

A Theory of the Intergenerational Dynamics of Inflation Beliefs and Monetary Institutions

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Abstract

We develop a stochastic overlapping-generations model with heterogeneous beliefs on the degree of inflation protection that can be provided by markets versus the monetary authority. A distinct feature is that it incorporates adaptive learning from inflation history and imperfect empathy in the cultural transmission of individual beliefs, allowing social beliefs to evolve endogenously. Analytical results on endogenous inflation beliefs and socially-optimal inflation are derived first in a within-generation voting equilibrium that defines a particular degree of inflation aversion of a society's monetary institution. Then, the intergenerational dynamics of inflation and inflation beliefs are analyzed, providing insights into the long-run evolution of population types and social institutions. The framework allows exploring the interactions of three mechanisms: the persistence of inflation, the degree of inflation aversion of the central bank and the recurrent irregular cycles of agent type proportions (and subsequent majority switches). It is thus shown how the endogenous transmission of inflation beliefs and monetary institutions in a stochastic economic environment can be understood as a process of intergenerational learning from history, combined with a political economy mechanism that amends legislation, and a socialization process that transmits experienced knowledge.

JEL classification: D72, D83, E31, E58, H41

Key words: evolving beliefs, inflation aversion, adaptive learning, voting equilibrium, cultural transmission, monetary institutions

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1 Introduction

There is a voluminous literature on the dynamics of higher-frequency inflation – i.e., observed monthly, quarterly or yearly – but not much is known on what drives lower-frequency inflation – i.e., observed within a lifetime. It is well-known that changes in the level and volatility of higher-frequency inflation have occurred over the last decades, particularly in developed economies. Sargent (1999) suggests this may have resulted from a learning process, through which central banks have managed a better control of inflation. But shifts in inflation dynamics may also be due to a variation in the degree of inflation aversion, as people in these countries have gone through high inflation within their own lifetime (Malmendier and Nagel, 2009). More generally, as a generation’s beliefs are strongly influenced by the ‘impressive years’ of growing up and becoming adult (Giuliano and Spilimbergo, forthcoming), after which they hardly change, inflation dynamics may be driven by the experience of a generation, and transmitted to the successive ones. This would explain the emergence of the ‘(anti-)inflation culture’ some societies have built through history, with the most prominent example surely being Germany (Hayo, 1998; Vaubel, 2003).

Hence, endogenizing individual beliefs with regard to inflation and modeling their aggregation and transmission across generations in a society is fundamental, but has hardly been explored. Differently from bioscience, where it takes a large number of generations for genes to mutate, in social science beliefs, values and behavior inherited as culture – and the resulting institutions – can be modified much faster, in a generation or two, as individuals and societies adapt in response to observation and experience. For example, Dessí (2008) relates individual internalization of cultural norms and values to the quality of the existing institutions, while Dohmen et al. (2012) report empirical evidence in favor of the intergenerational transmission within families of the willingness to take risk or trust others. However, Black et al. (2005) find limited within-family intergenerational spillover of human capital in their sample studying educational reforms in Norway in the 1960s. This points to the role played by variations in socialization efforts as well as outside-family influences we study later on. Of course, culture, attitudes and institutions are ultimately moulded by history, as relevant past experience – e.g., hyperinflationary episodes and the abrupt shifts in voting majorities and monetary institutions they cause – is then transferred as social inheritance to the next generations. It would, thus, appear natural that, as Scheve (2004) reports, there is significant cross-country variation in inflation aversion.

In this paper, we endogenize the beliefs on inflation protection in a society as being culturally transmitted from one generation to another. More precisely, we develop a stochastic overlapping-generations (OLG) framework with two types of agents distinguished by their beliefs on the degree of inflation protection to be provided by markets versus the monetary authority. Our framework incorporates three interdependent channels that drive the socioeconomic dynamics we aim to highlight. Transmission of beliefs operates through the first channel, ‘socialization’, a process whereby parents and peers affect the

adoption of inflation protection ‘priors’, as they experience the consequences of actual inflation during their lifetime. Then, at the threshold of adulthood, each next generation updates the conditional inflation forecast over the horizon of its own mature life. This is the second, ‘learning’, channel of socioeconomic dynamics captured by our set-up, whereby generations learn asymptotically the unconditional mean of low-frequency, or generational, inflation. Finally, given heterogeneity in types predetermined by socialization but the same lifetime inflation forecast due to common knowledge, agents optimally choose the degree of lifetime inflation protection they would wish to see enacted as mandate for the monetary authority. Voting equilibria at the beginning of mature life of every generation thus modify the evolving monetary institutions of a society, which is the third channel, ‘institutionalization’, affecting the degree of inflation aversion of the monetary authority. Combining the three channels of socioeconomic dynamics we highlight, learning, socialization and institutionalization, allows us to investigate the longer-run interdependent evolution of attitudes toward inflation protection and the degree of inflation aversion of monetary institutions.

In implementing this approach, we follow Bisin and Verdier (2000, 2001) who build on the literature within economics on endogenous preferences¹ to develop and analyze formal set-ups where preferences evolve across generations. Recently, Bisin and Verdier (2010) have extended their earlier analysis of preferences to beliefs; Alesina and Giuliano (2010) provide a related discussion of belief formation; and Guiso et al. (2010) propose a dynamic model of beliefs. Tabellini (2008 a) suggests that distant political and economic history matters for the functioning of current institutions; while Tabellini (2008 b) explains the range of situations in which individuals cooperate responding to incentives, but are also influenced by norms of good conduct inherited from earlier generations.

In this paper we apply the Bisin-Verdier (2000, 2001, 2010) framework of cultural transmission to a new issue, extending it accordingly in three dimensions. First, we replace the ‘cultural substitution’ assumption that causes convergence to an interior steady state, with both types coexisting over time in their purely deterministic framework, with adaptive learning in a dynamic-stochastic environment.² Second, we apply the model to study the resulting, endogenously evolving degree of inflation aversion embedded in monetary institutions, initially in a theoretical context within each generation, and then through simulations in an intergenerational long-run perspective. Third, no paper in the political economy or macroeconomics literature has linked the dynamics of beliefs to a learning model where socialization and institutionalization interact, reinforcing or eroding one another, which we do.

Analyzing such interactive socioeconomic dynamics constitutes our main theoretical contribution. We, in effect, devise a novel modeling framework appropriate to study

¹ Going back to Becker (1976), Hirshleifer (1977) and Rubin and Paul (1979); Becker (1996) is a widely cited book.

² For an early analysis of social learning and personality development in cognitive psychology, see, e.g., Bandura and Walters (1963); for a compact survey of learning models in economics, see, e.g., Sobel (2000).

the joint (low-frequency) dynamics of inflation and beliefs on inflation protection in two types in the population as well as the resulting degree of central bank inflation aversion, determined by a voting mechanism. Its fundamental ingredients are: (i) an equation for the dynamics of generational inflation; (ii) an adaptive learning rule by which agents uncover asymptotically the unknown mean of the inflation process; (iii) an equation for the intergenerational transmission of beliefs on inflation protection in the population; and (iv) a voting mechanism which determines a time-varying policy drift term that allows for a partial control of the monetary authority over the law of motion for inflation.

We show that when positive generational inflation dominates historically (and, hence, in expectation), and with costly market-based inflation protection instruments, the non-redistributive rate of inflation is positive, and is a function of the latter costs. We also show how preferences for the degree of inflation aversion of the central bank, and the corresponding socialization effort, arise endogenously within each generation. Simulations confirm that such endogenous belief transmission in a stochastic set-up – where generations learn and adapt – provides an alternative to the assumption of ‘cultural substitution’ that is common in the previous deterministic socialization literature. Interior equilibria with both types coexisting across time are generated by explicitly modeling the response of parents in their socialization effort to the change in the conditional mean of inflation they have observed. We thus show how the endogenous transmission of inflation beliefs and monetary institutions in a stochastic economic environment can be understood as a process of intergenerational learning from history and, subsequently, amending existing institutions. Moreover, our simulations provide insights into the principal forces at play and their relative dominance or balance under key parameterizations. Viewed from an empirical perspective, the framework we propose offers one possible explanation of the differences in the degree of inflation aversion and *de jure* independence of the monetary authority across countries as reflecting longer-run histories of inflation dynamics.

The paper is organized as follows. In the next section, we present our OLG model and derive our main theoretical results within a single generation lifetime. Section 3 illustrates the dynamics of the model across generations, with a focus on our key simulation results for alternative parameterizations. Section 4 summarizes our concluding remarks. Proofs of lemmas and further details on the special case of deterministic socialization are provided in the appendix.

2 Voting Equilibrium within Each Generation

A generation consists of a continuum of individuals, each living for two periods (childhood and adulthood) and having one child, the standard assumption in the cultural transmission literature. The population is thus constant, and the size of the mature generation in any period t is normalized to 1.

2.1 Belief Types

We consider two types of beliefs in the population, $i \in \{a, b\}$, defined on a private good c and a public good G , as is customary in the public economics literature. We assume that belief types are complete, or revealed, at the beginning of adulthood (in any period t), which coincides with the end of a socialization process during childhood (in the preceding period $t - 1$) described further below. To simplify, we assume throughout that types remain unchanged during adulthood.³ Agents start with an identical initial nominal endowment received by all, ϖ . Agent types, however, differ in their beliefs with regard to the degree of protection of the real value of this endowment to be ensured by markets versus the monetary authority.⁴

We compare two cases of observed inflation histories by the population. As a first step, we begin with a benchmark where inflation and deflation can arise with equal probability and market-based protection against inflation is costless. This benchmark model rationalizes why partial private protection against inflation may not necessarily be beneficial, although agents cannot be sure about it. Thus, one of the types may not be (willingly) ‘passing on’ or ‘adopting’ beliefs involving any degree $0 < \varphi^i < 1$ of market-based protection against inflation: this is implied by assuming, in our benchmark version, that $\varphi^a = 0$ and $0 < \varphi^b = \varphi < 1$ and that investing the fraction φ implies no costs.

We then analyze the empirically relevant case where the observed history implies positive inflation rates most of the time. This version of the model rationalizes why partial private protection against inflation, via $\varphi^i > 0$, would characterize *both* types. It also justifies why agents would accept to pay certain private costs, $\Phi(\varphi^b) > \Phi(\varphi^a) > 0$ for $1 > \varphi^b > \varphi^a > 0$.⁵ These private costs are predetermined for each type by its socialization. In addition, $G_t^*(\cdot)$ is the cost of public protection against inflation decided by a voting mechanism in the beginning of each period t that everybody has to pay in equal share, as will be discussed further down. To save on notation, we henceforth will mostly write $\Phi(\varphi^i)$ as Φ^i and $G_t^*(\cdot)$ as G_t^* .

Hence, the real value of the final-period endowment for type i at the end of t is, in its general version, given by

$$\varpi_t^i \equiv \varphi^i \varpi + \frac{(1 - \varphi^i) \varpi}{1 + \pi_t} - \Phi^i - G_t^*, \quad (1)$$

³As in Acemoglu and Jackson (2011), this can be rationalized by some prohibitively high cost to change one’s type and, hence, behavior, later in mature life. Moreover, Huggett et al. (2011) find that, as of age 23, differences in initial conditions account for more of the variation in lifetime earnings, lifetime wealth and lifetime utility than do differences in shocks received over the working lifetime.

⁴It is out of the scope of the paper to model any specific variety of such inflation-indexed financial instruments or inflation-protecting portfolio diversification strategies. To mention just some widely used real-world examples, one may think of the Treasury Inflation-Protected Securities (TIPS) issued in the US by the federal government since 1997 as well as of their private-sector analogue known as Corporate Inflation-Protected Securities (CIPS): see, e.g., Barney and Harvey (2009). Of course, financial, real estate, commodity and other relevant markets provide protection against inflation in a broader context.

