance expenditure $m(t)$, which are excluded from the basic life-cycle model. The values of both $\lambda_1$ and $\beta$ are determined by the data; but, anticipating the empirical results below, $\lambda_1 = 2.0$ and $\beta = 0.3$. In addition, $i(t)$ is taken to be the mortgage interest rate and $\theta(t)$ is the rate of mortgage interest tax relief, which was finally abolished in 2000, following a period of phasing out. (7'') is graphed in Figure 1d.

$$CC(t) = \{[(1 - \theta(t))i(t) - \pi(t) + \delta(t) + p_t(t) + m(t) - \beta \Delta_4 \ln[g(t - 1)] + \lambda_1 [\Delta \ln(M_D(t)) - \Delta \ln(M_S(t))]\}$$

$(7'')$

where:

$p_t(t)$  property tax rate (%);
$m(t)$  maintenance expenditures as percentage of the property value (%);
$\Delta_4$  annual difference operator (with quarterly data).

[Insert Table 2 about here]
The second column of Table 2 shows the results from estimating equation (11) over the period 1970Q1 to 2012Q4; solving for the long-run solution, equation (13) yields a long-run income elasticity of house prices of 2.5, a housing stock elasticity of -1.4 and a cost of capital semi-elasticity of -0.05. The diagnostic statistics show little evidence of model misspecification; there is no evidence of heteroscedastic disturbances and the residuals are normally distributed, although the Lagrange Multiplier test indicates some residual autocorrelation. Since the autocorrelation could arise from insufficient attention to the lag structure, the third column adds further lagged dependent variables. This eases the problem and, interestingly, these terms show some evidence of second-order positive autocorrelation\(^9\) followed by mean reversion at higher-order lags, consistent with international studies such as Englund and Ioannides (1997). However, the additional variables have little effect on the remaining coefficients. Since the equation appears to fit the data well, any missing variables are likely to be well-disguised or have unusual properties, not revealed by diagnostics conducted over the whole sample period.

\[
\ln[g(t)] = -6.93 + 2.49\ln[RY(t)/HH(t)] - 1.37\ln[H^S(t)/HH(t)] - 0.05[CC(t)] + \ldots
\]

(13)\(^{10}\)

### 4.3 Policy Implications

The previous section established the condition that, to stabilise affordability, the growth in housing supply must be faster than the growth in the number of households. In addition, the long-run solution to the price equation, (13), implies a relationship between the growth in aggregate income and the growth of the housing stock required to maintain constancy in affordability (measured by the ratio of house prices relative to per household income). For a given cost of capital, changes in affordability are determined by (14). Therefore, in order for affordability not to worsen, the housing stock must grow at the same rate as income. If the long-run growth in income is 3%, which is the average annual growth rate over the sample period, then the housing stock must grow by 3%. In

\(^{8}\) Note that the cost of capital is not expressed in logs since Figure 1d shows that (on this definition of expectations) the variable can take negative values temporarily.

\(^{9}\) The first-order autocorrelation coefficient was statistically insignificant.

\(^{10}\) The remaining terms from the long-run solution to Table 2 are excluded in equation (13) since they are not central to the subsequent argument.
practice, over the estimation period, the stock has only grown by 1.7%. However, the rule is parameter dependent; Muellbauer and Murphy (1997), for example, find the elasticity of house prices with respect to income and the housing stock to be 2.0 and -2.0 respectively, which yields (14'). In this case, the growth in the housing stock must be at least half of the growth in household income for stability in affordability. The condition has been met under these alternative coefficient values over the sample period, but in recent years the increase in the stock has been much lower than 1.7%; from 1980, the annual average growth rate was 1.4%, and only 1% from 1990. This is because new construction is a stationary process; Figure 2 shows British housing starts since 1973 and yields an ADF(4) test statistic of -4.37. Ball et al. (2010) show that construction in the US and Australia also has no trend. Therefore, the contribution of a stationary new supply to a non-stationary stock falls over time, equation (9), and so the growth rate of the stock declines. Thus, by themselves, housing supply policies, which have been the main focus since the 2004 Barker Review of Housing Supply, are unlikely to bring about stability in affordability. The conditions (14, 14') indicate that affordability will worsen over time.

\[
\frac{d \ln \left( \frac{g(t)}{RY(t)/HH(t)} \right)}{d \ln \left( \frac{R^Y(t)}{H^S(t)} \right)} \approx 1.5 \quad \text{and} \quad \frac{d \ln \left( \frac{H^S(t)}{HH(t)} \right)}{d \ln \left( \frac{R^Y(t)}{HH(t)} \right)} \approx 1.0 \quad \text{(14)}
\]

\[
\frac{d \ln \left( \frac{g(t)}{RY(t)/HH(t)} \right)}{d \ln \left( \frac{R^Y(t)}{HH(t)} \right)} \approx 1.0 \quad \text{and} \quad \frac{d \ln \left( \frac{H^S(t)}{HH(t)} \right)}{d \ln \left( \frac{R^Y(t)}{HH(t)} \right)} \approx 2.0 \quad \text{(14')}
\]

Equations (14, 14') hold the cost of capital constant; relaxing the assumption, from (13), the required change in the cost of capital to keep \(d \ln \left( \frac{R^Y(t)}{HH(t)} \right) = 0\) can be calculated and is given by (15).

\[
d[CC(t)] = \left( \frac{1.5}{0.05} \right) d \ln \left( \frac{R^Y(t)}{H^S(t)} \right) \quad \text{(15)}
\]

Therefore, if \(d \ln [R^Y(t)] = 0.03\) and \(d \ln [H^S(t)] = 0.01\) then the cost of capital would need to rise by approximately 0.6 percentage points per annum. However, from Figure 1d, the cost of capital is a stationary process and, so, there is no evidence that, historically, such changes have occurred over long periods. Were a permanent change to the growth in the cost of capital to take place, from the definition of the cost of capital, (7’’), it would need to occur either through a change in fiscal policy –
notably property taxes – or through changes to monetary policy. In practice, changes to property taxes, which on average across the UK are approximately 0.5% of the market value of the housing stock, would need to be unrealistically large to be feasible. Changes to interest rates are targeted towards wider inflation objectives and are, therefore, not available; increasingly stringent lending controls might be feasible, but this would represent a fundamental change in direction from the liberalisation of mortgage markets which began in the 1980s.

In summary, under stationarity of new housing construction and the cost of capital, affordability (measured by price to income ratios) cannot be stabilised in the conventional model; neither supply nor fiscal/monetary policies can bring about the desired changes. Consequently, the next section considers an alternative stabilisation mechanism, where the intuition is that the user cost is misspecified by the exclusion of a housing risk premium. It is shown that risk is positively related to the market value of the housing stock; therefore, as prices rise, the corresponding increase in risk leads to a rise in the user cost and, hence, to a reversal of the initial price increase.

5. The Three-Asset Expected Utility Model

5.1 Theory

The majority of empirical life-cycle house price studies do not explicitly include uncertainty, but shifting portfolio choices by investors between housing and risky financial assets need to be taken into account. The rise in Buy-to-Let mortgages since the mid-1990s, and therefore the expansion in private renting as an investment has been an important housing market innovation, reversing the long-run decline in the market that had taken place since the First World War. The attractiveness of this market at least partly reflects the weakness in returns from financial assets and, in order to analyse the choices fully, not only the relative expected returns, but also the variances and covariances need to be considered. As shown below, this leads to a revised definition of the user cost that allows for risk.

As noted, theoretical and empirical work in finance has traditionally been isolated from work on housing. Only very recently there have been attempts to integrate these two fields in portfolio
choice models that incorporate housing under varying degrees of institutional detail (Cocco, 2005; Flavin and Nakagawa, 2008; Case et al., 2010; Attanasio et al., 2012; Pelletier and Tunc, 2015). Financial theory originated with the mean-variance portfolio model (Markowitz, 1952) and evolved into the Capital Asset Pricing Model or CAPM (Sharpe, 1964; Lintner, 1965; Black 1972). The CAPM shows that the return of any risky asset is equal to a riskless rate plus a risk premium for the particular risky asset. However, the empirical support for the static CAPM was weak (Fama, 1970; Fama and French, 1993), which may reflect theoretical failings arising from many simplifying assumptions as well as difficulties in implementing valid tests of the model (Fama and French, 2004).

