

**THE UNIVERSITY OF READING  
DEPARTMENT OF ECONOMICS**

**Discussion Papers in Economics and Management**

**Series A  
Vol XIII (2000/01)**

**No. 429**

Variance of Growth and Size of Firm

by

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July 2001

## **Variance of growth and size of firm**

**Peter E. Hart and Nicholas Oulton**

### **1. Introduction**

Do large firms have smaller variations in growth rates than do small and medium-sized firms? Gibrat's (1931) law of proportionate effect implies that the mean and variance of growth is independent of size, which conflicts with the widespread belief that larger firms have smaller variance of growth because they have greater scope for offsetting favourable economic shocks against unfavourable shocks. A large firm may be regarded as a portfolio of small firms and if it can acquire two firms with negatively correlated sales (e.g. sausages and ice-cream) the variance of growth of both together is less than that of each of them. At the other extreme, if a parent company grows by acquiring other companies similar to itself with positively correlated sales, the variance of growth of the whole group will be much the same as that of the individual constituent firms. For a more detailed discussion, though in terms of rates of return rather than growth, see Prais (1976).

What are the facts? There have been many reports on the relationship between the variance of growth and size of firm since the early work of Hart and Prais (1956), Prais (1957), Hart (1962, 1965), Singh and Whittington (1968) for the UK, and Hymer and Pashigian (1962) and Mansfield (1962) for the USA. Different measures of size were used but the usual result was that the variance of growth decreased with increases in size of firm, contrary to Gibrat (1931). Over time the volume of data and the power of computers have increased remarkably so that it is now possible to examine the relationship between variations of growth and size for over 29 thousand companies, as shown in section 3. Before this it is necessary to outline the theoretical models commonly used to summarise the growth process, as in section 2. Section 4 provides a brief discussion.

## 2. Models

The Gibrat model may be written

$$y_{it} = y_{it-1} + \mathbf{e}_{it}, \quad \mathbf{e}_{it} \sim NID(0, \mathbf{s}^2) \quad (1)$$

where  $y_{it} = \ln(X_{it} / G)$  with  $X_{it}$  denoting the size of the  $i$ th firm at time  $t$  and  $G$  denoting the geometric mean. The variance of growth is given by

$$V(y_{it} - y_{it-1}) = \mathbf{s}^2 \quad (2)$$

and is independent of initial size, as is the variance of proportionate growth,  $V[(X_t - X_{t-1}) / X_{t-1}] = V(X_t / X_{t-1})$ . Since any linear function of normally distributed variables is normal, we should expect  $X_t / X_{t-1}$  to be lognormal. In practice, especially in the short period,  $X_t / X_{t-1}$  is approximately normal. This is because  $V(y_t) \approx V(y_{t-1})$  and the correlation,  $\mathbf{d}$ , between them is high (near unity) so that  $V(y_t - y_{t-1}) \approx 2V(y_{t-1})(1 - \mathbf{d})$  is low and hence the skewness and kurtosis of  $X_t / X_{t-1}$  are low enough to make its distribution nearly normal.

The Gibrat model is a special case of the Galton model

$$y_{it} = \mathbf{b}y_{it-1} + \mathbf{e}_{it} \quad (3)$$

with  $\mathbf{b} = 1$ . The joint distribution of  $y_{it}$  and  $y_{it-1}$  can be assumed to be bivariate normal and the regression is linear and homoscedastic. The OLS estimate,  $\mathbf{b}$ , of  $\beta$  is BLUE. If  $V(\mathbf{e}_{it})$  varies across  $i$ , the regression is heteroscedastic and  $\mathbf{b}$  is inefficient but remains unbiased. There has been an extensive literature on heteroscedasticity since the pioneer work of Prais and Houthakker (1955) on family budgets and many corrections for heteroscedasticity have been proposed. A common procedure is to assume that