⁵More generally, $\Phi(\varphi^i)$ could be a monotonically increasing (but bounded – by wealth) function of the respective degrees of private protection against inflation by type i .

where π_t denotes the (net) rate of low-frequency inflation over a mature-generation life span t . For comparability with high-frequency inflation dynamics, we measure it in % per annum.

2.2 Learning from Inflation History

The low-frequency, or generational, inflation dynamics in part depends on its own past, and in part on the contemporaneous institutionalized degree of inflation aversion (applicable as from the beginning of each period). The latter is implemented via the imperfect control a monetary authority exerts over inflation, $\Gamma(G_t^*)$.⁶ The law of motion for inflation also includes a disturbance process, ε_t . But differently from most of the literature, we assume more realistically that the latter process is unknown to the agents. One can write then, very generally, the actual law of motion (ALM) of generational inflation as $\pi_t = \pi(\pi_{t-1}, \pi_{t-2}, \dots; \Gamma(G_t^*); \varepsilon_t; \cdot)$, which we narrow down to a first-order autoregressive (AR(1)) stochastic process with a time-varying ‘inflation-aversion’ or ‘monetary-control’ drift, $\Gamma(G_t^*)$,

$$\pi_t = -\Gamma(G_t^*) + \rho\pi_{t-1} + \varepsilon_t, \quad (2)$$

where $0 \leq \rho \leq 1$ is the persistence parameter of this low-frequency rate of inflation and ε_t is an independently and identically distributed (i.i.d.) inflation-shock process, $\varepsilon_t \rightsquigarrow iid(\mu_\varepsilon, \sigma_\varepsilon^2)$. Both types of agents are assumed boundedly rational, in the sense of learning adaptively as history unfolds. They do not know this ALM and the properties of the exogenous shock, but know that they can reduce actual inflation to some extent (via the magnitude of the drift term, as we model it) by voting on G_t^* in the beginning of their mature life. However, at the end of every period t they observe π_t and update its conditional mean $\hat{\mu}_{\pi,t}$. Generations will, therefore, be converging to the true mean of the inflation process in (2) asymptotically, $\hat{\mu}_{\pi,t \rightarrow \infty} \rightarrow \mu_\pi$. Of course, this will be so as long as ε_t and π_t do not change (frequently) their respective laws of motion and as long as $\Gamma(G_t^*)$ does not vary too much either. Yet, in our set-up agents will not be able to apply recursively ordinary least squares (OLS), since in general $\mu_\varepsilon \neq 0$ and $\Gamma(G_t^*) \neq G^* = const$ as in (2). Hence they will not have a way to separate apart estimates of $\hat{\rho}_t$ and $\hat{\mu}_\varepsilon$ each period. In that our approach differs from the literature on least-squares learning (as expounded in Evans and Honkapohja, 2001), which relates to a more restrictive scenario assuming a (known) normal zero-mean *Niid* shock process, $\varepsilon_t \rightsquigarrow Niid(0, \sigma_\varepsilon^2)$, and a constant intercept, G^* .

In the broader context we study in this paper, in which $\mu_\varepsilon \neq 0$ and $\Gamma(G_t^*) \neq G^* = const$, agents will have to predict inflation over their mature lifetime t from its sample mean observed up through $t - 1$,

⁶More generally, $\Gamma(G_t^*)$ could be a monotonically increasing (but bounded) function of the legislated cost of such a public good, G_t^* , as will be discussed below.

$$E[\pi_t | \Omega_t] = \pi_t^e = \hat{\pi}_t = \hat{\mu}_{\pi, t-1} \equiv t^{-1} \sum_{s=0}^{t-1} \pi_s \quad (3)$$

conditional on the information set at the beginning of each t , $\Omega_t \equiv \{G_s^*, \pi_{s-1}\}_{s=0}^t$, that is common knowledge through historical record. It is well-known in the adaptive learning literature (see, e.g., Evans and Honkapohja, 2001, p. 62) that the sample mean in (3) can be equivalently written as a forecast rule according to which agents form expectations adaptively from past data with a *decreasing* gain deterministic sequence, t^{-1} , so that

$$\pi_t^e = \hat{\pi}_t = \pi_{t-1}^e + t^{-1} (\pi_{t-1} - \pi_{t-1}^e). \quad (4)$$

Such an adaptive learning process implies that all agents share the same forecast rule for (conditionally) expected inflation, $\hat{\pi}_t$.

2.3 Monetary Institutions

With view to our purpose here, the degree of control $\Gamma(G_t^*)$ of the monetary authority over current-period inflation π_t highlighted in (2) can simply be referred to as the central bank's degree of enacted inflation aversion. This is in line with the conservative and independent monetary authority proposed by Rogoff (1985).⁷ Each type i would optimally wish to enshrine its own preferred degree of central bank inflation aversion, $\Gamma(G_t^{i*})$, through legislation when voting at the beginning of t . This interpretation introduces a second channel in our model that contributes to mitigate the consequences of high inflation, a publicly chosen one. Majority voting in parliament through proportional representation at the beginning of mature life of each generation decides on a unique degree of central bank inflation aversion to be enacted throughout their adulthood.⁸ Such a modeling is consistent with the literature that, at least since Strotz (1955), has shown the importance of precommitment technologies in safeguarding the value of money, and boils down to considering central bank inflation aversion as a public good. On the contrary, the role of central banks in hyperinflations (see Fischer et al., 2002) provides a *reductio ad absurdum* argument that, once they deviate from this objective, central banks harm the economy, and provide a public 'bad'. Our low-frequency inflation dynamics in (2) thus captures the related evidence that the degree of central bank inflation aversion (in addition to its conservatism) tends to be negatively correlated with inflation (Crowe and Meade, 2007).

If there is a shift in the voting majority by type at the beginning of t , then $\Gamma(G_t^*)$ substantially differs from $\Gamma(G_{t-1}^*)$. One way to think about this – and model it in the simulations later as we do – is to assume that $\Gamma(G_t^{i*})$ reduces expected lifetime inflation $\hat{\pi}_t = \hat{\mu}_{\pi, t-1}$ by some amount (e.g., by its observed conditional mean or by a

⁷In effect, one could in a broader sense interpret $\Gamma(G_t^*)$ alternatively as the degree of central bank independence (CBI).

⁸Modeling the political system is out of the scope of this article, and we refer the reader to Faust (1996), Bullard and Waller (2004) or Berentsen and Strub (2009).

given percentage of it) which is not precisely inferable by agents. If the majority does not switch, $\Gamma(G_t^*)$ is just a minimal ‘update’ of $\Gamma(G_{t-1}^*)$ by the same type one generation ahead, after observing an additional realization of lifetime inflation. Thus, for the same value of the sample mean, $\hat{\mu}_{\pi,t-1}$, and hence of the inflation forecast $\hat{\pi}_t$, actual inflation π_t will be lower or higher depending on the legislated value of $\Gamma(G_t^*)$. Once $\Gamma(G_t^*)$ is voted, throughout t both types pay the social cost of the enacted degree of central bank inflation aversion, experience inflation, which redistributes their real endowments according to (1) and (2), socialize their children, consume up their remaining real endowments, and die at the end of t . The described sequence of events is illustrated in Figure 1.

[Figure 1 about here]

2.4 Voting Equilibria under Alternative Inflation Histories

Following Bisin and Verdier (2000), we assume that each adult chooses the total amount of the public good, G_t , knowing that everyone in the society, irrespective of preference type, will have to contribute an equal share, $g_t = \frac{G_t}{1}$, towards the cost of providing the public good.⁹ In our set-up, the benefits and costs of inflation or deflation, and hence of the legislated degree of inflation aversion for the central bank (or its degree of independence in a real-world context¹⁰), are captured captured by equations (1) and (2), and affect the final (i.e., real) endowment of each type, ϖ_t^i .

2.4.1 Inflation-Deflation Symmetry and Costless Private Protection

The benchmark version of our model assumes that inflation and deflation have the same probability of occurrence. This is the case when the unconditional mean of the inflation-shock process is zero, $\mu_\varepsilon = 0$,¹¹ although agents do not know that. Hence, they would be learning the unconditional mean of inflation by observing the available historical record, generation after generation. In the benchmark investing the fraction φ of type b ’s nominal endowment in inflation-protection instruments available in the market implies no costs, so that $\Phi(\varphi) \equiv \Phi = 0$.

An adult agent i ’s preferences are represented in an additively separable form,

$$u^i(c_t, G_t) = u^i(c_t) + \gamma_t^i v^i(G_t), \quad \text{with } i \in \{a, b\} \text{ and } \gamma_t^i > 0,$$

where $u(c_t)$ and $v(G_t)$ are strictly concave, increasing functions satisfying $u'(0) = v'(0) = \infty$, and γ_t^i measures the relative weight of the utility from the public good.

⁹The literature on the private provision of public goods allows a less restrictive setting where each agent chooses his contribution, in units of the consumption good, and the resulting amount of the public good equals the sum of all contributions. We leave this avenue for future research.

¹⁰Many studies have strongly emphasized such benefits (Berger et al., 2001; Crowe and Meade, 2007), so we avoid their discussion here, to focus on our point.

¹¹Starting from initial inflation and degree of inflation aversion of the central bank both equal to zero, $\pi_{-1} = 0$ and $\Gamma(G_0^*) = 0$: check the ALM (2).

If the fraction q_t^i , with $0 \leq q_t^i \leq 1$, of type $i \in \{a, b\}$ individuals at time t is more than a half, then $q_t^i > q_t^j$, and the voting equilibrium degree of central bank inflation aversion solves the maximization program of the type i (identical) agents,¹²

$$\max_{G_t^i} u^i(c_t^i, G_t^i) \quad \text{s.t.} \quad c_t^i + \frac{G_t^i}{1} \leq \varpi_t^i,$$

so that the corresponding unconstrained optimization problems by type can be written, respectively, as

$$\begin{aligned} \max_{G_t^a} u^a \left(\frac{\varpi}{1 + \hat{\pi}_t} - G_t^a \right) + \gamma_t^a v^a(G_t^a), \\ \max_{G_t^b} u^b \left[\varphi \varpi + \frac{(1 - \varphi) \varpi}{1 + \hat{\pi}_t} - G_t^b \right] + \gamma_t^b v^b(G_t^b), \end{aligned}$$

with FONCs:

$$\begin{aligned} u^{a'} \left(\frac{\varpi}{1 + \hat{\pi}_t} - G_t^{a*} \right) &= \gamma_t^a v^{a'}(G_t^{a*}), \\ u^{b'} \left[\varphi \varpi + \frac{(1 - \varphi) \varpi}{1 + \hat{\pi}_t} - G_t^{b*} \right] &= \gamma_t^b v^{b'}(G_t^{b*}). \end{aligned} \quad (5)$$

Equations (5) implicitly define the respective *optimal* social cost, $G_t^{a*}(\hat{\pi}_t, \gamma_t^a)$ and $G_t^{b*}(\hat{\pi}_t, \gamma_t^b; \varphi)$, which corresponds to the *preferred* degree of central bank inflation aversion by type i agents in any period t . It is a function of the common inflation forecast $\hat{\pi}_t$ as well as of the type-specific public-good weight γ_t^i and market protection against inflation φ . Plugging that optimal degree of central bank inflation aversion back into the utility yields the value function. For type a agents, it is:

$$\begin{aligned} V^a \left(\frac{\varpi}{1 + \hat{\pi}_t} - G_t^{a*} \right) &\equiv \arg \max_{G_t^a} u^a \left(\frac{\varpi}{1 + \hat{\pi}_t} - G_t^a \right) + \gamma_t^a v^a(G_t^a) \\ &= u^a \left[\frac{\varpi}{1 + \hat{\pi}_t} - G_t^{a*}(\hat{\pi}_t, \gamma_t^a) \right] + \gamma_t^a v^a[G_t^{a*}(\hat{\pi}_t, \gamma_t^a)]. \end{aligned}$$

Because of the optimality of $G_t^{a*}(\cdot)$ and the positivity of $\gamma_t^a > 0$,

$$V^a \left(\frac{\varpi}{1 + \hat{\pi}_t} - G_t^{a*} \right) > u^a \left(\frac{\varpi}{1 + \hat{\pi}_t} \right),$$

so that it is always in the interest of a type a mature agent to enjoy the public good, here her preferred degree of central bank inflation aversion, $G_t^{a*}(\cdot)$.