Consequently, the Consumption-based Capital Asset Pricing Model (C-CAPM) was developed by Lucas (1978) as a dynamic generalisation of the CAPM also linking it to the real macroeconomy. In contrast to the CAPM, it can theoretically be shown how economic fundamentals affect returns and prices of risky assets. Nevertheless, housing was not incorporated in asset allocation models of the C-CAPM type until Grossman and Laroque (1990). Furthermore, Flavin and Nakagawa (2008) find that their housing model, which adds nondurable consumption in the utility function to extend Grossman and Laroque (1990) beyond durable housing being the sole argument in utility, provides support for the C-CAPM. As noted above, Piazzesi et al. (2007) propose a nonseparable utility model aggregating ‘shelter’ and ‘consumption’ via a constant elasticity of substitution index and find that the change in the expenditure share emerges as a second factor, complementing the single-factor C-CAPM, in their ‘Housing C-CAPM’ (p. 540). In an empirical context, Case et al. (2010) develop what they term a Multifactor Housing Asset Pricing Model and find that house prices are not only determined by riskless assets and a theoretically derived risk premium, but also by idiosyncratic risk, momentum and a size effect.

The three-asset expected utility model we propose is a version of the C-CAPM with housing that extends the standard life-cycle housing model of the preceding section to incorporate uncertainty in both housing and financial assets. Our aim is to focus on the explicit derivation of house prices in this set up and to show that the housing risk premium is a key omitted component in the standard model. The three-asset expected utility model differs from equations (1)-(4) primarily through the budget constraint and the laws of motion. First, in (17), financial assets (net of mortgage loans) are partitioned into risky \( A_1 \) and risk-free \( A_2 \). Units of the former have a variable real market price \( p(t) \) whose distribution is given in (25) below. The risk-free asset (net of mortgages) yields an interest rate \( i(t) \); implicitly, the interest rate on mortgages is assumed to be the same as on the risk-free asset. The equations of motion (18)-(20) also allow for the partitioning of the financial

\[ Banz (1981) \text{ suggests that returns on small firms were relatively high.} \]
assets; the three-asset model with housing is set out in discrete time,\(^{12}\) although the key results are invariant to this choice. Similarly, utility in both (1) and (16) is expressed in terms of an infinite horizon, but the main features are unchanged in finite-horizon versions.\(^{13}\)

\[
E[U] = E \left[ \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \mu[H^D(t), C(t)] \right]
\]

\[C(t) + g(t)X(t) + p(t)AP_1(t) + AP_2(t) = (1 - \theta(t))RY(t) + (1 - \theta(t))D(t)A_1(t - 1)
+(1 - \theta(t - 1))i(t - 1) \cdot A_2(t - 1) \tag{17}
\]

\[X(t) - \delta(t - 1)H^D(t - 1) = H^D(t) - H^D(t - 1) \tag{18}
\]

\[AP_2(t) - \pi(t - 1)A_2(t - 1) = A_2(t) - A_2(t - 1) \tag{19}
\]

\[AP_1(t) = A_1(t) - A_1(t - 1) \tag{20}
\]

where:

- \(AP_1\) purchases of units of the risky asset;
- \(AP_2\) purchases of risk-free assets net of mortgages advances;
- \(p(t)\) real market price of the risky financial asset;
- \(D(t)\) real dividend per unit of the risky financial asset;
- \(A_1\) stock of units of the risky financial asset;
- \(A_2\) stock of risk-free assets net of mortgages;
- \(\theta(t)\) household marginal tax rate;
- \(i(t)\) risk-free rate, interest rate.

Wealth is now composed of three elements: housing, a risky financial asset and a safe financial asset. Households make investment decisions on the basis of expectations held at time \(t\) of the prices of housing and the risky financial asset (and its dividend yield). Households’ investment decisions can change in each period and, therefore, households only need to form expectations at time \(t\) for the

\(^{12}\) Nevertheless, we keep the time indexing of variables in brackets rather than as subscripts for more direct comparisons with the notation from the standard life-cycle housing model in continuous time introduced earlier. Furthermore, since the addition of uncertainty and expected utility complicates the expressions, we opt in what follows (including the appendixes) to drop explicit \(t\)-indexing for terms involving expectations, variances, covariances and correlations, despite the fact that these moments are conditional on information available at time \(t\).

\(^{13}\) Proofs are available upon request.
Endogenous UK Housing Cycles and the Risk Premium

following time period, \( t+1 \). In light of this, current and historical prices and returns are certain, and the future – more specifically, the next-period – prices and returns are uncertain.

In Appendix A, from the first-order conditions, equations (21) and (22) are first derived. Under the assumption of certainty, where the final term is ignored, equation (21) is identical to equation (5). Equation (22) now results from the combination of the first-order conditions for the risky asset and the risk-free asset and is equivalent to the key equation in the C-CAPM. Equations (23) and (24), derived by combining (21) and (22), show the generalised form of the expected marginal rate of substitution between housing and non-housing consumption.

\[
\frac{E[\mu_H(t)]}{E[\mu_C(t)]} \approx g(t) \left\{ (1 - \theta(t))i(t) - \pi(t) + \delta(t) - \frac{E[\mu_C(t+1)r_h(t+1)]}{E[\mu_C(t+1)]} \right\} \tag{21}
\]

\[
(1 - \theta(t))i(t) - \pi(t) = \frac{E[\mu_C(t+1)r_a(t+1)]}{E[\mu_C(t+1)]} \tag{22}
\]

\[
E[\mu_H(t)/\mu_C(t)] = g(t) \left[ (1 - \theta(t))i(t) - \pi(t) + \delta(t) - E[r_h(t+1)] + \tau(t) \right] \tag{23}
\]

\[
\tau(t) = \left[ E[r_a(t+1)] - \left[ (1 - \theta(t))i(t) - \pi(t) \right] \right] \frac{\text{Cov}(\mu_H(t+1), r_a(t+1))}{\text{Cov}(\mu_C(t+1), r_a(t+1))} \tag{24}
\]

where:

\[
r_h(t+1) = \frac{g(t+1) - g(t)}{g(t)} \text{ real capital return on housing;}
\]

\[
r_a(t+1) = \frac{p(t+1) + (1 - \theta(t+1))D(t+1) - p(t)}{p(t)} \text{ real return on risky financial assets.}
\]

Two additional assumptions are needed; first, since future prices and returns are unobservable, households’ expectations about the returns on housing, (25), and risky financial assets, (26), are assumed to be jointly normally distributed.

\[
r_h(t+1) \sim N \left( r_H^e(t+1), \sigma_h^2(t+1) \right) \tag{25}
\]

\[
r_a(t+1) \sim N \left( r_a^e(t+1), \sigma_a^2(t+1) \right) \tag{26}
\]
Second, a specific utility functional form is needed. As is common in finance, and following Dynan (1993), utility is initially assumed to take the CARA form, given by (27), although the sensitivity to the CRRA case is also discussed below. From the first-order conditions, the expected marginal rate of substitution between housing and the consumption good, \( E[\mu_H(t)/\mu_C(t)] \), and the user cost of capital derived fully in Appendix B, are given by (28) and (29).

\[
\mu(H^D(t), C(t)) = -e^{-\varphi H^D(t)} \cdot e^{-\varphi C(t)}
\]

(27)

where \( \varphi \) is the Pratt (1964)-Arrow (1971) parameter of risk aversion.

\[
E[\mu_H(t)/\mu_C(t)] = g(t)[(1 - \theta(t))i(t) - \pi(t) + \delta(t) - r^e_H(t + 1) + \tau(t)]
\]

(28)

\[
CC(t) = (1 - \theta(t))i(t) - \pi(t) + \delta(t) - r^e_H(t + 1) + \tau(t)
\]

(29)

where:

\[
\tau(t) = [r^e_H(t + 1) - (1 - \theta(t))\hat{g}(t) + \pi(t)]\frac{\rho_{ah}(t + 1)\sigma_{h}(t + 1)}{\sigma_{a}(t + 1)} \\
+ \varphi H(t)\left(1 - \rho^2_{ah}(t + 1)\right)\sigma^2_{h}(t + 1)
\]

(30)

\( r^e_H(t + 1) \) and \( \sigma^2_{a}(t + 1) \) are the expected returns on housing and the risky asset respectively; the former is equivalent to \( \hat{g}^e / g(t) \) in the continuous time case discussed earlier, and \( \rho_{ah} = \frac{\sigma_{ah}}{\sigma_{a}\sigma_{h}} \) is the (expected conditional) correlation coefficient between the return on housing and the risky financial asset.

