

Testing the Statistical Significance of Sector & Regional Diversification

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Abstract

The question as to whether it is better to diversify a real estate portfolio within a property type across the regions or within a region across the property types is one of continuing interest for academics and practitioners alike. The current study, however, is somewhat different from the usual sector/regional analysis taking account of the fact that holdings in the UK real estate market are heavily concentrated in a single region, London. As a result this study is designed to investigate whether a real estate fund manager can obtain a statistically significant improvement in risk/return performance from extending out of a London based portfolio into firstly the rest of the South East of England and then into the remainder of the UK, or whether the manager would be better off staying within London and diversifying across the various property types. The results indicating that staying within London and diversifying across the various property types may offer performance comparable with regional diversification, although this conclusion largely depends on the time period and the fund manager's ability to diversify efficiently.

Keywords: *Sector/Regional Diversification, Portfolio Performance Improvement.*

Testing the Statistical Significance of Regional & Sector Diversification

I Introduction

In deciding to expand a real estate portfolio within a domestic market a key point of interest is the relative benefits of sector (across property types within a region) versus regional (across the regions within a property type) diversification. A related question is whether overall diversification is to be preferred. However, the UK market presents a particular problem when considering sector and regional diversification. The investment institutions, insurance companies and pension funds, in the UK, together with foreign investors have a bias towards London, especially the City office market.

A number of reasons exist for this focus of investment in London by UK institutions. First there is more information on the real estate market of London, especially so for City offices, as London is the most researched region in the UK and Europe. Second in terms of the stock of offices London accounts for the majority of the total office space in the UK. Third although the speed and costs of transaction varies enormously from one property type to another and across properties of differing lot-sizes, London typically offers the greatest speed of execution, for properties of a similar lot-size, across all property sectors McNamara (1999). Consequently for most institutional investors in the UK property in London provides an easy entry into direct real estate investment market. As a result whilst the bias towards London has declined over time, due to the server market collapse of offices in London market in the early 1990's, at the end of 1999 the UK institutional investors still held over one-third of their investments in the London region.

Similar arguments can be made to explain overseas investor interest in London. Not only is London the most researched, potentially most liquid and contains a sufficient stock of the right quality the but London office market is the most mature market in Europe (Keogh and D'Arcy, 1994) and is on a par with those found in the USA (Gordon, 2000). Alongside this, property in the UK shows relatively longer lease lengths, a more the favourable lease-structure and relatively lower transaction costs compared with Europe. All of which makes London the first port-of-call for investment in Europe by almost all overseas investors, DTZ (2000). For example, over the period 1988-1996, 80% of UK purchases by overseas investors were concentrated in London. As a result overseas investors now hold one-fifth of the total office stock in the City, Lizieri et al (2000). Furthermore although overseas investors have started to diversify out of London (DTZ, 2000), 60% of purchases are still concentrated in London, and the capital still remains the focus of attention of overseas investment.

This implies that most UK institutional and overseas investor portfolios are relatively un-diversified. Thus the primary interest of the analyses here is to investigate the incremental contribution provided by sector and regional diversification in enhancing the risk/return profile of a real estate portfolio initially heavily concentrated in London.

However, to take full advantage of such an expansion it may well be that fund managers will have to design complex weighting schemes. If so, these potential benefits of sector and regional diversification may be unattainable without perfect foresight. Alternatively, perhaps the contribution of sector and regional diversification is attainable with even the most naïve of investment strategies. For example, Cheng and Liang (2000) in a comparison of the performance of regional diversification in the US find that although optimal portfolios do significantly outperform their equal-weighted alternative on an *ex post* basis, when the analysis was repeated on an out of sample basis, there proved to be no statistical difference in performance. Indeed in some cases the equal-weighted portfolios showed greater performance than portfolios constructed using mean-variance analysis.

In order to examine these issues we form a number of portfolios that can be directly compared to a number of benchmark portfolios, as well as to each other. Then using the statistical tests developed by Gibbons, Ross & Shanken (1989) and Jobson and Korkie (1981), we investigate whether the benefits that accrue from the differing diversification strategies are statistically significant or not.