Analogously, the value function of type b agents becomes:

¹²Note that the budget constraint that follows explicitly states a trade-off, usual in this literature, according to which providing G means foregoing c , for each type i .

$$\begin{aligned}
& V^b \left[\varphi \varpi + \frac{(1-\varphi)\varpi}{1+\hat{\pi}_t} - G_t^{b*} \right] \\
\equiv & \arg \max_{G_t^b} u^b \left[\varphi \varpi + \frac{(1-\varphi)\varpi}{1+\hat{\pi}_t} - G_t^b \right] + \gamma_t^b v^b(G_t^b) \\
= & u^b \left\{ \varphi \varpi + \frac{(1-\varphi)\varpi}{1+\hat{\pi}_t} - G_t^{b*}(\hat{\pi}_t, \gamma_t^b; \varphi) \right\} + \gamma_t^b v^b \left[G_t^{b*}(\hat{\pi}_t, \gamma_t^b; \varphi) \right].
\end{aligned}$$

Again, because of the optimality of $G_t^{b*}(\cdot)$ and the positivity of $\gamma_t^b > 0$,

$$V^b \left[\varphi \varpi + \frac{(1-\varphi)\varpi}{1+\hat{\pi}_t} - G_t^{b*} \right] > u^b \left[\varphi \varpi + \frac{(1-\varphi)\varpi}{1+\hat{\pi}_t} \right],$$

so that it is always in the interest of a type b mature agent as well to enjoy the public good, here her preferred degree of central bank inflation aversion, $G_t^{b*}(\cdot)$.

However, the *legislated* degree of central bank inflation aversion each period is unique, as it is determined by the dominant agent type via representation in parliament.

Lemma 1 *Assume that: (i) market-provided inflation protection is costless and $\varphi^b = \varphi > \varphi^a = 0$; (ii) inflation and deflation have the same probability of occurrence, $\mu_\varepsilon = 0$. Then, a -types arise endogenously as requiring a more inflation-averse central bank than b -types.*

Proof. See the Appendix. ■

Lemma 1 simply states that the optimal degree of publicly legislated inflation aversion of the central bank is shaped by the agent's expected real endowment over mature lifetime (which is affected by the conditionally expected inflation or deflation, given a particular realization of inflation history, and the degrees of market-based and central bank protection against inflation). If agents forecast inflation/deflation unanimously and optimally, as in this benchmark, it immediately arises that a -types will have a stronger preference than b -types for a monetary authority that enjoys a higher degree of inflation aversion through its legislated mandate.

Proposition 1 *Under the assumptions of Lemma 1, the unique socially-optimal (actual) net inflation rate, in the sense of being non-redistributive in terms of (final) real endowments across types, is zero: $\pi_t^* = 0, \forall t$.*

Proof. Follows directly from the proof of Lemma 1, replacing $\hat{\pi}_t$ by $\pi_t^* = 0$. ■

Note that any positive rate of inflation, $\pi_t > 0$, *harms* both agent types, but, by assumption, a -types *more* relative to b -types. Inversely, any negative rate of inflation, i.e., deflation, $\pi_t < 0$, *benefits* again both agent types, and again a -types *more* relative to b -types. Both agent types will be better-off under any deflation rate relative to any inflation rate, which is reminiscent of the Friedman (1969) rule. Yet only $\pi_t^* = 0, \forall t$ makes the real endowments of both types equal within each period and also equal to their

identical nominal endowments in any t , across generations, and thus eliminates the cause for the types being distinct intratemporally as well as intertemporally.¹³

2.4.2 Positive Inflation Expectations and Costly Private Protection

We now consider the case where the observed history implies *positive* inflation rates most of the time and both types pay costs $\Phi(\varphi^i)$ when investing a fraction φ^i of their nominal endowment in inflation-protection instruments available in the market. Type b differs again from type a in that she ‘chooses’ – with this ‘choice’ being predetermined by the outcome of socialization – to pay a *higher private* cost $\Phi^b > \Phi^a$ to get her endowment indexed (or diversified) against inflation to a *higher* extent, $1 > \varphi^b > \varphi^a > 0$. All agents thus value publicly legislated control over inflation by the monetary authority, but each type will prefer a different degree of central bank inflation aversion.

As before, if $q_t^i > q_t^j$, the voting equilibrium degree of central bank inflation aversion solves the maximization program of the type i , yet now with a market-based cost term, Φ^i , entering the budget constraint

$$\max_{G_t^i} u^i(c_t^i, G_t^i) \quad \text{s.t.} \quad c_t^i + \Phi^i + \frac{G_t^i}{1} \leq \varpi_t^i,$$

so that the corresponding unconstrained optimization problems by type can be written, respectively, as

$$\begin{aligned} \max_{G_t^a} u^a \left[\varphi^a \varpi + \frac{(1 - \varphi^a) \varpi}{1 + \hat{\pi}_t} - \Phi^a - G_t^a \right] + \gamma_t^a v^a(G_t^a), \\ \max_{G_t^b} u^b \left[\varphi^b \varpi + \frac{(1 - \varphi^b) \varpi}{1 + \hat{\pi}_t} - \Phi^b - G_t^b \right] + \gamma_t^b v^b(G_t^b), \end{aligned}$$

with FONCs:

$$\begin{aligned} u^{a'} \left[\varphi^a \varpi + \frac{(1 - \varphi^a) \varpi}{1 + \hat{\pi}_t} - \Phi^a - G_t^{a*} \right] &= \gamma_t^a v^{a'}(G_t^{a*}), \\ u^{b'} \left[\varphi^b \varpi + \frac{(1 - \varphi^b) \varpi}{1 + \hat{\pi}_t} - \Phi^b - G_t^{b*} \right] &= \gamma_t^b v^{b'}(G_t^{b*}). \end{aligned} \quad (6)$$

Again, equations (6) implicitly define the respective *optimal* social cost, $G_t^{a*}(\hat{\pi}_t, \gamma_t^a; \Phi^a)$ and $G_t^{b*}(\hat{\pi}_t, \gamma_t^b; \Phi^b)$, which defines the *preferred* degree of central bank inflation aversion by each type in any period t . Now, it also depends on the assumed fixed costs of market-based protection from inflation Φ^i , itself a function of the protected share φ^i , in our shorthand notation $\Phi^i \equiv \Phi(\varphi^i)$. The value function for types a now becomes:

¹³Of course, such a zero-inflation policy would be implementable only if the monetary authority enjoys a complete control over the inflation process.

$$\begin{aligned}
& V^a \left(\varphi^a \varpi + \frac{(1 - \varphi^a) \varpi}{1 + \hat{\pi}_t} - \Phi^a - G_t^{a*}(\cdot) \right) \\
& \equiv \arg \max_{G_t^a} u^a \left(\varphi^a \varpi + \frac{(1 - \varphi^a) \varpi}{1 + \hat{\pi}_t} - \Phi^a - G_t^a(\cdot) \right) + \gamma_t^a v^a(G_t^a(\cdot)) \\
& = u^a \left[\varphi^a \varpi + \frac{(1 - \varphi^a) \varpi}{1 + \hat{\pi}_t} - \Phi^a - G_t^{a*} \right] + \gamma_t^a v^a(G_t^{a*}(\cdot)).
\end{aligned}$$

As before, due to optimality of $G_t^{a*}(\cdot)$ and the positivity of $\gamma_t^a > 0$,

$$V^a \left(\varphi^a \varpi + \frac{(1 - \varphi^a) \varpi}{1 + \hat{\pi}_t} - \Phi^a - G_t^{a*}(\cdot) \right) > u^a \left(\varphi^a \varpi + \frac{(1 - \varphi^a) \varpi}{1 + \hat{\pi}_t} - \Phi^a \right),$$

so that it is always in the interest of a type a mature agent to enjoy the public good, $G_t^{a*}(\cdot)$.

Analogously, the value function of type b agents becomes:

$$\begin{aligned}
& V^b \left[\varphi^b \varpi + \frac{(1 - \varphi^b) \varpi}{1 + \hat{\pi}_t} - \Phi^b - G_t^{b*}(\cdot) \right] \\
& \equiv \arg \max_{G_t^b} u^b \left[\varphi^b \varpi + \frac{(1 - \varphi^b) \varpi}{1 + \hat{\pi}_t} - \Phi^b - G_t^b(\cdot) \right] + \gamma_t^b v^b(G_t^b(\cdot)) \\
& = u^b \left\{ \varphi^b \varpi + \frac{(1 - \varphi^b) \varpi}{1 + \hat{\pi}_t} - \Phi^b - G_t^{b*}(\cdot) \right\} + \gamma_t^b v^b(G_t^{b*}(\cdot)).
\end{aligned}$$

Again, due to the optimality of $G_t^{b*}(\cdot)$ and the positivity of $\gamma_t^b > 0$,

$$V^b \left[\varphi^b \varpi + \frac{(1 - \varphi^b) \varpi}{1 + \hat{\pi}_t} - \Phi^b - G_t^{b*}(\cdot) \right] > u^b \left[\varphi^b \varpi + \frac{(1 - \varphi^b) \varpi}{1 + \hat{\pi}_t} - \Phi^b \right],$$

so that it is always in the interest of a type b mature agent as well to enjoy her preferred degree of central bank inflation aversion, $G_t^{b*}(\cdot)$.

As before, the *legislated* degree of central bank inflation aversion each period is determined by the dominant agent type via representation in parliament.

Lemma 2 *Assume that: (i) market-provided inflation protection is costly, with fixed costs of differential protection predetermined by type and implied by $\Phi^b \equiv \Phi(\varphi^b) > \Phi(\varphi^a) \equiv \Phi^a > 0$ for $1 > \varphi^b > \varphi^a > 0$ and driven by the outcome of socialization; (ii) positive inflation expectations, $\hat{\pi}_t > 0$, prevail for most $t = 0, 1, 2, \dots$ (since $\mu_\varepsilon > 0$). Then, a -types arise endogenously again as requiring a more inflation-averse central bank than b -types.*

Proof. See the Appendix. ■

Now, although a -types still prefer endogenously a *more* inflation-averse central bank than b -types, a particular positive rate of inflation is socially optimal.

Proposition 2 *Under the assumptions of Lemma 2, the unique socially optimal (actual) net inflation rate is positive: $0 < \pi_t^* = \pi^* = \frac{(\varphi^a - \varphi^b)\varpi}{(\varphi^a - \varphi^b)\varpi + (\Phi^b - \Phi^a)} - 1 = \text{const} < 1, \forall t$.*

Proof. Follows from the proof of Lemma 2, replacing $\hat{\pi}_t$ by $\pi_t^* = \frac{(\varphi^a - \varphi^b)\varpi}{(\varphi^a - \varphi^b)\varpi + (\Phi^b - \Phi^a)} - 1$. ■

Whenever actual inflation turns out to be higher than π^* , $\pi_{t,H} > \pi^*$, both types lose but a -types *lose more* relative to b -types. Inversely, whenever actual inflation turns out to be lower than π^* , $\pi_{t,L} < \pi^*$, both types gain but a -types *gain more* relative to b -types. As a consequence, a -types endogenously emerge as requiring a more inflation averse central bank than b -types. Both agent types will be better-off under any deflation rate relative to any inflation rate (which is again reminiscent of the Friedman, 1969, rule), only $\pi_t = \pi^*$ makes the real endowments of both types equal within each period and also equal to their identical nominal endowments in any t , across generations.