The derivation of the user cost under uncertainty, (29), is a key theoretical contribution of our paper. The user cost is identical to the deterministic case in (7) except for the addition of the housing risk premium, \( \tau(t) \), defined in (30). This risk premium term consists of two parts. The first is analogous to the derived risk premium for an arbitrary asset or portfolio relative to the (mean-variance investor) efficient portfolio, empirically approximated by some market index, in the standard CAPM. Typically, if subscript \( a \) denotes a market index (a stock market index below) and subscript \( h \) denotes a particular asset (housing, in our case), then \( \rho_{ah} \cdot \sigma_{h}/\sigma_{a} \) is the systematic risk of asset \( h \) known as the Sharpe (1966) beta. The beta of asset \( h \) measures its marginal contribution to the overall portfolio risk an investor holds. It implies that the investor needs to be compensated by a higher expected return if she is to hold a particular asset exhibiting (observed) higher covariance with the market. Note that the first part of the housing risk premium vanishes, and the second part simplifies to
\( \varphi g(t)H(t)\sigma_h^2(t + 1) \) if housing and financial assets are perfectly uncorrelated.\(^{14}\) However, even in this case, the model shows that simply adding the variance of the housing market return (as is common in empirical studies) leads to misspecification. In the simulations in Section 6, our estimation of the CARA \( \varphi \) is consistent with the typical value ranges reported in Raskin and Cochran (1986).\(^{15}\)

Equations (28), (29) and (30) imply that: (i) as absolute holdings of housing wealth rise – whether through changes in house prices or the housing stock – the housing market risk premium increases, raising the user cost and pushing down house prices; thus, in contrast to the certainty case, the model has an inbuilt mechanism for market downturn as holdings of housing wealth rise; (ii) via the housing risk premium, the user cost is positively related to the Pratt-Arrow parameter of risk aversion, \( \varphi \), so that house prices will tend to decrease if households are more sensitive to housing market risk; (iii) the housing risk premium is positively related to the correlation between housing and risky financial assets and to the variance of housing capital returns, but negatively related to the variance of financial returns.

The alternative assumption of CRRA utility modifies the definition of the cost of capital, which is now given by (31).

\[
CC(t) \approx (1 - \theta(t))i(t) - \pi(t) + \delta(t) - \tau_h^c(t + 1) + \left[ \tau_d^c(t + 1) - (1 - \theta(t))i(t) + \pi(t) \right] \frac{\rho(t+1)\sigma_a(t+1)}{\sigma_h(t+1)} + \\
\gamma C^e(t) - \frac{\gamma - 1}{E[\mu_C(t)]} g(t)H(t)(1 - \rho(t + 1)^2)\sigma_h^2(t + 1) \tag{31}
\]

\(^{14}\) Flavin and Nakagawa (2008) identify the conditions under which their model remains tractable in a setting sufficiently general to incorporate variation in the price of housing relative to the nondurable good. The required assumption is that housing price risk is uncorrelated with financial assets, which they claim ‘seems to be reasonably consistent with the data’.

\(^{15}\) \( \varphi(x) \) is the standard Pratt (1960) – Arrow (1971) coefficient of absolute risk aversion (CARA) over any ‘outcome variable’ \( x \), defined as \( \varphi(x) \equiv -\frac{u''(x)}{u'(x)} = -\frac{d}{dx} \log(u'(x)) = -\frac{du'(x)}{dx}/u'(x) = -\frac{du'(x)}{dx}u''(x) \), whose interpretation and transformations to scale are discussed at length by Raskin and Cochran (1986). Note that the last of the equivalent representations suggests that the CARA may be interpreted as the percent change in marginal utility per unit of outcome space.
The difference from the CARA case occurs in the last term. If expectations are ignored, 
\[ \frac{\gamma C_e(t) - 1}{\mathbb{E}[\mu_c(t)]} = \frac{\gamma}{C(t)}, \]
so that risk increases with a relative rise in \( \frac{g(t)H(t)}{C(t)} \). The latter is the ratio of the real housing wealth to the real consumption numeraire, which is analogous to the ratio of the real relative quantities of housing services and the consumption numeraire derived in Piazzesi et al. (2007) under their nonseparable CRRA.

In the empirical work in the next section, we concentrate on the CARA case for simplicity.

### 5.2 Empirical Estimation

Empirical implementation of the risk premium, \((30)\), requires estimates of expectations, variances and the covariance between the return on housing and the risky asset. The consensus in the literature is that housing markets are not fully efficient (see Barkham and Geltner, 1996, for UK evidence); house price movements are autocorrelated and so past changes contribute to an explanation of current returns (see Englund and Ioannides, 1997, for an international comparison); expectations are not forward-looking (see Case and Shiller, 1988, for an extensive discussion of housing market expectations and volatility). Therefore, expectations may be modelled adaptively (Nordvik, 1995), given by \((32)\).

\[ r_h^g(t + 1) = (1 - \alpha_h) \cdot r_h^g(t) + \alpha_h \cdot r_h(t) \]  
\[ (32) \]

However, there is little guidance from the literature for the appropriate value of \( \alpha_h \), but if \( \alpha_h = 1 \) then expectations are determined entirely by actual returns in the previous time period; this is the implicit assumption used in \((7'')\) above. However as \( \alpha_h \to 0 \), the influence of past-period expectations increases. Therefore, tests of the appropriate value of \( \alpha_h \) can be based on the significance of the lagged variables. As discussed above, these are shown in the third column of Table 2 where lagged values of \( \Delta \ln (g) \) are added. These additional lagged values make only a modest contribution. Overall the evidence is that house price expectations are affected primarily by recent price changes.

Risky assets are approximated by the stock market and the real annual return is given by the sum of the growth in the FT All Shares Ordinary Price Index and its conjoined dividend yield (deflated by the consumers’ expenditure deflator, see Figure 3). The correlation coefficient, \( \rho_{ah} \), between \( r_h(t) \) and
The first row indicates that over the whole sample period, the correlation is close to zero (0.08), but differs substantially between the five year sub-periods; particularly noticeable is the behaviour from the beginning of the boom period in 1996. Between 1996 and 2005 the correlation was strongly negative, but strongly positive from 2006 onwards. The only other period of strong positive correlation was between 1981 and 1985, a further period in which housing returns were weak. Over the whole sample period, since \( \rho_{ah} \equiv 0 \), the risk premium could be reduced to (33);\(^{16}\) alternatively, the correlation increases market volatility in the short run and raises the possibility of a market collapse, but has limited effect in the long run as a determinant of house prices. This may explain why the diagnostic statistics in Section 4 revealed little evidence of model misspecification.

\[ \tau(t) = \varphi g(t)H(t)\sigma_h^2(t + 1) \]  

Nevertheless, even in the long term, (33) reveals that risk depends on three factors – the market value of the housing stock, the variance of housing returns and the degree of risk aversion. The first implies that there are built-in seeds of market collapse as prices rise.