Specifically, the comparisons undertaken here should help in providing answers to a number of questions;

1. Is it better to diversify out of London across the regions within a property type or to stay within London and diversify across the sectors?
2. Are the benefits of sector or regional diversification dependent on the weighting scheme employed? In other words is mathematical programming required to reap the benefits of diversification, or can relatively simple investment strategies be applied without any real loss in diversification benefits?
3. How much diversification is needed to produce portfolios that display risk/return characteristics, which are statistically indistinguishable from that displayed by optimal diversification?

The remainder of the paper is organised as follows. The next section reviews previous research in the UK on real estate portfolio diversification. Section three gives details of the methodology and the research design employed. Section four describes the data and presents the results of an initial analysis covering the whole period. Section five then tests the stability of the initial results over two sub-periods, representing different types of market performance. Finally, section six concludes the paper.

II. Studies of Sector and Regional Diversification in the UK

There have been a number of studies of sector and regional diversification within the UK. Two main approaches have been adopted to investigate this issue, the construction of efficient frontiers using modern portfolio theory (MPT) and/or by the use of clustering

techniques. Extensive literature reviews are to be found in Hamelink, et al (2000) and Lee and Byrne (1998).

For example, Eichholtz et al (1995) find that, in terms of diversification, regional diversification varies from region to region and that the greatest benefits accrue the further the region is from London. In the case of property type diversification the authors find that the correlations are strongest between the office and industrial markets and less significant between retail and the other two sectors. This suggests a limited scope for diversification within a region across property types. Results borne out by constructing efficient frontiers based on the tenets of MPT. The authors find that in the UK office market diversification was optimised over the North and South regions or just the London market. Indeed for the riskiest portfolios in the UK, diversification across property types within London proved to be almost as effective as countrywide diversification by region and property type. In contrast, the retail and industrial sectors provided results consistent with results in the US, with total diversification across sectors and regions as the optimum strategy.

Lee and Byrne (1998) investigated the issue of sector and regional diversification within the UK by comparing a variety of efficient frontiers. In particular the authors compared the efficient frontiers produced by sector diversification against three types of regional portfolios; those based on the standard administrative regions of the UK, a '3 super-regional' classification and economically defined regions, based on travel-to-work areas. In line with the previous work the authors find that sector portfolios generally dominates the regional portfolios, however defined, and that certain functional groups outperformed the standard regional classification. However, the 'SuperRest' region, which contains all other 'peripheral' areas other than London and the Southeast, would have outperformed almost all other diversification strategies. This confirms the observation of Eichholtz et al (1995) that diversification benefits increase the further away from London and that the simple 3x3 classification scheme presents a reasonable investment strategy. Nonetheless the authors note that some of the functionally based 'regions' produced comparable results to those of the 'SuperRest' region and may be preferred by fund managers as they provide more insight into the reasons for a region's performance than that presented by the standard geographical areas.

Byrne and Lee (2000) find similar results when investigating the extent of risk reduction that can be achieved across the sectors and regions. Using annual returns data from 392 local authorities in the *IPD Local Markets Report* the authors find that the greatest percentage reduction in total risk, from naïve diversification, across the three sectors, retail office and industrial and the four regions, London, the South East, the South West and the North (the rest of the regions) occurs within the regional portfolios spread across the three sectors. In contrast the sector portfolios spread across the four regions showed only minor reductions in risk, with the office sector showing the worse performance. Byrne and Lee attribute this to the fact that the average correlations within a region were lower than the correlation across the sectors, the lowest average correlation, and so the greatest risk reduction potential, occurring in the regions the further away from London. The authors therefore conclude that this indicates that two properties in the same sector,

but in different regions, are closer substitutes than two different property types in the same region. As a consequence diversification within a region across the sector is preferable in terms of risk reduction from diversification across regions within a sector, a result confirming the previous studies by Eichholtz et al (1995) and Lee and Byrne (1998).

The alternative approach to MPT is to use cluster analytic techniques on the returns data of individual property markets (essentially towns) to try identify the extent to which the urban areas cluster by sector or regional. That is if the regional dimension plays an important role in return determination property markets will cluster by location. Then as the aggregation of the data continues 'standard' regions should be formed. In contrast, if the returns in the local property market are more determined by property type, sector clusters might be expected.

Cullen (1993), using cluster analysis techniques on 5500 properties from the IPD database, finds that industrial property is relatively homogenous across the UK, while retail properties are partitioned more on ownership and lease terms rather than on any regional basis. In contrast the office market displayed a distinct geographical structure, with City offices showing the greatest difference compared with the rest of the UK.