3 Intergenerational Dynamics of Beliefs and Institutions

As agents live for a single period of adulthood, voters choosing the political economy equilibrium change every period t . That is, the same within-period socioeconomic structure and sequence of decisions is played by each generation of agents, with the fractions of types q_t^a and q_t^b attained by socialization in the previous period as the initial condition. This allows, differently from strategic interactions in a classical or evolutionary game theory set-ups, to derive the law of motion of q_t^a over time. We essentially do that by focusing on the mechanism of belief transmission across generations, building upon the deterministic OLG set-up of Bisin and Verdier (2000, 2001, 2010). We, however, extend it to adaptive learning under stochastic dynamics, replacing their cultural substitution assumption.

3.1 Belief Transmission through Imperfect Empathy

Children are born without well-defined beliefs, but acquire them through observation, imitation and adoption of ‘cultural models’ with which they are matched. This matching, termed ‘socialization’, naturally comes in two steps and is influenced to some extent by economic choices, but also by parents. Children are first exposed to their parents model (type a or b), and are thus ‘matched’ with their family, in what is termed ‘direct vertical transmission’. If they do not adopt their parent’s type, they are then exposed to the influence of other individuals of the old generation (e.g., teachers, peers, role models) and adopt the belief type of some among these, i.e., ‘oblique vertical transmission’.¹⁴

¹⁴This terminology originates in the anthropological and psychological literature and was introduced by Cavalli-Sforza and Feldman (1981). ‘Horizontal transmission’, which we do not model, occurs within a generation.

Imperfect empathy, a particular form of myopia we assume throughout the paper, further implies that parents always want to socialize their children to their own beliefs.¹⁵

To examine the mechanism driving the intergenerational transmission of beliefs through the socialization channel we assume that a child adopts his parent's type with an endogenous probability $\tau^i(\cdot)$, to be made more precise later, with $0 \leq \tau^i(\cdot) \leq 1$, $i \in \{a, b\}$. If not, with probability $1 - \tau^i(\cdot)$, the child is then matched randomly with another individual of the old generation and adopts her type of beliefs.

The transition probabilities at time t , P_t^{ij} , that a parent of type i has a child adopting a preference of type j are then:

$$\begin{aligned} P_t^{aa} &= \tau^a(\cdot) + (1 - \tau^a(\cdot)) q_t^a, \\ P_t^{ab} &= (1 - \tau^a(\cdot)) (1 - q_t^a), \\ P_t^{bb} &= \tau^b(\cdot) + (1 - \tau^b(\cdot)) q_t^b = \tau^b(\cdot) + (1 - \tau^b(\cdot)) (1 - q_t^a), \\ P_t^{ba} &= (1 - \tau^b(\cdot)) (1 - q_t^b) = (1 - \tau^b(\cdot)) q_t^a. \end{aligned}$$

Given these transition probabilities and in a purely *deterministic* set-up as in the cultural transmission literature, the fraction q_t^a of adult individuals of type a in period $t + 1$ evolves according to:

$$\begin{aligned} q_{t+1}^a &= q_t^a P_t^{aa} + q_t^b P_t^{ba} \\ &= q_t^a P_t^{aa} + (1 - q_t^a) P_t^{ba} \\ &= q_t^a + q_t^a (1 - q_t^a) (\tau^a(\cdot) - \tau^b(\cdot)) \\ &= \left[1 + (1 - q_t^a) (\tau^a(\cdot) - \tau^b(\cdot)) \right] q_t^a. \end{aligned}$$

It is clear that the fraction of type- a agents in the old generation may stay constant across time only if the term in square brackets is equal to 1. This would occur if either (i) $q_t^a = 1$ or (ii) $\tau^a(\cdot) = \tau^b(\cdot)$ or (iii) both. However, case (i) – and, hence, case (iii) – is excluded by assumption for the initial condition ($0 < q_t^a < 1$), i.e., of type heterogeneity. Therefore, only case (ii) remains as a potentially relevant, symmetric option to consider; yet, it defines a steady state for any initial condition, without any evolution of the relative proportions of beliefs in the society, and so is uninteresting.

In all other deterministic cases, different from (i), (ii) and (iii), the intergenerational dynamics of beliefs depends on two factors: first, the *proportion* of type- a agents inherited from past history, q_t^a , relative to q_t^b ; second, the *sign* of the difference of the vertical transmission probabilities, $\tau^a(\cdot) - \tau^b(\cdot)$, which determines the *direction* of belief convergence:

¹⁵ Imperfect empathy is a common assumption in the emerging socialization literature within economics. It implies that parents can perceive the welfare of their children only through the filter of their own preferences.

toward type a , if positive, or b , if negative. Writing the equation above as

$$q_{t+1}^a = q_t^a + \left[q_t^a - (q_t^a)^2 \right] \left(\tau^a(\cdot) - \tau^b(\cdot) \right) \quad (7)$$

delivers a first-order non-linear sequence, which does not admit any general solution even for constant τ^a and τ^b . However, as the appendix shows, the deterministic benchmark can easily be represented in a standard phase diagram for any choice of parameters τ^a and τ^b . It always leads to dynamics whereby one of the types is extinguished and the other perpetuates forever, so coexistence never obtains.

3.2 Stochastic Endogenous Socialization under Learning

We now endogenize the direct vertical transmission of beliefs in a *stochastic* socialization set-up, linked with the law of motion for low-frequency inflation, (2). To do so, we simulate the theoretical framework presented thus far, embedding in it a mechanism that generates endogenous socialization effort by each agent type in response to experienced loss (or gain) from inflation relative to the other type. In general, any simulation of our model will depend on how close π^* is to the assumed mean of the inflation shock process, μ_ε , and – as clear from Proposition 2 – will, therefore, ultimately reflect the chosen values of the share parameters, φ^i , and the functional form of costs, $\Phi(\varphi^i)$, defining π^* . To focus on the learning process from inflation experience and the socialization effort in cultural transmission, we will assume henceforth that the costs of market-based protection from inflation are zero, $\Phi(\varphi^i) = 0$ (or negligible), which reduces the expression in Proposition 2 to $\pi^* = 0$.

Since agents do not know the true mean of the generational inflation process, in each period t they update their common knowledge of its conditional mean through historical observation. That is, actual inflation each period is $\pi_t = \hat{\pi}_t + \eta_t$, where η_t is the forecast error and the common inflation forecast across types is $\hat{\pi}_t \equiv \hat{\mu}_{\pi,t-1}$. The next-period inflation forecast then becomes $\hat{\pi}_{t+1} \equiv \hat{\mu}_{\pi,t}$, and so forth.

Assumption 1 *Given (i) costless market-based protection from inflation ($\Phi(\varphi^i) = 0$) and (ii) the unanimous conditional inflation forecast in any t ($\hat{\pi}_t$), if $\pi_t > \hat{\pi}_t$ leading to $\hat{\pi}_{t+1} - \hat{\pi}_t > 0$, a -types lose more from the positive ‘surprise generational inflation’ and therefore socialize stronger relative to b -types; and vice versa, if $\pi_t < \hat{\pi}_t$ leading to $\hat{\pi}_{t+1} - \hat{\pi}_t < 0$, b -types lose more from the negative ‘surprise generational inflation’ and therefore socialize stronger relative to a -types.*

This assumption highlights the behavioral mechanism through which socialization and the longer-run evolution of inflation interact.

3.3 Calibration and Simulation Design

The design and implementation of the simulations incorporates several additional features.¹⁶

First, we assume that the probability of vertical socialization to the parent's type i , τ_t^i , is a positive function of the effort at time t the parent exerts to socialize her child to her own belief, e_t^i : $\tau_t^i(e_t^i, \cdot)$, with $\frac{\partial}{\partial e_t^i} \tau_t^i(e_t^i, \cdot) > 0$.

Second, according to Lemma 2 and Assumption 1, socialization efforts, e_t^i , are in turn a positive function of the change in the conditional mean of inflation ($\hat{\pi}_t - \hat{\pi}_{t-1}$) a particular generation t observes over its adulthood, i.e., therefore a function of cumulative historical record up through t . Thus, implementing Assumption 1, we write: $e_t^i(\hat{\pi}_t - \hat{\pi}_{t-1}, \cdot)$, with $\frac{\partial e_t^a(\hat{\pi}_t - \hat{\pi}_{t-1}, \cdot)}{\partial(\hat{\pi}_t - \hat{\pi}_{t-1})} > 0$ if $\hat{\pi}_t - \hat{\pi}_{t-1} > 0$, while $\frac{\partial e_t^b(\hat{\pi}_t - \hat{\pi}_{t-1}, \cdot)}{\partial(\hat{\pi}_t - \hat{\pi}_{t-1})} > 0$ if $\hat{\pi}_t - \hat{\pi}_{t-1} < 0$. That is, types a react to an observed *rise* in the conditional mean of lifetime inflation by increasing their socialization efforts, while types b react to an observed *fall* in the conditional mean of lifetime inflation by increasing their socialization efforts. Consequently, in addition to the institutionalization channel, which operates via $\Gamma(G_t^*)$, we now explicitly distinguish a second channel of transmitting inflation experience by the adult generation to the young through socialization, via $e_t^i(\hat{\pi}_t, -\hat{\pi}_{t-1}, \cdot)$.

Third, in implementing the simulations we assume $\Gamma(G_t^*) \equiv \chi^i \hat{\mu}_{\pi, t-1}$, with $0 \leq \chi^i \leq 1$ and $\hat{\mu}_{\pi, t-1}$ denoting the conditional mean of inflation observed through generation $t-1$. We further impose $\chi^a = 1$ and $\chi^b = 0.5$ so that $\Gamma(G_t^{*a}) = \hat{\pi}_t$ and $\Gamma(G_t^{*b}) = 0.5\hat{\pi}_t = 0.5\Gamma(G_t^{*a})$ in (2), with the initial condition for inflation being $\pi_0 = \mu_\varepsilon$. The latter initialization implies that those of the alternative simulation cases where the mean of the inflation shock is assumed zero, $\mu_\varepsilon = 0$, eliminate (by construction, as above, and in the codes) the drift term in (2) and, thus, the feedback from the institutional channel embodying central bank inflation aversion. Our assumption here linking $\Gamma(G_t^{*a})$ and $\Gamma(G_t^{*b})$ to the observed conditional mean $\hat{\pi}_t$ bears some realism and intuitive power, in the sense that each agent type will have some idea of the order of magnitude to which the monetary authority will be able to affect actual inflation π_t over t via the legislated $\Gamma(G_t^*)$ and given (2). Moreover, as generations learn (asymptotically) from historical experience the unconditional mean of inflation, the control over actual inflation (asymptotically) increases too.

Using these assumptions, we substitute back into equation (7), to obtain:

$$q_{t+1}^a = q_t^a + \left[q_t^a - (q_t^a)^2 \right] \begin{pmatrix} \tau_t^a [e_t^a(\hat{\pi}_t - \hat{\pi}_{t-1}, \cdot), \cdot] \\ -\tau_t^b [e_t^b(\hat{\pi}_t - \hat{\pi}_{t-1}, \cdot), \cdot] \end{pmatrix}. \quad (8)$$

¹⁶Note that we consider here immediately the endogenous stochastic dynamics case. We also explored the exogenous stochastic dynamics case, by specifying stochastic processes directly for the probabilities of vertical transmission (τ_t^i). In the simplest context, assuming them random variables, e.g., drawing from Uniform $(0, 1)$ independent distributions each period requires to rewrite (9) as $q_{t+1}^a = q_t^a + [q_t^a - (q_t^a)^2] (\tau_t^a - \tau_t^b)$. Our simulations of this equation with stochastic τ_t^i 's from different initial fractions q_0^a generate ultimate convergence to either of the types, as in the deterministic exogenous case, within 20 to 100 periods depending on the particularly materialized sequences of $\tau_t^a - \tau_t^b$.