Two final operational problems have to be taken into account. The first concerns the time periods over which the variances and covariance are calculated. From Table 3, the covariance is not constant and so the chosen time period affects the results. In practice, the series are heavily smoothed in order to avoid sharp changes, but other assumptions are possible. The two parts of the risk premium, \( \tau_1(t) \) and \( \tau_2(t) \) defined below, are shown in Figure 4. Second, the risk parameter is identified only up to a scalar (see section 6.1 and Raskin and Cochran, 1986) and so the absolute level of risk is not defined in Figure 4, only its time path.

\[ \tau_1(t) = [r_a^e(t + 1) - (1 - \theta(t))i(t) + \pi(t)] \frac{\rho_{ah}(t + 1) \cdot \sigma_h(t + 1)}{\sigma_a(t + 1)} \]

\[ \tau_2(t) = \varphi g(t)H(t) \left(1 - \rho_{ah}^2(t + 1)\right) \sigma_h^2(t + 1) \]

\(^{16}\) Our findings on UK quarterly data are, thus, similar to those on US quarterly data reported in Flavin and Nakagawa (2008).
With these caveats, the figure indicates that $\tau_2(t)$ provides the biggest contribution and its upward trend reflects the rise in the market value of the housing stock. The historically high degree of housing risk in the build-up to the GFC is particularly noticeable. The effects of the risk premium can now be examined as an addition to the equations in Table 2. In column (4), the term is included separately and, then in column (5), incorporated into the full cost of capital definition. The first shows that risk takes the expected negative coefficient and is statistically significant; note that the coefficient on $\tau$ and $CC$ are similar in size, justifying the combining of the two terms. Compared with the riskless case, there is a modest improvement in equation fit (given the properties of $\tau$, major changes are not expected) and the remaining coefficients are little changed. Nevertheless, as demonstrated in the next section, this modest change has fundamental implications for the dynamics of housing markets and endogenous cycles.

### 6. Simulation Design and Results

#### 6.1 Baseline Calibration of the Key Parameters Used in the Simulations

We now proceed to analysis and interpretation of the role that the housing risk premium plays in dynamic stochastic simulations of our model. To summarise, the model consists of two equations - a house price equation, (Table 2, column 5), and the definition of the risk adjusted cost of capital, equation (29), where the risk premium is defined by (30). But since, from Figure 4, the risk premium is, in practice, dominated by $\tau_2$, simulations are run under the reasonable simplifying assumption that $\rho_{ah} \equiv 0$, so that the risk premium reduces to (33). In addition, the long-run solution to the price equation, (34) is used, rather than the full dynamic specification\(^{17}\); this is in order to abstract from cycles induced from the equation dynamics – Section 1 showed that lagged adjustment provides an alternative explanation for cycles in the literature.

\[
\ln[g(t)] = -7.44 + 2.73 \ln[RY(t)/HH(t)] - 1.61 \ln[H^s(t)/HH(t)] - 0.05[CC(t)] + 0.215 \ln[RW(t)] + 3.850 [WSH(t)]
\]

(34)

Figure 5 provides a stylised representation of how the cycles arise; in the first phase, where the (log) ratio of incomes to the housing stock exceeds the user cost, affordability will worsen through a rise

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\(^{17}\) Note the similarity to the key parameters in equation (13). The wealth ($RW$) and the wage share ($WSH$) coefficients are also shown in (34), but are not central to the main conclusion and so are not discussed further.
in house prices. However, the rise in prices raises the risk premium so that eventually the user cost dominates and the market turns down; this continues until the market value of housing falls sufficiently for the cycle to be repeated.

However, the full simulations illustrate the different dynamics of the endogenous variables in the standard life-cycle housing model compared with the expected utility model. In particular, we focus on how the endogenous variables evolve over time following unexpected large or persistent shock sequences in the growth rate of a key driving force, real income, resembling the GM-GFC cycle. Following most of the housing literature, real income is assumed exogenous; the large standard deviation of the income growth rate (see Table 4) relative to the other exogenous variables in (34), i.e. the housing stock and the number of households, supports the choice of this variable as a key driver. The quarterly growth rate in real household income is estimated to follow a first-order autoregressive process set out in the legend to Table 4. In simulation, temporary shocks are drawn from a log-Normal distribution for the growth rate of real income. This table also reports the starting values, means and standard deviations for the main variables. The table might, at first sight, suggest that the CARA parameter is very small (0.000368); however, this is purely a question of scaling, arising from the units in which housing wealth is expressed and is consistent with the results in Raskin and Cochrane (1986, Theorem 1). As an illustration, normalising the wealth stock on a value of unity, yields a CRRA parameter of approximately 1.5.

6.2 Sample Dynamics under a Single Replication

As an initial illustration, the six frames of Figure 6 plot one realisation for the key variables – income, real house prices, affordability, the variance of the return on housing, the risk premium and the user cost - over 200 quarters, where the stochastic real income growth rate has been calibrated to its mean and standard deviation over the historical period. At this stage, the events of the GM-GFC period have not been imposed on the stochastic income path. Figure 6 shows three cases: (i) where income simply grows at its long-run trend; (ii) where income grows stochastically, but there is no risk premium in the user cost; (iii) where income grows stochastically and the user cost includes a risk premium. Cases (ii) and (iii) allow a comparison of the standard life cycle model against our enhanced version.
Concentrating on affordability (measured by the ratio of house prices to incomes), shown in the top right-hand frame, Case (i) illustrates a worsening of affordability over time, consistent with equation (14); if income grows faster than the housing stock, then affordability continues to worsen, since there is no stabilising mechanism in the standard model. Case (ii) shows that this position does not fundamentally change when income is assumed to grow stochastically; although affordability is more volatile, it still exhibits a strong upward trend. However in Case (iii), where the risk premium is introduced, the upward trend in affordability disappears. The reason becomes clear by comparing the cost of capital (Frame 6) under Cases (ii) and (iii); in the latter the cost of capital (in contrast to Figure (1d)) is no longer a stationary process and acts as the market stabiliser.

[Insert Figure 6 about here]

6.3 A 100 Sample Replication and Simulation of the GM-GFC Period

The results from a single replication could occur by chance; therefore, this sub-section repeats the results, averaging over 100 replications. In addition, we attempt to model the effects of the strong positive income growth in the GM period, followed by the slump in the GFC. To replicate the GM-GFC dynamics, we introduce a particular sequence of two simulated shocks in real income growth, the first positive and not large in magnitude but more persistent (resembling the GM) and the second large and negative but transient (resembling the GFC).\(^{18}\) We superimpose these two big shocks onto the otherwise mild stochastic setting for the growth rate of real income, typified in the previous sub-section. Such a simulation design aims to reproduce the essential features of the UK housing environment over the 1990s and 2000s, and to trace down the effect of these income growth shocks on the dynamics of the real house price and other variables of central relevance to our model.

[Insert Figure 7a about here]

As a comparison, Figure 7a repeats Figure 6, but averages the results over 100 replications; the conclusions are fundamentally unchanged, but the averaging reduces the degree of volatility in

\(^{18}\) For more detail, see Table 4.
affordability and the other variables. Again, the key finding is that the addition of the risk premium heavily reduces any worsening of affordability.

Figure 7b adds in the GM-GFC shocks. The latter is particularly evident in the top-left panel for income. Since the elasticity of house prices with respect to income exceeds two, (equation 34), unsurprisingly, the addition of the income cycle increases the amplitude of the house price and affordability cycles considerably and is consistent with the house price cycle observed since the mid-1990s. But, once again, the key feature remains that affordability exhibits a weaker trend when the risk premium is included.

Concluding Remarks

This paper provides a number of lessons for policy and the next housing crisis (although it is not possible to predict when it will occur). A commonly held view is that house prices, at least in nominal terms, rarely fall and, therefore, housing is a safe investment. Historical experience appears to bear this out; between the mid-1950s and 1990, the annual growth rate in house prices was negative in only one quarter. Furthermore, conventional models appear to support the optimism; from equation (14), if real incomes grow faster than the housing stock, house prices rise relative to incomes. The speed of change depends on the income and price elasticities of housing demand.

But the myth of permanent price increases was first dispelled in the recession of the early 1990s and, then, during the GFC. However, this paper demonstrates formally that a failure to allow for risk in the definition of the user cost of capital meant that conventional models were unlikely to pick up the possibility of a market collapse and the models were misspecified. Once risk is taken into account, then housing markets have a built-in stabiliser that prevents price to income ratios increasing without bound. The risk premium is not adequately captured by the variance of house price changes alone, but also depends on the market value of the housing stock, the variance in the return on financial assets and the covariance in returns between financial assets and housing. In the case where the covariance is zero, which has approximately been the case in the long run, the definition of the risk premium simplifies. We find empirical support for our definition of the risk
premium when added to a conventional house price equation with the sign and coefficient size in line with prior expectations.