$V(\mathbf{e}_{it}) = \mathbf{s}^2 Y_{it-1}^2$ ,  $Y_{it-1} = \ln X_{it-1}$  and dividing the regression equation by  $Y_{it-1}$  yields efficient estimates of  $\beta$ , providing the initial assumption is correct. How  $V(\mathbf{e}_{it})$  varies across  $i$  depends on the data. Increasing  $V(\mathbf{e}_{it})$  across  $i$  may be a reasonable assumption in the context of family budgets, but in the case of firm sizes it might be more appropriate to assume that it decreases across  $i$ . In which case another correction for heteroscedasticity must be made, possibly following Stanley *et al* (1996) outlined in section 4. However, in the UK context, with over 29 thousand observations, the standard errors are likely to be low whether or not corrections for heteroscedasticity are made. The relevant data are summarised in the next section.

### 3. The Facts

The mean and standard deviation of growth (measured by  $Y_{it} - Y_{it-4}$ ) of 29,230 independent companies in the UK 1989-93 are reported in Table 1 for each of 12 equi-logarithmic size classes. Size is measured by employment and was taken from the accounts of companies in the enormous OneSource database described in detail in Hart and Oulton (1996a), (1996b), (1998a) and (1998b).

The period covered the recession 1989-92 and although gross national product at constant market prices began to increase again in 1992-93, there was still a fall in geometric mean employment by some 2.5% between 1989 and 1993, as shown by mean  $Y_{it} - Y_{it-4} = -0.0248$  in natural logarithms. Companies in the three smallest size classes up to 16 employees increased their geometric mean employment. Above this size, the larger the company, the larger was the decrease in geometric mean employment, at least up to 2048 employees. Even then, there were still decreases in geometric mean employment.

The dispersion around these means is also shown in Table 1. For all 29,230 companies, the variance of growth was 0.693. Traditionally, this estimate of  $V(\mathbf{e}_{it})$  in (1) has been used to indicate the size mobility of companies. The lower is  $V(\mathbf{e}_{it})$ , the greater is the correlation between the sizes of

companies at times  $t$  and  $t - 4$ , and the smaller is the extent of overtaking or leap frogging. The low values of  $R^2$  in Table 1 suggest there was a substantial degree of size mobility, or movement of companies up and down the size distribution, over the period 1989-93. Moreover, this was also true for each size class.

The variance of growth for each size class is plotted in Figure 1. It decreases with increases in size for the 52% of companies in the bottom four classes up to 32 employees, and for the very largest companies in the top three classes above 2048 employees. But for the central 46% of the size distribution there was a clear tendency for the variance of growth to *increase* with increases in size. Clearly, there was no simple negative relationship between the variance of growth and size of company.

However, for size classes 4 to 12 with negative mean growth, there was a clear negative relationship between the dispersion of growth, measured by the variance, and mean growth, as shown by Figure 2: the larger was the mean decrease in employment, the larger was the variance. For the three smallest size classes 1 to 3, which had positive mean growth, the larger the mean growth, the larger was the variance. This might be explained along the following lines. Suppose that within each size class above 16 employees approximately the same small proportion of companies managed to survive the recession without downsizing. But for the remainder, those companies which suffered most in the recession not only reduced the mean growth rate of their size class, but also increased the standard deviation of growth as a result of their extreme downsizing. In terms of absolute size, there was an increase in the skewness of the size distribution within each size class.

Extreme downsizing is linked to early retirement schemes financed by pension funds, to the conversion of former employees to self-employed contract workers, and to general increases in outsourcing. Since such sophisticated manpower policies are more likely to occur in larger than in the smaller companies below 16

employees, there is a tendency for the variance of the proportionate fall in employment to increase with increases in the size of company for most of the size classes. In the case of the very largest companies above 2048 employees, this tendency was outweighed by other factors, especially the greater scope for

offsetting unfavourable shocks by less unfavourable shocks. The result was that although these largest companies reduced their employment, the mean proportionate decline was somewhat lower than in companies between 16 and 2048 employees, and the variance around this mean also fell.