Hoesli, et al (1997) and Hamelink, et al (2000) using quarterly data from 1977-1995 find similar results to those of Cullen in that there appears to be a geographical dimension to the office and industrial property types but none for retail property. In particular the central London office market, especially the City office market, behaves differently from the Southeast and the rest of the UK, the distinctiveness of London becoming stronger in the second half of the analysis period. A similar conclusion was found for the industrial property sector, which displayed a split into a London cluster, the fringe immediately around London and all other 'peripheral' markets. In contrast the retail property markets clustered into a single group and failed to show a distinct London dimension. A conclusion maintained even with a more refined regional classification scheme. Finally the authors confirm the findings of Eichholtz et al (1995) that the 9-group classification provides a useful framework for the real estate portfolio construction process.

Based on the results of numerous studies using MPT, in the UK and other countries of the world, the general consensus is that property type (sector) dominates geographical (regional) diversification. When clustering methodology is applied to the real estate returns, in order to discover the sector/regional 'dimensions' within the data, two further points of interest have been identified. First in line with the results from the studies using MPT the property markets appear to cluster by property-type rather than region, a result most noticeable in the retail sector. Second, London offices, and especially the City office market, is somehow 'different' from the other property types and regions and forms a distinct cluster, see Hamelink, et al (2000). Finally both the results of MPT and cluster analysis show that the UK real estate market can be effectively grouped by a 3x3 classification scheme for portfolio investment purposes, with little to be gained from a more refined classification scheme. That is property market data can be classified by the

three-property sectors; office, retail and industrial across three geographical areas; London, the remainder of the south east and the rest of the UK.

This paper extends this previous work in a number of ways. First this analysis investigates the issue of sector and regional diversification starting from a realistic portfolio structure, i.e. one heavily concentrated in London. Secondly by using portfolios, which differ only in their complexity, the impact of diversification is examined to see whether the benefits accrue from the spreading of assets across different market segments or from the optimality of the allocation. Third the data used here is based on actual property data from the Investment Property Databank (IPD) monthly index. In contrast the data used in previous work by Eichholtz et al (1995), Hoesli, et al (1997) and Hamelink, et al (2000) is hypothetical, based on properties of a particular specification, were the rents quoted are not actual proven rents but rather a valuer's view of the open market rent based on market knowledge. The applicability of the results to actual property portfolios is therefore unclear. Next the data used is monthly over the period 1987 to 1998; previous studies used quarterly or biannually data from 1977 up to 1995 in the latest study. We can test therefore whether their conclusions are maintained over a different time period and with a different data source. Finally the use of monthly data provides sufficient data points so that we can investigate the stability of the results over a number of different time periods.

III. Methodology

Research Design

In order to evaluate the benefits of sector and regional diversification a number of portfolios were constructed all starting from a portfolio that initially was solely invested in a market sector of London and was then expanded across various sectors and regions. In the first case the tests are run on the basis of examining whether a real estate fund manager can obtain a statistically significant improvement in performance by extending a portfolio out of London into firstly the rest of the south east of England and then into the remainder of the UK on an equal-weighted basis. In the second strategy we initially start with a London based portfolio and then calculate the optimal portfolio that maximises the new portfolio's Sharpe ratio by first adding the south east region and then the rest of the UK. The two strategies therefore only differ in the complexity of the weighting scheme. Hence, in any sector/regional portfolio, which is made up of the same assets, any reduction in diversification benefits from less than optimal weighting can be identified. The analysis was then repeated within the London region to examine the benefits of sector diversification, with the analysis undertaken using the optimal and naïve investment strategies.

The purpose of such an approach is to isolate the role that sector or regional spread and the weighting schemes play in the diversification process. In other words since each portfolio can be directly compared to another which differs only in one dimension (sector/regional or weighting) we are able to control for one factor so the influence of the others can clearly be identified. As a consequence analysing these different portfolios

structures helps in assessing whether the benefit of diversification lies in the complexity of the weighting schemes or the sector and regional spread.

To obtain a measure that can be used for comparisons between the different portfolio combinations, we construct that optimum portfolio that achieves the greatest risk/return performance over a particular time period, i.e. the one with the highest Sharpe ratio. Thus the identification of the optimum portfolio is crucial to this analysis since this theoretical portfolio captures the maximum diversification benefits over a particular time frame. Clearly, this portfolio is not realistically achievable; but instead, provides the most stringent benchmark against which all other less diversified portfolios can be compared.