Stochastic simulations demonstrate the differences between the effects of income shocks in the conventional life-cycle housing model and the risk-adjusted version; in the former, affordability is strongly trended, but the trend disappears in the latter. Furthermore simulations that approximate the GM-GFC period demonstrate the high degree of housing market volatility induced through macroeconomic drivers, in particular real income growth fluctuations.

From a policy perspective, attempts to improve affordability have concentrated heavily on increasing housing supply. Although higher levels of house building are certainly desirable, the paper shows that there is a limit to what can be achieved by this route. The required increase in supply to stabilise the price to income ratio in the standard model is not feasible - permanent increases in construction would be required that have never been achieved in history. But, equally, the model shows that price to income ratios are likely to stabilise even without major increases in supply, although adjustment could take the form of an undesirable market collapse.
Figure 1a. House Prices Relative to Consumer Prices (annual % change, 1970-2014)

Figure 1b. House Prices Relative to Household Disposable Income (2011=100, 1970-2014)
Figure 1c. Net Mortgage Advances (deflated by house prices, £m, moving average 1970Q1-2015Q1)

Figure 1d. Cost of Capital (% 1969Q1-2012Q4)
Figure 2. Private Sector Housing Starts (000s, 1973Q1-2014Q4)
Figure 3: Annual Growth in Real Share Prices (RFT) plus dividend yield (DY), and Annual Growth in Real House Prices (RPH), (1969Q2 – 2014Q2)

Figure 4: The Two Components of the Housing Risk Premium (τ)
1. $1.5 \ln(\frac{R Y}{H^S}) > 0.05 \text{ CC}$
2. $1.5 \ln(\frac{R Y}{H^S}) = 0.05 \text{ CC}$
3. $1.5 \ln(\frac{R Y}{H^S}) < 0.05 \text{ CC}$

Figure 5: A Stylised Representation of the Housing Cycle
Figure 6: Dynamics under a Single Replication
Figure 7a: Average Dynamics of 100 Stochastic Replications without Imposing Large Shocks to Simulate the GM-GFC Boom-Bust Cycle in UK House Prices
Figure 7b: Average Dynamics of 100 Stochastic Replications Imposing Two Large Shocks to Simulate the GM-GFC Boom-Bust Cycle in UK House Prices
Table 1: Results from Unit Root Tests; ADF(4), PP(4) and GLS-DF (whole sample, 1969Q3 – 2012Q4)

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g</td>
</tr>
<tr>
<td>ADF (Intercept)</td>
<td>-1.00</td>
</tr>
<tr>
<td>Levels</td>
<td></td>
</tr>
<tr>
<td>First difference</td>
<td>-5.28</td>
</tr>
<tr>
<td>PP (Intercept)</td>
<td>-0.27</td>
</tr>
<tr>
<td>Levels</td>
<td></td>
</tr>
<tr>
<td>First difference</td>
<td>-7.26</td>
</tr>
<tr>
<td>GLS-DF (Intercept)</td>
<td>-0.16</td>
</tr>
<tr>
<td>Levels</td>
<td></td>
</tr>
<tr>
<td>First difference</td>
<td>-5.29</td>
</tr>
</tbody>
</table>

Note: ADF is Augmented Dickey-Fuller test; PP is Phillips-Perron test; GLS-DF is Dickey-Fuller GLS test. g denotes the real house price; RY is real income; RW is real wealth.

Table 2. Modelling House Prices, Dependent Variable: Δln(g)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.707 (4.9)</td>
<td>-0.725 (4.9)</td>
<td>-0.782 (5.5)</td>
</tr>
<tr>
<td>ln (g)_{t-1}</td>
<td>-0.102 (6.9)</td>
<td>-0.102 (7.0)</td>
<td>-0.103 (7.0)</td>
</tr>
<tr>
<td>ln (RW)_{t-1}</td>
<td>0.022 (2.2)</td>
<td>0.025 (2.4)</td>
<td>0.022 (2.3)</td>
</tr>
<tr>
<td>ln (HS/HH)_{t-1}</td>
<td>-0.140 (3.0)</td>
<td>-0.140 (2.8)</td>
<td>-0.171 (3.7)</td>
</tr>
<tr>
<td>ln (RY/HH)_{t-1}</td>
<td>0.254 (4.9)</td>
<td>0.257 (5.0)</td>
<td>0.290 (5.5)</td>
</tr>
<tr>
<td>CC_{t-2}</td>
<td>-0.005 (12.3)</td>
<td>-0.006 (6.5)</td>
<td>-0.005 (12.6)</td>
</tr>
<tr>
<td>WSH_{t-1}</td>
<td>0.340 (2.4)</td>
<td>0.363 (2.6)</td>
<td>0.425 (3.0)</td>
</tr>
<tr>
<td>Δln(RY/HH)_{t}</td>
<td>0.257 (3.1)</td>
<td>0.257 (3.1)</td>
<td>0.268 (3.3)</td>
</tr>
<tr>
<td>Δln(g)_{t-2}</td>
<td>-0.006 (5.7)</td>
<td>-0.004 (3.1)</td>
<td>-0.006 (5.7)</td>
</tr>
<tr>
<td>Δln(g)_{t-3}</td>
<td></td>
<td>0.162 (2.0)</td>
<td></td>
</tr>
<tr>
<td>Δln(g)_{t-4}</td>
<td></td>
<td>-0.118 (1.6)</td>
<td></td>
</tr>
<tr>
<td>Δln(g)_{t-5}</td>
<td></td>
<td>-0.111 (1.6)</td>
<td></td>
</tr>
<tr>
<td>τ_{t-1}</td>
<td></td>
<td>-0.130 (2.2)</td>
<td></td>
</tr>
<tr>
<td>(CC+τ)_{t-1}</td>
<td></td>
<td></td>
<td>-0.005 (12.6)</td>
</tr>
<tr>
<td>Δ(C+C+τ)_{t}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj R²</td>
<td>0.73</td>
<td>0.76</td>
<td>0.74</td>
</tr>
<tr>
<td>Equation standard error</td>
<td>0.0160</td>
<td>0.0154</td>
<td>0.0157</td>
</tr>
<tr>
<td>Lagrange Multiplier (serial correlation)</td>
<td>Prob(F_{4,153})=0.02</td>
<td>Prob(F_{4,146})=0.15</td>
<td>Prob(F_{4,152})=0.08</td>
</tr>
<tr>
<td>ARCH (heteroscedasticity)</td>
<td>Prob(F_{1,169})=0.14</td>
<td>Prob(F_{1,166})=0.05</td>
<td>Prob(F_{1,169})=0.09</td>
</tr>
<tr>
<td>Ramsey RESET</td>
<td>Prob(F_{1,156})=0.61</td>
<td>Prob(F_{1,150})=0.11</td>
<td>Prob(F_{1,153})=0.51</td>
</tr>
<tr>
<td>Jarque-Bera (Residual Normality)</td>
<td>Prob = 0.15</td>
<td>Prob = 0.18</td>
<td>Prob = 0.03</td>
</tr>
</tbody>
</table>

Note: The equation includes seasonal dummies and dummies to reflect the abolition of double mortgage tax relief in 1988. t-values in brackets.
Table 3. Correlations between the Returns on Housing and Risky Assets

<table>
<thead>
<tr>
<th>Time period</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970-2012</td>
<td>0.080</td>
</tr>
<tr>
<td>1970-1975</td>
<td>0.154</td>
</tr>
<tr>
<td>1976-1980</td>
<td>-0.171</td>
</tr>
<tr>
<td>1981-1985</td>
<td>0.822</td>
</tr>
<tr>
<td>1986-1990</td>
<td>0.049</td>
</tr>
<tr>
<td>1991-1995</td>
<td>0.169</td>
</tr>
<tr>
<td>1996-2000</td>
<td>-0.330</td>
</tr>
<tr>
<td>2001-2005</td>
<td>-0.661</td>
</tr>
<tr>
<td>2006-2012</td>
<td>0.691</td>
</tr>
</tbody>
</table>

Note: Bold figures highlight the largest values.