#### 4. Discussion

There are several themes in the standard economics literature on the relationship between the size of firm and the variance of its proportionate growth. The first stems from the Gibrat model which implies that this variance is the same for small, medium, and large firms. Formally, the joint size distribution at two states is bivariate lognormal and hence the logarithmic regression is linear and homoscedastic. The many empirical studies over the years have actually found that this variance decreases with increases in size of firm and so this part of the Gibrat model is not accepted. A second theme involves the use of  $V(\mathbf{e}_{it})$  to measure the size mobility of firms up and down the size distribution. A third theme is part of the extensive literature on heteroscedasticity and involves specifying the precise relationship between firm size and variance of growth. The regression is then weighted to counteract heteroscedasticity in order to obtain WLS regressions which are BLUE. A fourth theme regards a large enterprise as the sum of several smaller businesses, often resulting from mergers and acquisitions, and investigates the relationship between the variance of the growth of the parent enterprise and those of its constituent businesses. For example, see Scherer *et al* (1975), Prais (1976), Kattuman (1996).

In addition to the standard economics literature, there is an interesting development of the fourth theme by a team of American physicists, Stanley *et al* (1996). They studied all US manufacturing companies quoted on US securities markets, thereby excluding the many thousands of unquoted companies and using severely truncated size distributions. Truncated lognormal distributions do not have the simple reproductive properties of complete lognormal distributions so Stanley *et al* (1996) find their distributions of  $\ln(X_t / X_{t-1})$  are not normal but exponential. These symmetric distributions suggest that their dispersion decreases with increases in initial size. Indeed, they find that  $\mathbf{s}_i$  decreases in accordance with a power law:

$$\mathbf{s}_i = \mathbf{q} \exp[-\mathbf{g}Y_{t-1}] = \mathbf{q}X_{t-1}^{-\mathbf{g}} \quad (4)$$

whether size is measured by sales or by employment. In principle, this result could be used to correct for heteroscedasticity in a weighted least squares procedure, following Prais and Houthakker (1955). Unfortunately, the American results are not supported by Table 1 and Figure 1, possibly because we include over 27 thousand unquoted companies and cover a four year period 1989-1993, whereas Stanley *et al* (1996) exclude unquoted companies and cover 16 annual changes in the period 1975-1991.

In the UK over the period 1989-93, it was found that the variance of growth varied widely across size classes of company. In the very smallest and very largest size classes it declined with increases in size of company. But for most size classes it increased with increases in size of company. While there is no simple relationship between size of company and variance of growth, there is a clear negative relationship between this variance and mean growth for most companies. Tentative explanations are suggested in terms of company manpower policies, but detailed case studies of the companies involved in extreme downsizing are required before a complete explanation can be provided.