Equations (2) and (8) thus form an interdependent recursive dynamic system in two state variables, π_t and q_{t+1}^a . They also highlight the two channels through which societies transmit beliefs and institutions from a generation to the next, socialization and institutionalization, respectively. Starting from some initial conditions π_0 and q_0^a , implying also a corresponding initial value for G_0^* , the shock realization ε_1 gives π_1 from (2); from (8), then, $\hat{\pi}_1 - \hat{\pi}_0$, will first impact the socialization effort across types, next the belief transmission probabilities, and ultimately q_1^a ; and so on and so forth in subsequent periods. This chain of effects constitutes the mechanism that generates the intergenerational evolution of inflation beliefs and monetary institutions. While the conventional deterministic socialization literature relies on the assumption of cultural substitution to ensure coexistence of types in the population, our stochastic model highlights an alternative mechanism: namely, endogenous socialization efforts in response to relative losses from observed lifetime inflation can lead to shift cycles in inflation beliefs that reverse each other, as illustrated further below by a summary of our key simulation results.¹⁷

To explore this mechanism, we simulated our dynamic stochastic OLG model embodied in the recursive system (2) and (8) over 1000 periods, so that we could also judge conclusively about very long-term coexistence of types versus convergence to one of them. Our simulations involved a ‘grid’ of alternative parameters and shock processes assuming three cases concerning inflation dynamics, each in a low- and high-variance version:¹⁸

1. $\pi_0 = 0\% = \mu_\varepsilon$ (measured, for comparability with the usual inflation literature, as an annual average over mature lifetime) and $\sigma_\varepsilon^2 = 1\%$ or $\sigma_\varepsilon^2 = 2\%$, i.e., a zero-mean inflation regime, or one consistent with zero-inflation steady states in theoretical models;
2. $\pi_0 = 2\% = \mu_\varepsilon$ and $\sigma_\varepsilon^2 = 1\%$ or $\sigma_\varepsilon^2 = 4\%$, i.e., a low-inflation regime, or one broadly typical for advanced economies over the most recent generation span; and
3. $\pi_0 = 6\% = \mu_\varepsilon$ and $\sigma_\varepsilon^2 = 3\%$ or $\sigma_\varepsilon^2 = 12\%$ (all these three parameters three times higher than in case 2), i.e., a high-inflation regime with higher inflation volatility, or one broadly typical for emerging markets over the most recent generation span.

Moreover, all three cases were simulated for three alternative values of the parameter measuring low-frequency inflation persistence,¹⁹ $\rho = \{0.1, 0.5, 0.9\}$, and for four endogenous vertical probability differentials, $|\tau_t^a e_t^a(\hat{\pi}_t - \hat{\pi}_{t-1}, \cdot), \cdot| - \tau_t^b [e_t^b(\hat{\pi}_t - \hat{\pi}_{t-1}, \cdot), \cdot]| =$

¹⁷Note that ‘nature’ also plays a role in the dynamics of the system, by ‘drawing’ the inflation shock every period. Observe as well that the control society has over nature is imperfect, operating along both transmission channels, socialization and institutionalization, only in addition to the draw of nature.

¹⁸Our codes using R and MATLAB and a full account of our results are available as a *.zip archive.

¹⁹Note that in our context persistence of the inflation process at (mature) generation spans (t of the order of 25-30 years) may not necessarily correspond to measured short-run (annual or quarterly t) inflation persistence in the abundant literature. Also, $\rho \rightarrow 0$ captures a normal stochastic process for inflation with drift, while with $\rho \rightarrow 1$ it approaches random walk with drift. Thus, while modeled and simulated as a stochastic AR(1) with drift, our low-frequency inflation dynamics is rather general though remaining simple.

$\{0.02, 0.1, 0.2, 0.5\}$. The latter translate the reaction of agent types to their relative losses or gains due to observed variation in the conditional mean of generational inflation into corresponding socialization effort and, ultimately, probability differential of passing over the parent's belief to the child across the two types.²⁰

3.4 Simulation Results

Our simulations lead to the following key conclusions.

First, whenever the vertical transmission probability differential is sufficiently high – of the order of 0.5 in absolute value or more, convergence to one of the types occurs very quickly (sometimes in less than 10 generations). This conclusion remains valid even when starting from an equal initial share in the population, $q_0^a = 0.5 \pm$.

Second, the main insight from the simulations highlights the possibility of irregular cycles in beliefs, manifested in a sequence of interior values for the fraction of types which does not converge to any of the two corner steady states, even after 1000 generations, as illustrated in the subset of figures we discuss further below. The conditions which lead to such dynamics are two: (i) the endogenous vertical probability differential should be low – of the order of 0.1 in absolute value or less; and (ii) the initial fraction should be close to the mid-point, $q_0^a \approx q_0^b$. The first condition appears to be the more influential one in ensuring coexistence of both types with recurrent cycles in their fractions, unless the initial fractions are too distant. The second condition is of interest as it potentially facilitates reversals at irregular intervals in the voted degree of central bank inflation aversion too, that is, when the institutionalization channel is also operative, in addition to the socialization channel.

For these reasons and because our focus is on the coexistence of both types and the shifting majorities, we illustrate the flavor of our simulation results selecting exactly the case of a close ‘balance of power’ between the types and their socialization effort: $q_0^a = 0.5 \pm$ and $|\tau_t^a e_t^a(\hat{\pi}_t - \hat{\pi}_{t-1}, \cdot) - \tau_t^b [e_t^b(\hat{\pi}_t - \hat{\pi}_{t-1}, \cdot), \cdot]| = 0.02$. Our essential findings under this ‘balance of power’ parameterization are documented in the figures we discuss next to gain deeper insights. To analyze their meaning, it is convenient to think in terms of the degree of persistence ρ of the inflation process (2) – distinguishing in the subsections below low ($\rho = 0.1$), medium ($\rho = 0.5$) and high ($\rho = 0.9$) persistence – as one force. It is also useful to consider first the benchmark of equal probability of (or symmetry between) inflation and deflation to occur, i.e., the first row in the panels of graphs that follow. In these cases, as we mentioned, the monetary control term in (2) capturing the legislated degree of central bank inflation aversion each period is, logically, not present, $\Gamma(G_t^*) \equiv 0$. Then, it is also enlightening to let this latter drift term in (2) operate, as a second force in the *opposite* direction of persistence, in the second and third rows in the panels of graphs

²⁰The simulations also assume a *symmetric* socialization effort by the two types, in the sense that, for example, when $\tau_t^a(\cdot) = 0.55$ and $\tau_t^b(\cdot) = 0.45$ after an observed increase in the conditional mean of inflation, then $\tau_t^b(\cdot) = 0.55$ and $\tau_t^a(\cdot) = 0.45$ after an observed decrease in the conditional mean of inflation: this results into $|\tau_t^a [e_t^a(\pi_t - \pi_{t-1}, \cdot), \cdot] - \tau_t^b [e_t^b(\pi_t - \pi_{t-1}, \cdot), \cdot]| = 0.1$.

that follow.

3.4.1 Low-Persistence Generational Inflation Process

We first look at a generational inflation process with low persistence, of 0.1, in Figures 2 (under lower shock variance) and 3 (under higher shock variance, but the same respective means).

[Figures 2 and 3 about here]

(a) When the inflation shock is distributed as $N(0, 1)$ and there is no monetary control drift (by construction), as in the first row of Figure 2, the agents tend to learn fast (relative to the later simulation graphs) the mean of actual inflation, which converges to the zero mean of the inflation shock after about 75 periods. This simulation case (a), as we noted, corresponds to the theoretical case of inflation-deflation symmetry. With a history of close to zero inflation, the fraction of a -types is very stable. It remains within the range of 0.4 – 0.6 over the 1000 simulated periods and, therefore, shifting majorities occur frequently (relative to the later simulation graphs). Increasing the variance of the inflation shock twice, but keeping its zero mean, as in the first row of Figure 3, slows down the learning process about five times and results in a decreasing trend in a -types. Since the inflation history reflected in the mid-graph of the first row of Figure 3 exhibits low and stable inflation converging to zero after around period 350, the fraction of the more inflation-averse type a declines from that point on, but is not close to extinction even by the end of the simulation sample.

(b) Next, the inflation shock is distributed as $N(2, 1)$ and there is now a central bank inflation aversion drift legislated each period (by construction). The effect of the latter drift term is visible in the mid-graph of the second row of Figure 2 by driving the mean of observed inflation, converging to about 1.35% (as an average in annual terms), *below* the mean of the inflation shock, 2%. Agents again quickly learn this inflation mean: about period 25, three times faster than in case (a). Essentially, the downward pressure on actual inflation exerted by the inflation aversion drift dominates the upward pressure of above-mean inflation shocks under low persistence of the generational inflation process, shifting the mean of observed inflation down relative to the mean of the inflation shock. If there was no monetary control drift in the law of motion of inflation, the observed inflation would have converged back to the mean of the inflation shock, as in case (a). Furthermore, because mean inflation stays persistently at about 1.35% p.a. without decreasing, and even with a minimal upward trend in the last third of the sample in this particular simulation draw, the more inflation-averse type a increases steadily – but not much – its proportion in the population after about period 650. Doubling the variance of the inflation shock, and now increasing its mean from 0% to 2% (p.a. on average), as in the second row of Figure 3, slows down the learning process again and results in more volatility of the fraction of a -types.

(c) Finally, the inflation shock is distributed as $N(6, 3)$, and there is again a monetary control drift. A similar outcome to case (b) is visible in the third row of Figure 2: the control drift, i.e., the inflation-averse central bank, drives the mean of observed inflation down, converging quickly, after 33 periods, to about 4.3% (as an average in annual terms), *below* the mean of the inflation shock, 6%. As was in case (b), the downward effect of the central bank inflation aversion drift dominates the upward effect of above-mean shock realizations under low persistence of the inflation shock. As in the preceding two cases, (a) and (b), the more inflation-averse type a fluctuates close to the majority tipping point of 0.5, but in case (c), differently, majority switches are rare. Increasing the variance of the inflation shock now four times, to 12% (and keeping the same mean of 6%), as in the third row of Figure 3, introduces noise and slows down the learning process again, and similarly results in a higher volatility of the fraction of a -types. Yet, the dynamics of the fraction of both types oscillates far from extinction.

3.4.2 Medium-Persistence Generational Inflation Process

We next consider our simulation results under a mid-point persistence of the inflation process, of 0.5, in Figures 4 (under lower shock variance) and 5 (under higher shock variance).

[Figures 4 and 5 about here]

(a) Similarly to our analysis under low-persistence of generational inflation in the preceding subsection, when the inflation shock is distributed as $N(0, 1)$ and there is no monetary control drift (that is, under inflation-deflation symmetry), the agents tend to learn the mean of observed inflation as it converges to the zero mean of the inflation shock, but now slower: see the mid-graph in the first row of Figure 4. Again, the inflation history depicted in the same graph is characterized by low and stable inflation converging to zero. Consequently, the fraction of the more inflation-averse type a declines overall, but with irregular cycles. Doubling the variance of the inflation shock (but keeping its zero mean), as in the first row of Figure 5, slows down tremendously the learning process, yet does not affect dramatically the evolution of the proportion of a -types.

(b) When the inflation shock is distributed as $N(2, 1)$ and the inflation process includes a monetary control drift, the mid-graph in the second row of Figure 4 resembles the same graph in Figure 2 in terms of fast learning; but not in driving the mean of observed inflation below the mean of the inflation shock, 2%. By contrast, the downward effect on inflation of the control drift is now *offset* by the upward effect of the above-mean inflation shock realizations under *mid-point* persistence here of the inflation shock. Type a fluctuates once again within the range of 0.4–0.6. Doubling the variance of the inflation shock, and now increasing its mean from 0% to 2%, as in the second row of Figure 5, slows down learning as before, but does not affect again in any essential way the evolution of a -types.

(c) When the inflation shock is distributed as $N(6, 3)$ and an inflation-aversion drift of the central bank is included, similarly to the preceding case (b) agents converge to learn the actual inflation mean at the level implied by the mean of the inflation shock, 6%: see the mid-graph in the third row of Figure 6. Now the higher fluctuations in inflation (plotted in the bottom left graph of the same figure) provoke waves of rising and declining trends in the proportion of types, generating irregular but persistent cycles and causing the most frequent switches of majority in our simulations, at times in successive generations and often several times over 5 – 10 periods. While this particular simulation draw features the most frequent majority switches, their incidence is still quite low, with implied probability of less than 0.1, given our calibration.