Table 4. Calibration of Key Values used in the Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - \theta) i$</td>
<td>Net-of-tax nominal mortgage interest rate (pa)</td>
<td>0.1047</td>
<td>UK sample average; Meen (2013)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Inflation rate (pa)</td>
<td>0.0578</td>
<td>UK sample average</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>CARA absolute risk aversion parameter</td>
<td>0.000368</td>
<td>Model- and UK estimate-implied</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Housing depreciation rate (pa)</td>
<td>0.01</td>
<td>See Meen (2013)</td>
</tr>
<tr>
<td>$R_Y(0)$</td>
<td>Initial value of households’ real income</td>
<td>203324</td>
<td>Base period (2000Q1)</td>
</tr>
<tr>
<td>$H_S(0)$</td>
<td>Initial value of the housing stock</td>
<td>17039</td>
<td>Base period (2000Q1)</td>
</tr>
<tr>
<td>$H_H(0)$</td>
<td>Initial number of households</td>
<td>23664</td>
<td>Base period (2000Q1)</td>
</tr>
<tr>
<td>$r_{RY}$</td>
<td>Quarterly growth rate of real income</td>
<td>0.006756</td>
<td>UK sample average</td>
</tr>
<tr>
<td>$r_{HS}$</td>
<td>Quarterly growth rate of housing stock</td>
<td>0.004227</td>
<td>UK sample average</td>
</tr>
<tr>
<td>$r_{HH}$</td>
<td>Quarterly growth rate of the number of households</td>
<td>0.002113</td>
<td>UK sample average</td>
</tr>
<tr>
<td>$\sigma_{RY}$</td>
<td>SD of the growth rate of real income</td>
<td>0.016228</td>
<td>UK sample statistic</td>
</tr>
<tr>
<td>$\sigma_{HS}$</td>
<td>SD of the growth rate of the housing stock</td>
<td>0.002555</td>
<td>UK sample statistic</td>
</tr>
<tr>
<td>$\sigma_{HH}$</td>
<td>SD of the growth rate of population</td>
<td>0.000819</td>
<td>UK sample statistic</td>
</tr>
</tbody>
</table>

Note: The estimated AR(1) process for real income growth is: $r_Y(t) = 0.008615 - 0.254524 \cdot r_Y(t-1) + e(t)$, with the empirical distribution of the error term approximately distributed as $N(0,0.015629^2)$. When reproducing the GM-GFC shock sequence in our simulations, we have additionally made the following assumptions. A persistent positive shock to real income that lasts for 60 quarters (quarters 81-140) – similar to the Great Moderation (GM) – is imposed equal to 20% of the sample mean. A large negative shock that lasts for 8 quarters (quarters 141-148) – similar to the Global Financial Crisis (GFC) – is, then, assumed to reduce real income by 2 standard deviations. Further details and the complete replication of our results are available via our MATLAB simulation codes, to be provided upon request.
Appendix A: Derivation of the User Cost and the Risk Premium

The expected lifetime utility and the constraints from the main text\textsuperscript{19} are:

$$E[U] = E \left[ \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \mu[H(t), C(t)] \right]$$

(16)

$$C(t) + g(t)X(t) + p(t)AP_1(t) + AP_2(t) = (1 - \theta(t))RY(t) + (1 - \theta(t))D(t)A_1(t - 1) + (1 - \theta(t - 1))i(t - 1) \cdot A_2(t - 1)$$

(17)

$$X(t) - \delta(t - 1)H(t - 1) = H(t) - H(t - 1)$$

(18)

$$AP_2(t) - \pi(t - 1)A_2(t - 1) = A_2(t) - A_2(t - 1)$$

(19)

$$AP_1(t) = A_1(t) - A_1(t - 1)$$

(20)

Maximizing the Lagrangian function:

$$L(t) = E \left[ \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left\{ \mu[H(t), C(t)] + \lambda(t) \left[ (1 - \theta(t))RY(t) - g(t)[H(t) - H(t - 1) + \delta(t - 1)H(t - 1)] + (1 - \theta(t))(1 - \pi(t)) \right] A_2(t - 1) - A_2(t - 1) - \lambda(t) + (1 - \theta(t))D(t)A_1(t - 1) - p(t)[A_1(t) - A_1(t - 1)] \right\} \right]$$

The first-order conditions are:

$$\frac{\partial L(t)}{\partial H(t)} = E \left\{ \frac{1}{(1+r)^t} \mu_H(t) - \frac{1}{(1+r)^t} \lambda(t)g(t) + \frac{1}{(1+r)^t} \lambda(t + 1)g(t + 1)(1 - \delta(t)) \right\} = 0$$

$$\frac{\partial L(t)}{\partial C(t)} = E \left\{ \frac{1}{(1+r)^t} \mu_C(t) - \frac{1}{(1+r)^t} \lambda(t) \right\} = 0$$

$$\frac{\partial L(t)}{\partial A_2(t)} = E \left\{ -\frac{1}{(1+r)^t} \lambda(t) + \frac{1}{(1+r)^t} \lambda(t + 1)[1 + (1 - \theta(t))i(t) - \pi(t)] \right\} = 0$$

$$\frac{\partial L(t)}{\partial A_1(t)} = E \left\{ -\frac{1}{(1+r)^t} \lambda(t)p(t) + \frac{1}{(1+r)^t} \lambda(t + 1)[p(t + 1) + (1 - \theta(t + 1))D(t + 1)] \right\} = 0$$

implying:

$$C: \quad E[\mu_C(t)] = E[\lambda(t)]$$

and simplifying:

$$H: \quad \frac{1}{(1+r)^t} E[\mu_H(t)] = \frac{1}{(1+r)^t} E[\mu_C(t)]g(t) - \frac{1}{(1+r)^t} (1 - \delta(t))E[\mu_C(t + 1)g(t + 1)]$$

(1a)

$$A_2: \quad \frac{1}{(1+r)^t} E[\mu_C(t)] = \frac{1}{(1+r)^t} E[\mu_C(t + 1)] [1 + (1 - \theta(t))i(t) - \pi(t)]$$

(2a)

$$A_1: \quad \frac{1}{(1+r)^t} E[\mu_C(t)] = \frac{1}{(1+r)^t} E \left[ \mu_C(t + 1)[p(t + 1) + (1 - \theta(t + 1))D(t + 1)] \right]$$

(3a)

\textsuperscript{19} Note that the ‘D’ superscript has been dropped in the notation for housing demand compared with the main text, since the distinction from housing supply is not relevant here.
Notice that the current values \(g(t)\) and \(p(t)\) are certain but the future values \(g(t+1), p(t+1)\) and \(D(t+1)\) are uncertain.

From (3a), \(E[\mu_C(t)] = \frac{1}{(1+r)^t}E[\mu_C(t+1)]\left[1 + (1 - \theta(t))i(t) - \pi(t)\right] \quad (4a)\)

From (1a),
\[
\frac{1}{(1+r)^t}E[\mu_H(t)] = \frac{1}{(1+r)^t}E[\mu_C(t)]g(t) - \frac{1}{(1+r)^{t+1}}(1 - \delta(t))E[\mu_C(t+1)g(t+1)]
\]
\[
E[\mu_H(t)] = E[\mu_C(t)]g(t) - \frac{1}{(1+r)}(1 - \delta(t))E[\mu_C(t+1)g(t+1)]
\]

Substituting (4a) into the above equation,
\[
E[\mu_H(t)] = [\mu_C(t)]g(t) - \frac{(1 - \delta(t))E[\mu_C(t+1)g(t+1)]}{E[\mu_C(t+1)][1 + (1 - \theta(t))i(t) - \pi(t)]}E[\mu_C(t)]
\]
\[
\frac{\mu_H(t)}{\mu_C(t)} = \frac{g(t)}{\mu_C(t)} \left[1 - \frac{(1 - \delta(t))E[\mu_C(t+1)g(t+1)]}{E[\mu_C(t+1)][1 + (1 - \theta(t))i(t) - \pi(t)]}\right]
\]
\[
\frac{\mu_H(t)}{\mu_C(t)} \approx \frac{g(t)}{\mu_C(t)} \left[1 - \frac{\mu_C(t+1)[1 + r_h(t+1) - \delta(t) - (1 - \theta(t))\pi(t)]}{E[\mu_C(t+1)]}\right]
\]
\[
\frac{\mu_H(t)}{\mu_C(t)} \approx \frac{g(t)}{\mu_C(t)} \left[(1 - \theta(t))i(t) - \pi(t) + \delta(t) - \frac{\mu_C(t+1)r_h(t+1)}{E[\mu_C(t+1)]}\right] \quad (5a, \text{see (21) in the main text})
\]

where: \(r_h(t+1) = [g(t+1) - g(t)]/g(t)\), which is the real housing capital return.