**Table 1 Growth rates of employment over 4 years, 1989-93, by initial size summary statistics**

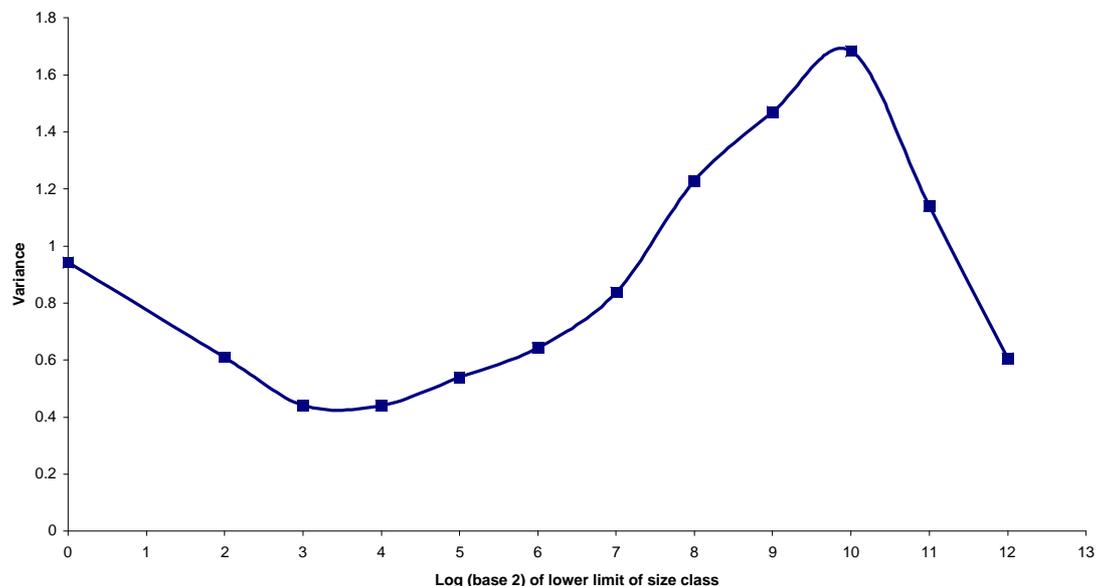
**group:**

Size class in year $t-4$	Employment in year $t-4$	$N$	Mean	S.D.	$R^2$	RMSE
1	$\leq 4$	2,534	0.4438	0.9708	0.0721	0.9374
2	$>4 \text{ \& } \leq 8$	3,028	0.1850	0.7805	0.0493	0.7625
3	$>8 \text{ \& } \leq 16$	4,522	0.0773	0.6645	0.0497	0.6486
4	$>16 \text{ \& } \leq 32$	5,071	-0.0113	0.6634	0.0535	0.6462
5	$>32 \text{ \& } \leq 64$	4,991	-0.0726	0.7344	0.0428	0.7194
6	$>64 \text{ \& } \leq 128$	4,335	-0.1892	0.8019	0.0809	0.7698
7	$>128 \text{ \& } \leq 256$	2,313	-0.2698	0.9152	0.0324	0.9026
8	$>256 \text{ \& } \leq 512$	1,161	-0.3566	1.1083	0.0370	1.0933
9	$>512 \text{ \& } \leq 1,024$	533	-0.4490	1.2125	0.0455	1.1982
10	$>1,024 \text{ \& } \leq 2,048$	313	-0.4819	1.2977	0.0391	1.2973
11	$>2,048 \text{ \& } \leq 4,096$	162	-0.2598	1.0678	0.0995	1.0498
12	$>4,096$	267	-0.1797	0.7785	0.0828	0.7629
	TOTAL	29,230	-0.0248	0.8326	0.0120	0.8277

Note  $R^2$  and RMSE are from regression of growth of employment,  $Y_t - Y_{t-4}$ , on constant, 9 SIC Division dummies and 3 accounting year dummies.

