University of Reading

Combinatorics Colloquium, May 2006

Titles and Abstracts

The Computer System GRAPHOGRAPH

Vadim Zverovich
University of the West of England, Bristol

The aim of this talk is to draw the attention of the scientific community to an ambitious project to develop multi-purpose software for graph theory. The basic idea of this computer system is to verify graph-theoretic properties for sets of graphs. A user specifies a property using a simple Formalised Graph Language. No programming skills are required. The next step is to specify a graphbase, a set of graphs that can be chosen either randomly or from a huge built-in database of graphs, or specified by the user in Graph Editor. A computer then verifies the property for each graph from the graphbase and constructs a new set consisting of graphs satisfying the property. The resulting set of graphs is visualised and can be analysed by the user. One of the possible applications of this computer system is to verify whether a certain conjecture is true for all graphs of small order. The software would also be helpful in proving theoretical results. Some examples will be demonstrated during the talk. In order to provide the ability to specify a wide range of graph properties, a bank of basic graph algorithms will be developed. To achieve this, the algorithmic part of the project is divided into modules designed for different branches of graph theory. Any graph theorist interested in taking part in developing one of the modules is invited to co-operate (please contact: vadim.zverovich@uwe.ac.uk). All such developers will become co-authors of the software.

A topological method in matching theory

Ron Aharoni
Technion

I will survey the developments, over the last 5 years, of a topological method in matching theory that was introduced in a paper by Haxell and myself. This method is based on a topological proof of Hall's theorem, which yields far reaching extensions of the theorem. For example, extensions to hypergraphs. In particular, it yields a proof of the first open case of Ryser's conjecture, that of 3-uniform hypergraphs: in a 3-partite 3-uniform hypergraph the covering number is no larger than 2 times the matching number.

Referee squares

Ian Anderson
Glasgow University

Referee squares were introduced by Anderson , Hilton and Hamilton at Reading in 1986. I shall describe how they are constructed, and indicate open problems.

The Rado graph and the Urysohn space

Peter Cameron
Queen Mary, University of London

Rado's universal graph, published in 1964, is the unique countable "random graph", and has many remarkable properties, involving automorphisms, decompositions, first-order properties, Ramsey properties, amenability of groups, etc. I will discuss some of these.

The graph is also one of a family of structures, of which perhaps the first to be recognised was a remarkable Polish (complete and separable) metric space found by Urysohn: this is the unique Polish space which is universal (it embeds all Polish spaces isometrically) and homogeneous (every isometry between finite subsets extends to the whole space).

Dichotomy for the minimum cost graph homomorphism problem

Gregory Gutin
Royal Holloway, University of London

For graphs G and H, a mapping $f : V(G) \to V(H)$ is a homomorphism of G to H if $uv \in E(G)$ implies $f(u)f(v) \in E(H)$. If, moreover, each vertex $u \in V(G)$ is associated with costs $c_i(u), i \in V(H)$, then the cost of the homomorphism f is $\sum_{u \in V(G)} c_{f(u)}(u)$. For each fixed graph H, we have the *minimum cost homomorphism problem*, written as MinHOM(H). The problem is to decide, for an input graph G with costs $c_i(u)$, $u \in V(G), i \in V(H)$, whether there exists a homomorphism of G to H and, if one exists, to find one of minimum cost. Minimum cost homomorphism problems encompass (or are related to) many well studied optimization problems. We describe a dichotomy of the minimum cost homomorphism problems for graphs H, with loops allowed. When each connected component of H is either a reflexive proper interval graph or an irreflexive proper interval bigraph, the problem MinHOM(H) is polynomial time solvable. In all other cases the problem MinHOM(H) is NP-hard. This solves an open problem from an earlier paper.

Joint work with Pavol Hell, Arash Rafiey and Anders Yeo.

Edge weights and vertex colours

Andrew Thomason
Cambridge University

Given a graph, is it possible to assign an integer from the set 1,2,3 to each edge so that, by then assigning each vertex the sum of its incident edge values, we get a proper vertex colouring?

Clearly not if the graph consists of a single edge, or has a component which is a single edge. It is conjectured that in every other case the answer is "yes". This simple and universal conjecture remains open, to the best of my knowledge.

Some ideas will be described that lead to proofs of weak versions of the conjecture. The work is joint with Karonski and Luczak, and with Addario-Berry, Dalai, McDiarmid and Reed.

Rigidity of Molecular Frameworks

Bill Jackson
Queen Mary, University of London

A d-dimensional *framework* (G, p) is a graph $G = (V, E)$ together with a map $p : V \to \mathbb{R}^d$. The framework is *generic* if the co-ordinates of all the points $p(v)$, $v \in V$, are algebraically independent over $\mathbb{Q}$. Let (G, p) and (G, q) be frameworks. Then:

- (G, p) and (G, q) are *equivalent* if $|p(u) - p(v)| = |q(u) - q(v)|$ for all $uv \in E$.
- (G, p) and (G, q) are *congruent* if $|p(u) - p(v)| = |q(u) - q(v)|$ for all $u, v \in V$.
- (G, p) is *rigid* if there exists an $\epsilon > 0$ such that every framework (G, q) which is equivalent to (G, p) and satisfies $|p(v) - q(v)| < \epsilon$ for all $v \in V$, is congruent to (G, p). (This is equivalent to saying that there is no 'continuous deformation' of (G, p) which preserves the lengths of all its edges.)

It is known that the rigidity of a generic framework (G, p) depends only on the graph G and not the particular map p. Hence we say a graph G is *rigid in* $\mathbb{R}^d$ if some, or equivalently all, generic frameworks (G, p) are rigid. The problem of characterizing which graphs are rigid in $\mathbb{R}^d$ has been solved when $d = 1, 2$ but it is a difficult open problem for $d \geq 3$. There is some evidence, however, that the problem may become tractable for squares of graphs in $\mathbb{R}^3$. Indeed, Tay and Whiteley have conjectured a combinatorial characterisation for when such graphs are rigid. Their conjecture is known as the 'Molecular Conjecture' since molecules can be modelled as squares of graphs in $\mathbb{R}^3$. I will describe the conjecture and give some partial results. This work is joint with Tibor Jordán.