From (5a), since \(E[AB] = E[A]E[B] + Cov(A, B)\),
\[
\frac{\mu_H(t)}{\mu_C(t)} \approx \frac{g(t)}{\mu_C(t)} \left[(1 - \theta(t))i(t) - \pi(t) + \delta(t) - E[r_h(t+1)] - \frac{Cov(\mu_C(t+1), r_h(t+1))}{E[\mu_C(t+1)]}\right] \quad (6a)
\]

From (3a),
\[
\frac{1}{(1+r)^t}p(t)E[\mu_C(t)] = \frac{1}{(1+r)^{t+1}}E \left[\mu_C(t+1)p(t+1) + (1 - \theta(t+1))D(t+1)\right]
\]
\[
p(t)E[\mu_C(t)] = \frac{1}{(1+r)^{t+1}}E \left[\mu_C(t+1)p(t+1) + (1 - \theta(t+1))D(t+1)\right]
\]
\[
E[\mu_C(t)] = \frac{1}{(1+r)^{t+1}}E \left[\mu_C(t+1)[1 + r_a(t+1)]\right] \quad (7a)
\]

where: \(r_a(t+1) = [p(t+1) + (1 - \theta(t+1))D(t+1) - p(t)]/p(t)\), which is the real risky financial asset return.
Substituting (4a) into (7a),

\[ E[\mu_C(t+1)]\left[1 + (1 - \theta(t))i(t) - \pi(t)\right] = E[\mu_C(t+1)]\left[1 + r_a(t+1)\right] \]

\[ (1 - \theta(t))i(t) - \pi(t) = \frac{E[\mu_C(t+1)r_a(t+1)]}{E[\mu_C(t+1)]} \]

(8a, see (22) in the main text)

From (8a),

\[ E[\mu_C(t+1)] = -\frac{\text{Cov}(\mu_C(t+1), r_a(t+1))}{E[r_a(t+1)] - [(1 - \theta(t))i(t) - \pi(t)]} \]

(9a)

Substituting (9a) into (6a),

\[ \frac{E[\mu_H(t)]}{E[\mu_C(t)]} \approx g(t) \left\{ (1 - \theta(t))i(t) - \pi(t) + \delta(t) - E[r_h(t+1)] \right\} \]

\[ + \left[ E[r_a(t+1)] - [(1 - \theta(t))i(t) - \pi(t)] \right] \frac{\text{Cov}(\mu_C(t+1), r_a(t+1))}{\text{Cov}(\mu_C(t+1), r_h(t+1))} \]

(10a, see (24) in the main text)
Appendix B: Derivation of the User Cost and the Risk Premium under CARA Utility

To obtain an explicit solution, we assume (i) CARA utility and (ii) that $r_h(t)$ and $r_a(t)$ are jointly normally distributed.

Recalling that:

$$r_h(t+1) \sim N\left(r_h^e(t+1), \sigma_h^2(t+1)\right) \quad (25)$$

$$r_a(t+1) \sim N\left(r_a^e(t+1), \sigma_a^2(t+1)\right) \quad (26)$$

$$u(H(t), C(t)) = -e^{-\phi_H(t)} - e^{-\phi_C(t)} \quad (27)$$

where:

$$\phi(z_1, z_2)$$ is the joint standard normal probability density function for $z_1, z_2$, given by

$$\phi(z_1, z_2) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}[z_1^2 - 2\rho z_1 z_2 + z_2^2]\right\}$$

$\rho$ is the correlation coefficient between $z_1$ and $z_2$;

$$\rho_{ah} = \frac{\sigma_{ah}}{\sigma_a \sigma_h}$$ is the correlation coefficient between $r_h$ and $r_a$.

The general process is that, given the specific CARA utility function and the distribution of households’ expectations, one substitutes the solution of (8a) into the expansion of (5a).

$$\frac{E[\mu_C(t)]}{E[\mu_C(t+1)]} \approx g(t) \left\{(1 - \theta(t))i(t) - \pi(t) + \delta(t) - \frac{E[\mu_C(t+1)r_h(t+1)]}{E[\mu_C(t+1)]}\right\} \quad (5a, \text{see (21) in the main text})$$

$$(1 - \theta(t))i(t) - \pi(t) = \frac{E[\mu_C(t+1)r_a(t+1)]}{E[\mu_C(t+1)]} \quad (8a, \text{see (22) in the main text})$$

**Step (1): Derivation of $E[\mu_C(t+1)]$**

Given the specific CARA utility,$$
\mu_c(t+1) = \varphi e^{-\varphi C(t+1)} \equiv \varphi \cdot \exp[-\varphi C(t+1)]
$$

$$E[\mu_c(t+1)]$$

$$= \int_{-\infty}^{+\infty} \mu_c(t+1) \phi(z_1, z_2)dz_1dz_2$$

$$= \int_{-\infty}^{+\infty} \varphi \exp[-\varphi C(t+1)] \phi(z_1, z_2)dz_1dz_2$$
\[
\begin{align*}
E[\mu_c(t + 1)] & = \int_{-\infty}^{+\infty} \phi_{\varphi} \cdot \exp[-\varphi C^e(t + 1)] \cdot \exp[-\varphi [p(t)A_2(t)\sigma_a(t + 1)z_2 + g(t)H(t)\sigma_h(t + 1)z_1]] \cdot \phi(z_1, z_2)dz_1dz_2 \\
\end{align*}
\]

where \( C^e(t + 1) = E[C(t + 1)] \)

\[
E[(1 - \theta(t + 1))R_Y(t + 1) - g(t + 1)[H(t + 1) - H(t) + \delta(t)H(t)]] + [(1 - \theta(t))i(t) - \pi(t)]A_2(t) - [A_2(t + 1) - A_2(t)] + (1 - \theta(t + 1))D(t + 1)A_1(t) - p(t + 1)[A_1(t + 1) - A_1(t)]
\]

\[
\approx E[(1 - \theta(t + 1))R_Y(t + 1) - g(t + 1)H(t + 1) + (1 + r_h(t + 1) - \delta(t))g(t)H(t) - p(t + 1)A_4(t + 1) + (1 + r_g(t + 1))p(t)A_4(t) - A_2(t + 1) + [(1 - \theta(t))i(t) - \pi(t)]A_2(t)]
\]

\[
E[\mu_c(t + 1)] = \int_{-\infty}^{+\infty} \varphi \cdot \exp[-\varphi C^e(t + 1)] \cdot \exp[-\varphi [p(t)A_2(t)\sigma_a(t + 1)z_2 + g(t)H(t)\sigma_h(t + 1)z_1]] \cdot \phi(z_1, z_2)dz_1dz_2 \\
\end{align*}
\]

\[
\begin{align*}
\varphi \cdot \exp[-\varphi C^e(t + 1)] & \cdot \int_{-\infty}^{+\infty} \frac{1}{2\pi \sqrt{1 - \rho^2}} \exp[-\frac{1}{2(1 - \rho^2)} \left( z_1^2 - 2\rho z_1 z_2 + z_2^2 \right)] \exp[-\varphi p(t)A_2(t)\sigma_a(t + 1)z_2 - \varphi g(t)H(t)\sigma_h(t + 1)z_1 - \frac{1}{2(1 - \rho^2)} \left( z_1^2 - 2\rho z_1 z_2 + z_2^2 \right)] dz_1dz_2 \\
\end{align*}
\]

\[
\begin{align*}
\varphi \cdot \exp[-\varphi C^e(t + 1)] & \cdot \exp \left[ -\frac{\rho(t + 1)\varphi^2}{1 - \rho(t + 1)^2} p(t)A_2(t)\sigma_a(t + 1) + \rho(t + 1)g(t)H(t)\sigma_h(t + 1) \right] \\
\end{align*}
\]

\[
\begin{align*}
\varphi \cdot \exp[-\varphi C^e(t + 1)] & \cdot \exp \left[ -\frac{\rho(t + 1)\varphi^2}{1 - \rho(t + 1)^2} p(t)A_2(t)\sigma_a(t + 1) + \rho(t + 1)g(t)H(t)\sigma_h(t + 1) \right] \\
\end{align*}
\]

\[
\begin{align*}
\textbf{Step (2): Derivation of } E[r_h(t + 1)\mu_c(t + 1)]
\end{align*}
\]