3.4.3 High-Persistence Generational Inflation Process

Finally, we move to our simulation results under a generational inflation process with high persistence, of 0.9, in Figures 6 (under lower shock variance) and 7 (under higher shock variance).

[Figures 6 and 7 about here]

(a) Once again, when the inflation shock is distributed as $N(0, 1)$ and there is no monetary control drift, agents learn the mean of observed inflation which converges to the zero mean of the inflation shock with no huge delay (relative to most other graphs): see the mid-graph in the first row of Figure 6. Again, doubling the variance of the inflation shock (but keeping its mean at zero), as illustrated in the first row of Figure 7, confuses and slows down learning. However, with high persistence here, the higher variance also increases the amplitude of fluctuations in the proportion of types a .

(b) When the inflation shock is distributed as $N(2, 1)$ and there is now a monetary control drift in the inflation dynamics, the effects we observed under *low* persistence are now *reversed*: under *high* persistence here of the inflation process, it is the upward pressure on inflation of above-mean inflation shock realizations that dominates the downward pressure of the central bank inflation aversion drift. This dominance consequently drives the mean of observed inflation, converging to about 3.4%, not below but now *above* the mean of the inflation shock, 2%. Agents again tend to learn this inflation mean over time, but with more fluctuations induced by the high persistence. As we noted, the latter force combined with the cycling trends in the observed inflation history in Figure 6 provoke dynamics of the fractions of types that manifest cyclical patterns with high amplitude. The amplitude of fluctuations is even higher with higher volatility of the inflation shock (and keeping its respective mean in the rows the same), as can be seen in Figure 7.

(c) Finally, when the inflation shock is distributed as $N(6, 3)$ with a monetary control drift present in the modeled dynamics of inflation, the highly persistent inflation process induces again a fluctuating inflation mean at around 10%, much *above* the mean of the inflation shock, 6%. With four times higher shock variance (and the same mean), as in

Figure 7, agents cannot easily learn the inflation mean over time. As was in the other high-persistence cases, the cyclical patterns in the observed inflation history induce dynamics of the fractions of types mirroring them, and with an exceptionally high amplitude. Because inflation is too high and volatile now (relative to the implied mean of the inflation shock as well as to all other cases we discussed so far), the types may come nearer to extinction under particular histories of shock realizations.

Viewed from an empirical perspective, our simulations bear relevance to the existing literature on monetary institutions. It has revealed different impacts of the degree of legislated inflation aversion (i.e. *de jure* central bank independence, see, e.g., Crowe and Meade, 2007) in developed and developing countries. These can be explained by the different dynamics of the respective inflation processes and the resulting socialization and institutionalization channels our theory and simulations have highlighted.

4 Concluding Comments

In this paper we addressed theoretically the question of what drives the interdependent long-run evolution of inflation beliefs and monetary institutions. To do so, we extended the OLG framework of Bisin and Verdier (2000, 2001, 2010) to explore endogenously derived and transmitted inflation beliefs, dropping the assumption of cultural substitution and replacing their deterministic model with a dynamic stochastic environment of adaptive learning. In the simplest cases, also examined in the earlier literature, where the vertical transmission probabilities are either (i) exogenously fixed or (ii) endogenous but deterministic, there is a clear ‘separation’ of results. In the first case, only one of the types survives while the other is extinguished, and convergence depends on the direction and the speed of changes in the structure of the population. In the second case, convergence to an interior equilibrium with both types surviving is achieved at the cost of assuming cultural substitution.

We showed that in a pure endowment economy with equal probability for inflation and deflation and available costless market-based inflation protection, the socially-optimal rate of inflation, in the sense of not redistributing real wealth across the agent types, is zero; but with positive inflation dominating historically and, hence in expectation, and costly market-based inflation protection, this non-redistributive rate of inflation is positive, and is a function of the latter costs. We also showed how preferences for the degree of inflation aversion of the central bank and the corresponding socialization efforts by type arise endogenously within each generation, even under a shared conditional inflation forecast and costless market-based inflation protection. Our simulations further confirmed that such endogenous belief transmission in a dynamic stochastic set-up where generations learn and adapt provides an alternative to the assumption of cultural substitution. Another theoretical contribution to the literature thus consists in showing how the endogenous transmission of inflation beliefs and monetary institutions in a long-run stochastic economic environment can be understood as a process of intergenerational learning from

history and subsequently amending existing institutions, more radically after switches in majority between the agent types. Moreover, the simulations reflected in our graphical analysis provide insights into the key forces at play and their relative dominance or balance under the studied parameterizations. Our results can also shed light on the empirics of central bank independence and on the evolution of the degree of inflation aversion embedded in monetary institutions in the real world.

Our model could be extended in several directions. On the theoretical side, allowing for population growth and technological progress, alternative modeling of the endogenous types and/or a higher heterogeneity could provide valuable insights, as well as the examination of more explicit financial market structures or different processes guiding generational inflation dynamics. More could be explored as well with regard to alternative simulation parameterizations, as we duly noted. Potential empirical implications are certainly interesting, but appear much harder to study. The reason is that low-frequency inflation data and proxies of the degree of monetary control or inflation aversion of central banks do not exist across generation spans, the more so on a comparable cross-country basis.

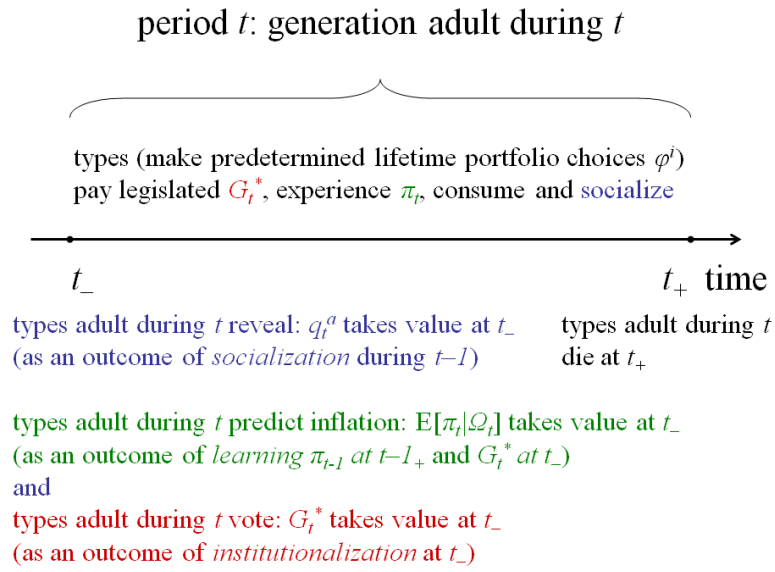


Figure 1: Sequencing of Events in Any Period t

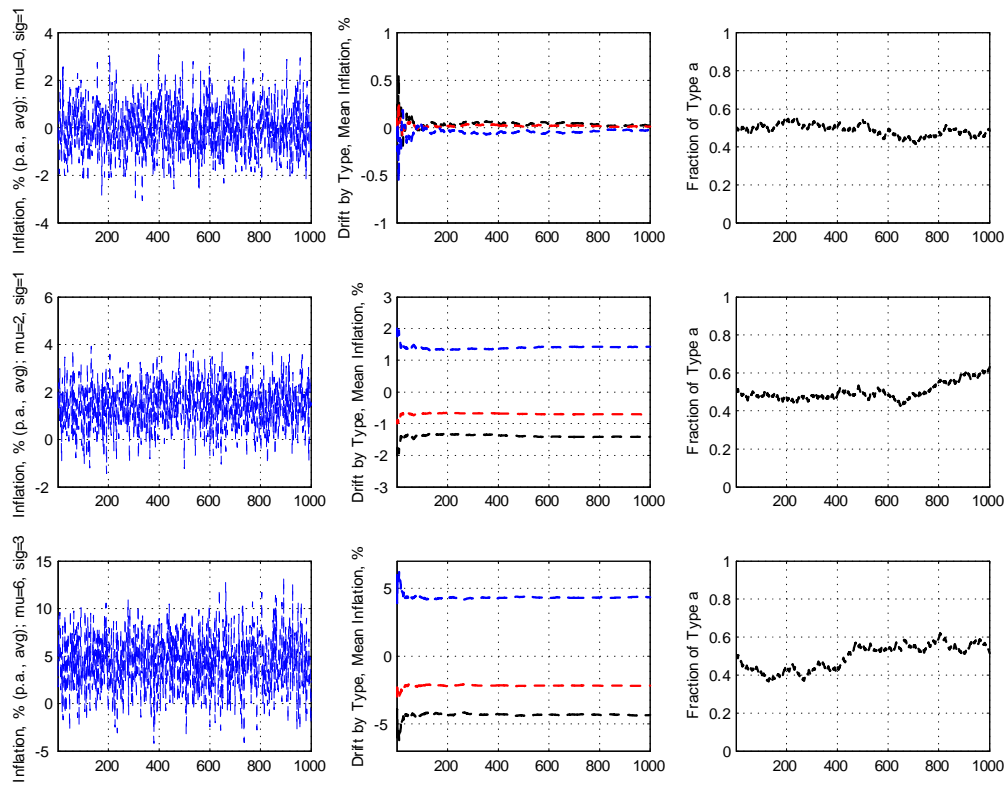


Figure 2: Stochastic Endogenous Coexistence of Types under Low Vertical Probability Differential, $|\tau_t^a(\cdot) - \tau_t^b(\cdot)| = 0.02$, Low Inflation Persistence, $\rho = 0.1$, and Low Variance of the Inflation Shock, σ_ε^2 (see the left-hand side axis for the precise values of the mean (μ) and variance (σ) of this shock for each of the rows)

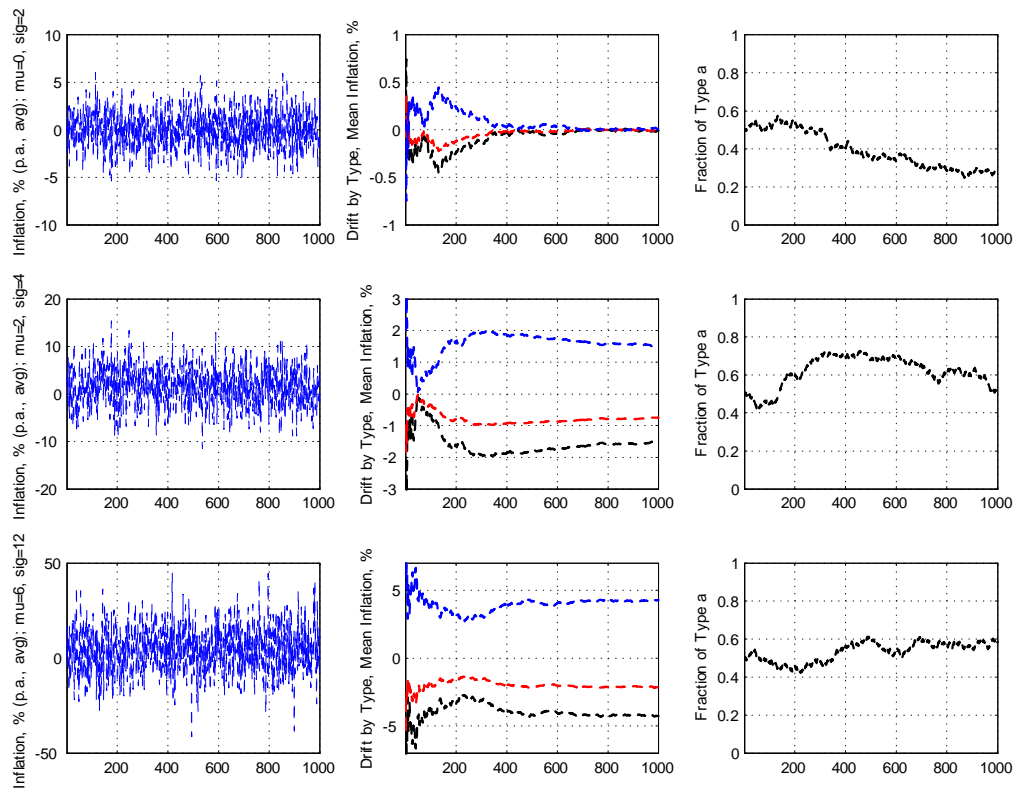


Figure 3: Stochastic Endogenous Coexistence of Types under Low Vertical Probability Differential, $|\tau_t^a(\cdot) - \tau_t^b(\cdot)| = 0.02$, Low Inflation Persistence, $\rho = 0.1$, and High Variance of the Normally Distributed Inflation Shock, σ_ε^2 (see the left-hand side axis for the precise values of the mean (μ) and variance (sig) of this shock for each of the rows)