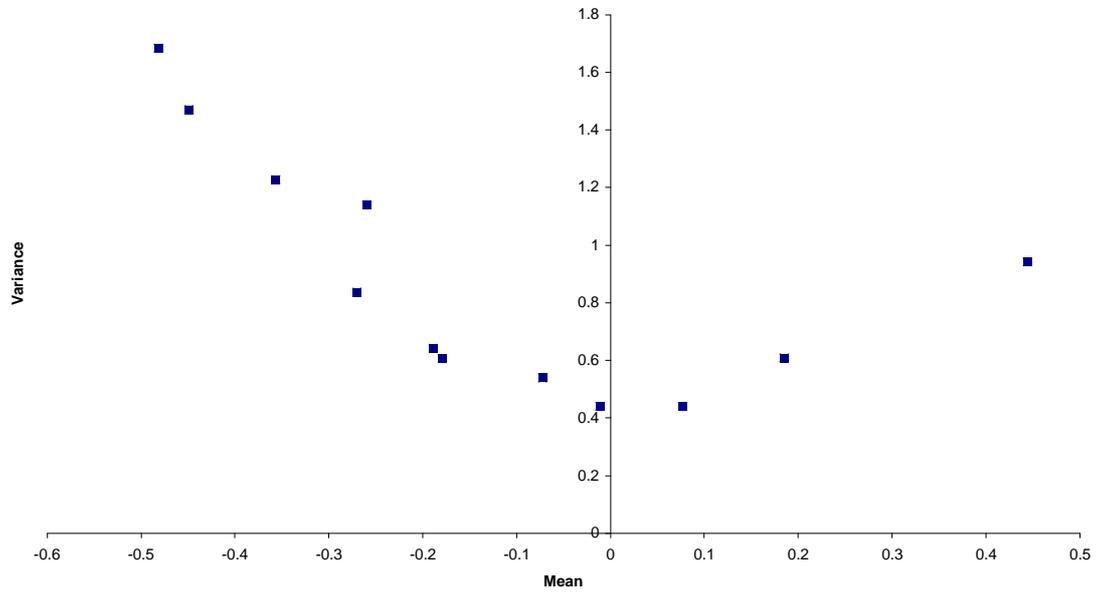
**Figure 1**

**Variance of growth versus log of size**



**Figure 2**

Variance versus mean growth



## REFERENCES

- Gibrat, R. (1931) *Les Inégalités Economiques*. Sirey, Paris.
- Hart, P.E. (1962) The size and growth of firms. *Economica*, NS.29, 29-39.
- Hart, P.E. (1965) *Studies in Profit, Business Saving and Investment in the United Kingdom 1920-1962*. Vol. 1. Allen and Unwin, London.
- Hart, P.E. and Oulton, N. (1996a). Growth and size of firms. *Economic Journal*, 106,. 1242-1252.
- Hart, P.E. and Oulton, N. (1996b). Job creation and variations in corporate growth. *National Institute of Economic and Social Research Discussion Paper No. 95*.
- Hart, P.E. and Oulton, N. (1998a). Job Creation and Destruction in the Corporate Sector: the Relative Importance of Births, Deaths and Survivors. *National Institute of Economic and Social Research Discussion Paper No. 134*.
- Hart, P.E. and Oulton, N. (1998b). Job generation in the UK corporate sector 1986-1995. *National Institute of Economic and Social Research, Discussion Paper No. 139*.
- Hart, P.E. and Prais, S.J. (1956) The analysis of business concentration: a statistical approach, *Journal of the Royal Statistical Society, Series A*, 119, 150-91.
- Hymer, S. and Pashigian, P. (1962) Firm size and rate of growth, *Journal of Political Economy*, 70, 556-69.
- Kattuman, P.A. (1996) On the size distribution of businesses of large enterprises: UK manufacturing. *Structural Change and Economic Dynamics*, 7, 479-94.
- Mansfield, E. (1962) Entry, innovation and the growth of firms. *American Economic Review*, 52, 1023-51.
- Prais, S.J. (1957) The financial experience of giant companies. *The Economic Journal*, 67, 249-64.
- Prais, S.J. (1976) *The Evolution of Giant Firms in Britain*. Cambridge University Press, Cambridge.
- Prais, S.J. and Houthakker, H.S. (1955) *The Analysis of Family Budgets*, Cambridge University Press, Cambridge.
- Scherer, F.M., Beckenstein, A., Kaufer, E., and Murphy, R.D. (1975) *The Economics of Multi-Plant Operation*. Harvard University Press, Cambridge, Mass: USA.
- Singh, A. and Whittington, G. (1968) *Growth, Profitability and Valuation*, Cambridge University Press, Cambridge.
- Stanley, M.H.R., Amaral, L.A.N., Buldyrev, S.V., Havlin, S., Leschorn, H., Maass, P., Salinger, M.A., and Stanley, H.E. (1996) Scaling behaviour in the growth of companies. *Nature*, 379, 804-806.