(noticing that \( \mu_c \) is a function of \( \eta_h \) and \( r_a \))

\[
\begin{align*}
E[r_h(t + 1)\mu_c(t + 1)] & = \int_{-\infty}^{+\infty} E[r_h(t + 1)] + \sigma_h(t + 1)z_1] \mu_c(t + 1) \cdot \phi(z_1, z_2)dz_1dz_2 \\
\end{align*}
\]
\[
\begin{align*}
&= \int_{-\infty}^{+\infty} [E[r_n(t + 1)] + \sigma_n(t + 1) z_1] \varphi \cdot \exp[-\varphi C^e(t + 1) \cdot \varphi[p(t)A_2(t)]\sigma_n(t + 1)z_2 + g(t)H(t)\sigma_n(t + 1)z_1] \phi(z_1, z_2)dz_1dz_2 \\
&= \int_{-\infty}^{+\infty} [E[r_n(t + 1)] + \sigma_n(t + 1) z_1] \varphi \cdot \exp[-\varphi C^e(t + 1)] \cdot \varphi[p(t)A_2(t)]\sigma_n(t + 1)z_2 + g(t)H(t)\sigma_n(t + 1)z_1] \phi(z_1, z_2)dz_1dz_2 \\
&= \int_{-\infty}^{+\infty} E[r_n(t + 1)] \varphi \cdot \exp[-\varphi C^e(t + 1)] \cdot \varphi[p(t)A_2(t)]\sigma_n(t + 1)z_2 + g(t)H(t)\sigma_n(t + 1)z_1] \phi(z_1, z_2)dz_1dz_2 \\
&= E[r_n(t + 1)]E[\mu_c(t + 1)] + \sigma_n(t + 1) \int_{-\infty}^{+\infty} z_1 \varphi \cdot \exp[-\varphi C^e(t + 1)] \cdot \varphi[p(t)A_2(t)]\sigma_n(t + 1)z_2 + g(t)H(t)\sigma_n(t + 1)z_1] \phi(z_1, z_2)dz_1dz_2 \\
&= E[r_n(t + 1)]E[\mu_c(t + 1)] + \sigma_n(t + 1) \int_{-\infty}^{+\infty} z_1 \varphi \cdot \exp[-\varphi C^e(t + 1)] \cdot \varphi[p(t)A_2(t)]\sigma_n(t + 1)z_2 + g(t)H(t)\sigma_n(t + 1)z_1] \frac{1}{2\pi(1-\rho^2)} \exp[-\frac{1}{2(1-\rho^2)}(z_1^2 - 2\rho z_1 z_2 + z_2^2)] dz_1dz_2 \\
&= E[r_n(t + 1)]E[\mu_c(t + 1)] + \sigma_n(t + 1) \cdot \varphi[p(t)A_2(t)]\sigma_n(t + 1) \cdot [\rho(t + 1)g(t)H(t)\sigma_n(t + 1) + p(t)A_2(t)\sigma_n(t + 1)] \cdot \int_{-\infty}^{+\infty} z_1 f_z(z_1)dz_1 \\
\text{where: } & f_z(z_1) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2} (z_1^2 + \rho(t)\varphi p(t)A_2(t)\sigma_n(t + 1) + g(t)H(t)\sigma_n(t + 1))^2\right\} \\
\text{Therefore, } & E[r_n(t + 1)\mu_c(t + 1)] = \\
& E[r_n(t + 1)]E[\mu_c(t + 1)] + \varphi \cdot \exp[-\varphi C^e(t + 1)] \cdot \sigma_n(t + 1) \cdot \\
& \exp\left\{-\frac{\rho(t + 1)^2}{1-\rho^2} [\rho(t + 1)p(t)A_2(t)\sigma_n(t + 1) + g(t)H(t)\sigma_n(t + 1)] \cdot [\rho(t)g(t)H(t)\sigma_n(t + 1) + p(t)A_2(t)\sigma_n(t + 1)] \cdot \rho(t)\varphi p(t)A_2(t)\sigma_n(t + 1) - g(t)H(t)\sigma_n(t + 1)] \right\} \\
& \text{(2b)} \\
\text{Step (3): Derivation of } E[r_n(t + 1)\mu_c(t + 1)] \\
\text{Similarly,}
\end{align*}
\]
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\[ E[r_a(t+1)\mu_c(t+1)] = \\
E[r_a(t+1)]E[\mu_c(t+1)] + \sigma_a(t+1) \cdot \exp\left[-\varphi C^e(t+1)\right] \cdot \left[\rho(t+1)g(t)H(t)\sigma_h(t+1) + p(t)A_2(t)\sigma_a(t+1) - \rho(t+1)\varphi g(t)H(t)\sigma_h(t+1)\right] \]

(3b)

\[ \text{Step (4):} \]

Substituting (1b) into (3b),

\[ E[r_a(t+1)\mu_c(t+1)] = E[r_a(t+1)]E[\mu_c(t+1)] - \sigma_a(t+1)[\varphi p(t)A_2(t)\sigma_a(t+1) + \rho(t+1)\varphi g(t)H(t)\sigma_h(t+1)]E[\mu_c(t+1)] \]

(4b)

Substituting (4b) into (8a),

\[ (1-\theta(t))i(t) - \pi(t) = E[r_a(t+1)] - \sigma_a(t+1)[\varphi p(t)A_2(t)\sigma_a(t+1) + \rho(t+1)\varphi g(t)H(t)\sigma_h(t+1)] \]

\[ \varphi p(t)A_2(t)\sigma_a(t+1) = \frac{E[r_a(t+1)] - [(1-\theta(t))i(t) - \pi(t)]}{\sigma_a(t+1)} - \rho(t+1)\varphi g(t)H(t)\sigma_h(t+1) \]

(5b)

Substituting (1b) into (2b),

\[ E[r_h(t+1)\mu_c(t+1)] = E[r_h(t+1)]E[\mu_c(t+1)] - \sigma_h(t+1)[\rho(t+1)\varphi p(t)A_2(t)\sigma_a(t+1) + \varphi g(t)H(t)\sigma_h(t+1)]E[\mu_c(t+1)] \]

(6b)

Substituting (6b) into (5a),

\[ \frac{E[\mu_h(t)]}{E[\mu_c(t)]} \approx \frac{g(t)[(1-\theta(t))i(t) - \pi(t) + \delta(t) - E[r_h(t+1)] + \sigma_h(t+1)[\rho(t+1)\varphi p(t)A_2(t)\sigma_a(t+1) + \varphi g(t)H(t)\sigma_h(t+1)]]}{\sigma_a(t+1)} \]

(7b)

Substituting (5b) into (7b),

\[ \frac{E[\mu_h(t)]}{E[\mu_c(t)]} \approx g(t)\left\{(1-\theta(t))i(t) - \pi(t) + \delta(t) - E[r_h(t+1)] + \left[E[r_a(t+1)] - [(1-\theta(t))i(t) - \pi(t)]\right] - \rho(t+1)\varphi^2 g(t)H(t)\left(1 - \rho^2(1+1)\right)\sigma^2_h(t+1)\right\} \]
References


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