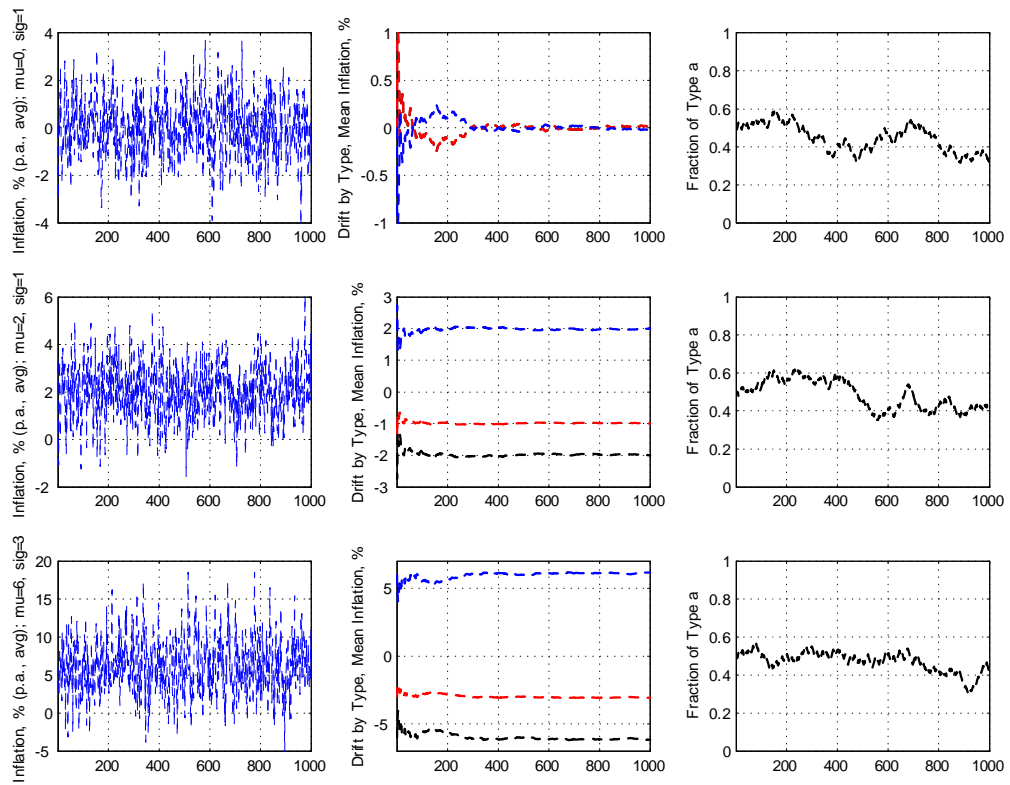


Figure 4: Stochastic Endogenous Coexistence of Types under Low Vertical Probability Differential, $|\tau_t^a(\cdot) - \tau_t^b(\cdot)| = 0.02$, Medium Inflation Persistence, $\rho = 0.5$, and Low Variance of the Normally Distributed Inflation Shock, σ_ε^2 (see the left-hand side axis for the precise values of the mean (mu) and variance (sig) of this shock for each of the rows)

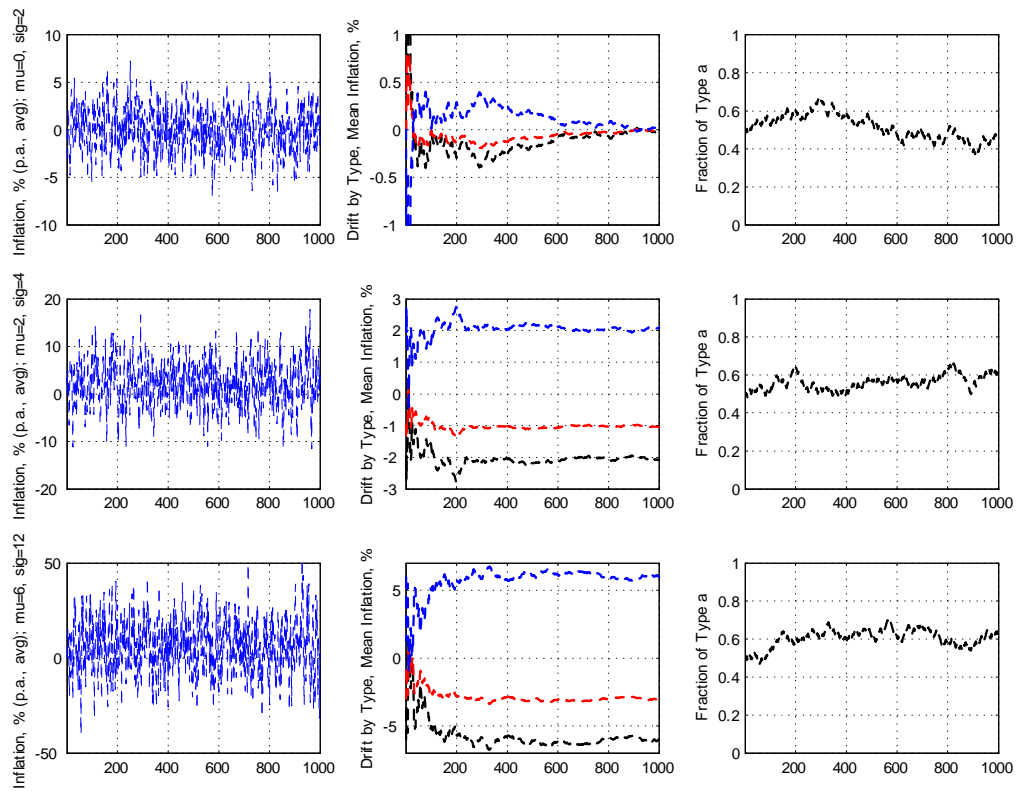


Figure 5: Stochastic Endogenous Coexistence of Types under Low Vertical Probability Differential, $|\tau_t^a(\cdot) - \tau_t^b(\cdot)| = 0.02$, Medium Inflation Persistence, $\rho = 0.5$, and High Variance of the Normally Distributed Inflation Shock, σ_ε^2 (see the left-hand side axis for the precise values of the mean (μ) and variance (sig) of this shock for each of the rows)

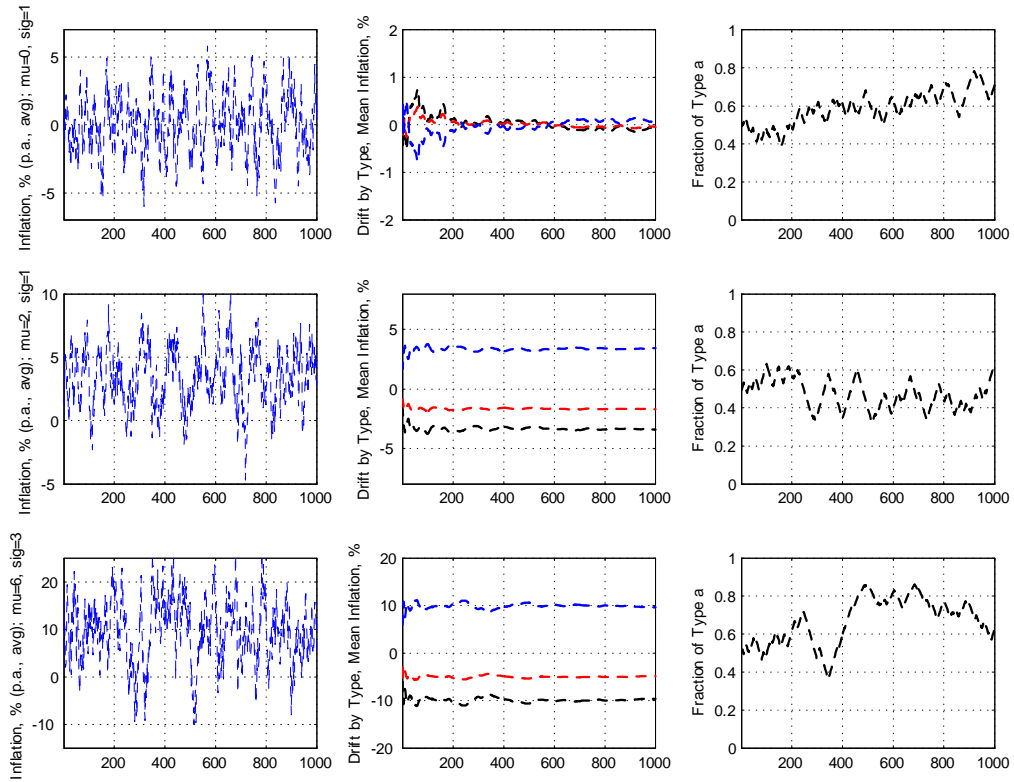


Figure 6: Stochastic Endogenous Coexistence of Types under Low Vertical Probability Differential, $|\tau_t^a(\cdot) - \tau_t^b(\cdot)| = 0.02$, High Inflation Persistence, $\rho = 0.9$, and Low Variance of the Normally Distributed Inflation Shock, σ_ε^2 (see the left-hand side axis for the precise values of the mean (μ) and variance (σ) of this shock for each of the rows)

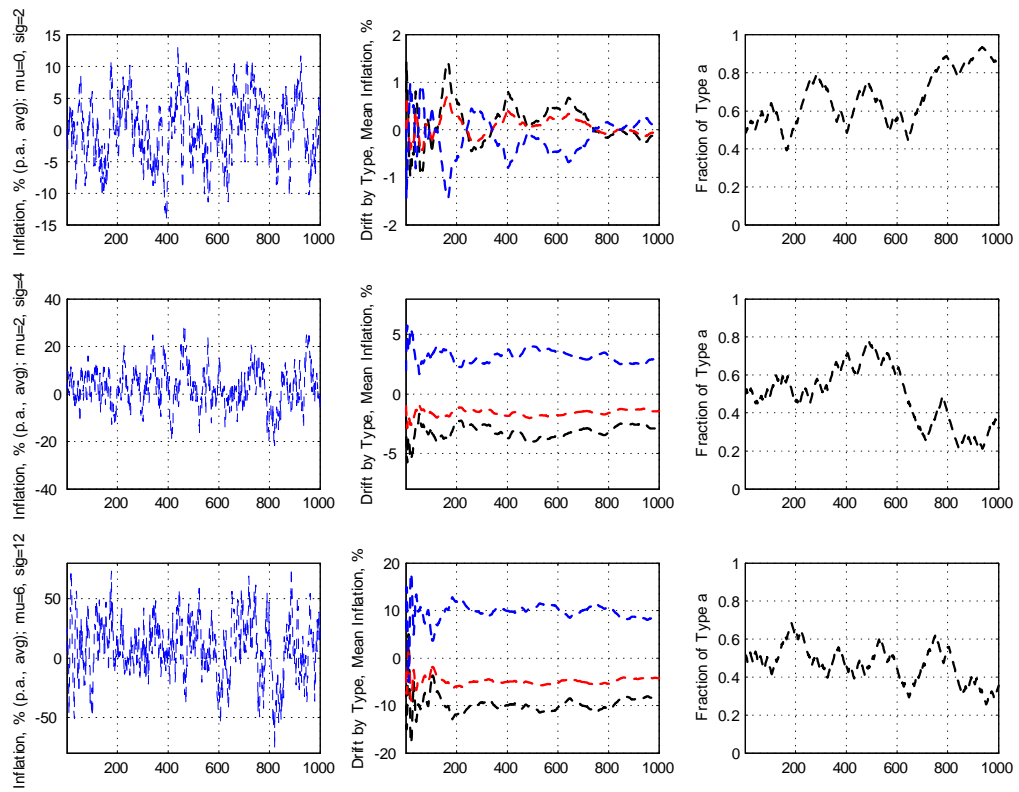


Figure 7: Stochastic Endogenous Coexistence of Types under Low Vertical Probability Differential, $|\tau_t^a(\cdot) - \tau_t^b(\cdot)| = 0.02$, High Inflation Persistence, $\rho = 0.9$, and High Variance of the Normally Distributed Inflation Shock, σ_ε^2 (see the left-hand side axis for the precise values of the mean (mu) and variance (sig) of this shock for each of the rows)

Appendix

This appendix provides proofs of the lemmas in the main text and further details on the special case of deterministic socialization.

A Proofs of Lemmas

A.1 Proof of Lemma 1

Proof. We proceed in three steps, each considering conditional inflation forecasts for some particular period t given legislated G_t^* that are, respectively, (i) positive, $\hat{\pi}_t > 0$, (ii) negative, $\hat{\pi}_t < 0$, or (iii) zero, $\hat{\pi}_t = 0$.

(i) Under $\hat{\pi}_t > 0$ (i.e., if inflation in the past has prevailed over deflation) we can write the inequality

$$\text{for } \hat{\pi}_t > 0 : \frac{\varpi}{1 + \hat{\pi}_t} - G_t^* < \varphi\varpi + \frac{(1 - \varphi)\varpi}{1 + \hat{\pi}_t} - G_t^*,$$

so that a -types *expect to lose more* from inflation than b -types and, hence, a -types endogenously *require a more inflation-averse* central bank than b -types:

$$G_t^{a*} > G_t^{b*}.$$

(ii) Now under $\hat{\pi}_t < 0$ (i.e., if deflation in the past has prevailed over inflation), we obtain

$$\text{for } \hat{\pi}_t < 0 : \frac{\varpi}{1 + \hat{\pi}_t} - G_t^* > \varphi\varpi + \frac{(1 - \varphi)\varpi}{1 + \hat{\pi}_t} - G_t^*,$$

so that a -types *expect to gain more* from deflation than b -types and, hence, a -types endogenously *require a more inflation-averse* central bank than b -types:

$$G_t^{a*} > G_t^{b*}.$$

(iii) Finally, under $\hat{\pi}_t = 0$ (i.e., if zero inflation has prevailed in the past), we obtain $G_t^{a*} = G_t^{b*}$, and the two types of beliefs, a and b , will thus be *indistinguishable* in terms of their preferred degree of central bank inflation aversion. ■

A.2 Proof of Lemma 2

Proof. For the expected real endowments to be equal under prevailing *positive* conditional expectations of inflation, $\hat{\pi}_t > 0$, a particular inflation forecast $\hat{\pi}_t^*$ must exist such that:

$$\varphi^a\varpi + \frac{(1 - \varphi^a)\varpi}{1 + \hat{\pi}_t^*} - \Phi^a - G_t^* = \varphi^b\varpi + \frac{(1 - \varphi^b)\varpi}{1 + \hat{\pi}_t^*} - \Phi^b - G_t^*.$$

Solving for it,

$$0 < \hat{\pi}_t^* = \hat{\pi}^* = \frac{(\varphi^a - \varphi^b) \varpi}{(\varphi^a - \varphi^b) \varpi + (\Phi^b - \Phi^a)} - 1 = \text{const} < 1.$$

Given the above benchmark non-redistributive forecast $\hat{\pi}^*$, we proceed again in three steps.

(i) If $\hat{\pi}_t \equiv \hat{\pi}_{t,H} > \hat{\pi}_t^*$, then

$$\varphi^a \varpi + \frac{(1 - \varphi^a) \varpi}{1 + \hat{\pi}_t} - \Phi^a - G_t^* < \varphi^b \varpi + \frac{(1 - \varphi^b) \varpi}{1 + \hat{\pi}_t} - \Phi^b - G_t^*,$$

so that

$$G_t^{a*} > G_t^{b*}.$$

That is, *a*-types endogenously require a *higher degree of inflation aversion of the central bank* than *b*-types.

(ii) If conversely $\hat{\pi}_t \equiv \hat{\pi}_{t,L} < \hat{\pi}_t^*$, then

$$\varphi^a \varpi + \frac{(1 - \varphi^a) \varpi}{1 + \hat{\pi}_t} - \Phi^a - G_t^* > \varphi^b \varpi + \frac{(1 - \varphi^b) \varpi}{1 + \hat{\pi}_t} - \Phi^b - G_t^*,$$

so that, again,

$$G_t^{a*} > G_t^{b*}.$$

(iii) Finally, under $\hat{\pi}_t \leq 0$, now with market costs of inflation protection, the two types of beliefs, *a* and *b*, still endogenously arise, in the sense that case (ii) just above still applies. ■

B Deterministic Benchmarks

To situate our work in a broader context, we next summarize the much simpler, deterministic case that the cultural transmission literature has generally considered.

B.1 Exogenous Vertical Preference Transmission

With *exogenous constants* τ^a and τ^b , the law of motion of *a*-types as a fraction in the total population, (7), becomes

$$q_{t+1}^a = q_t^a + \left[q_t^a - (q_t^a)^2 \right] \left(\tau^a - \tau^b \right). \quad (9)$$

Given the standard assumptions with regard to probabilities (as in the main text), namely that τ^a and τ^b are both between 0 and 1, we know that the stability points of this function are 0 and 1.²¹ The conditions for convergence then with *exogenously constant*

²¹In the case of exogenous constants τ^a and τ^b , our belief transmission model is the logistic map. Let $\tau = \tau^a - \tau^b$; $x_t = \frac{\tau}{1+\tau} q_t^a$ and $r = 1 + \tau$; then equation (9) becomes $x_{t+1} = r x_t (1 - x_t)$. The logistic map

vertical transmission probabilities τ^a and τ^b are obvious:

- If $\tau^a < \tau^b$, then for any initial condition q_0^a , $q_{t \rightarrow \infty}^a \rightarrow 0$: social beliefs will converge towards an economy with only type- b agents, i.e., a lower degree of central bank inflation aversion.
- If $\tau^a > \tau^b$, then for any initial condition q_0^a , $q_{t \rightarrow \infty}^a \rightarrow 1$: social beliefs will converge towards an economy with only type- a agents, i.e., a higher degree of central bank inflation aversion.

Since by the initial condition of preference heterogeneity, $0 < q_0^a < 1$, none of the above two cases can be ruled out, convergence in this deterministic exogenous preference dynamics equation, (9), will depend on the relative size of τ^a and τ^b . To illustrate this result, we present phase diagrams for the two opposite cases.

[Figures 8 and 9 about here]

As can be seen in Figure 8, if the sign of the vertical belief transmission probability differential between types a and b , $\tau^a - \tau^b$, is positive, then the intergenerational dynamics of the fraction of type a converges to the steady state S with coordinates $(1, 1)$ for any initial condition q_0^a . The process is driven by the *concavity* of the phase diagram curves, drawn for different magnitudes of the mentioned probability differential. This leads to an ultimate adoption of type a beliefs – which is the only type to survive, while the other type is extinguished. Conversely, Figure 9 shows that if the probability differential $\tau^a - \tau^b$ is negative, then beliefs in the society converge to type b at the steady state S' with coordinates $(0, 0)$ for any initial condition q_0^a . The *convexity* of the phase diagram curves in this case directs convergence to an ultimate equilibrium where only type b survives.

Interestingly, the *speed* of the belief convergence process depends on (the absolute value of) the *magnitude* of the vertical transmission probability differential, itself determining the *curvature* of the path of the fraction of type- a beliefs in our two phase diagrams. The larger (the modulus of) this differential (e.g., compare the graphs for 0.9 versus 0.1 in Figure 8 and for -0.9 versus -0.1 in Figure 9), the more curved the path and the quicker the convergence process.

B.2 Endogenous Vertical Preference Transmission

Differently from the situations depicted in figures 8 and 9, real-world heterogeneity of beliefs, values and norms of behavior does not seem to necessarily exhibit such convergence to an ultimate survival of one of the types, with the other extinguished (as in evolutionary selection mechanisms). Instead, an equilibrium where different belief types coexist would rather be sustained. Certain conditions on the transmission mechanism that induce heterogeneity in the long-run stationary distribution of beliefs in the population have been

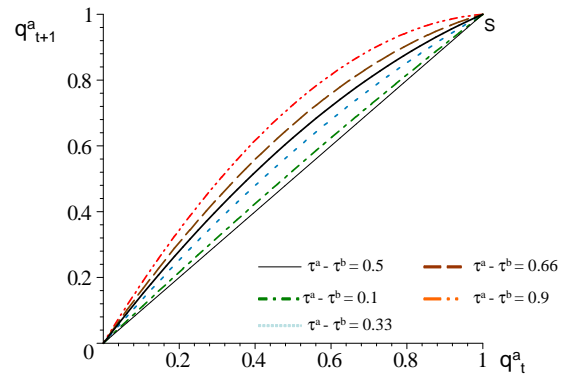
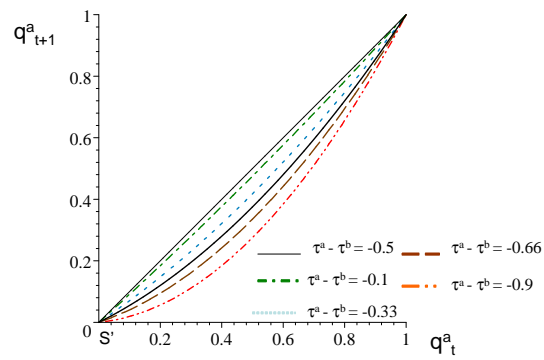
is well understood (at least in the range $0 \leq r \leq 2$, which is implied by $0 \leq \tau^i \leq 1$): here, the known behavior is equivalent to what we assert about the q_t^i processes.

examined by Bisin and Verdier (2001). However, in their set-up this analysis comes at the cost of imposing ‘cultural substitution’, an assumption also followed in Sáez-Martí and Sjögren (2008), which may be restrictive. Cultural substitution means that the vertical socialization of children *inside* the family and *outside* the family act as substitutes in the cultural transmission mechanism. Then, there can exist a heterogeneous distribution of beliefs in the population which is globally stable. Intuitively, oblique transmission acts as a cultural substitute for direct transmission when parents have less incentives to socialize their children once their own values are widely dominant in the population.

We could have assumed cultural substitution too: the probability of direct vertical socialization to the parent’s trait i , τ^i , will be a negative function of the attained level of the fraction in the population with that same trait, q_t^i , at time t ; that is, we can write $\tau^i(q_t^i)$, with $\frac{d}{dq_t^i}\tau^i(q_t^i) < 0$. Then (7) becomes:

$$q_{t+1}^a = q_t^a + \left[q_t^a - (q_t^a)^2 \right] \left[\tau^a(q_t^a) - \tau^b(1 - q_t^a) \right]. \quad (10)$$

In our context, equation (10) will have the same consequences as in the quoted papers, i.e., converging to an *interior* equilibrium. However, we have argued in section 2 that inflation beliefs could more realistically be thought of as a gradual outcome of past- and own-generation experience with inflation, accordingly modulating socialization effort and amending inherited monetary institutions. This led us to address in the main text the richer, stochastic environments where learning from history consistent with our theoretical results drives the transmission of beliefs and institutional change across generations.

Figure 8: Deterministic Exogenous Convergence to Type- a PreferencesFigure 9: Deterministic Exogenous Convergence to Type- b Preferences

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