However, while the maximisation of a portfolio's risk/return trade-off is consistent with financial theory, a property portfolio fund manager may need to consider other aspects of property investment not captured in returns. These other factors include: the large lot-size and indivisibility of property, the lack of a centralised market place, limited information, long transaction periods and high transaction costs. Naturally the large lot-size and indivisibility of assets hinder the construction of a fully diversified portfolio. But even where a more diversified portfolio is sought, there will inevitably be some concentration. The high transaction costs, long transaction times and lack, or poor quality, of information will inevitably slow or even undermine any attempt to implement a systematic diversification strategy. Consequently fund managers will naturally gravitate towards an investment strategy that builds on their skills in the real estate market. This will lead fund managers to focus on certain sectors or regions of the market in which they have specialised knowledge or expertise. It is not surprising therefore to see institutional investors and especially overseas fund managers focusing their investments on a particular sector and region of the market, i.e. London offices. This market segment is the most researched, provides the greatest number of trophy buildings and is possibly the most liquid market segment in the UK. Thus although a London office based portfolio may not be the most diversified, such a portfolio may offer the fund manager other benefits not found in a property portfolio spread across the sectors and regions of the UK. Nonetheless we rely on mathematical programming to find the optimum portfolio over a particular time period against which all other portfolios can be evaluated.

Expressed mathematically the objective function is to:

$$\text{Max(SharpeRatio)} = \frac{\bar{R}_p - R_f}{\sigma_p} \quad (1)$$

where: \bar{R}_p is the expected return on portfolio p, R_f is the risk-free rate of return and σ_p is the standard deviation of the portfolio. In conducting the analysis the portfolio weights were constrained to be positive (i.e., no short sales were allowed), with the fractions summing to one. For simplicity the risk-free rate of return was assumed to be zero.

Two other benchmark portfolios are also considered. The first is an equal-weighted naïve portfolio of all 9 assets. The second is the IPDMI a market-weighted index that represents the ‘consensus’ performance of all funds in the IPD monthly universe. Unlike the unrealistic and unconstrained portfolio identified by maximising the Sharpe Ratio, these two benchmarks of performance are closer in spirit to the portfolios against which fund performance would be compared. In other words out-performance by the sector/regional portfolios is possible in comparison with these two portfolios, unlike the case above. These two benchmarks of performance provide a more realistic if less stringent measures of performance.

Testing the Significance of Portfolio Improvement

We use two alternative approaches to assess the improvement in performance. The first is that proposed by Gibbons, Ross & Shanken (1989), which compares the Sharpe ratios obtained for both the original data and the extended portfolios. The statistic has a F -distribution with $(T - N_2, N)$ degrees of freedom and can be displayed as:

$$F = \frac{\frac{(T - N_2)}{N} (\hat{\theta}_2^2 - \hat{\theta}_1^2)}{(1 + \hat{\theta}_1^2)} \quad (2)$$

Where $\hat{\theta}_1$ is the initial maximum Sharpe ratio, $\hat{\theta}_2$ is the maximum Sharpe ratio from the expanded data set, T is the number of observations, N_1 is the number of core assets, N_2 is the number of total assets and N can be defined as $N_2 - N_1$.

Using this statistic Rubens et al (1998) tested the benefits of adding international investments to a domestic mixed-asset portfolio, the authors finding little improvement in performance. In a similar way Cheng and Liang (2000) find little statistical difference in performance of regional portfolios constructed using MPT compared with an equal weighted strategy. However, in both cases the authors use the statistic unconstrained, i.e. they implicitly assume short selling can occur, an assumption that is unrealistic within most capital market but especially in real estate markets. If no short selling is assumed, however, this means that the F distribution is unknown and needs to be approximated using simulations. In order to approximate the distribution we adopt the approach utilised in studies such as Glen & Jorion (1993) and Stevenson (2000). Initially, the historical returns, variances and covariances are calculated, with the returns modified so that the null hypothesis concerning the mean-variance efficiency of the initial set of assets is satisfied. The optimal portfolio of N_1 assets that maximizes the Sharpe ratio is then calculated, under the assumption of no short selling. The expected returns on the additional assets are forced to be proportional to their beta relative to this market, thus ensuring that the optimal portfolio is the same for the sample of N_1 and N_2 assets. The set of simulated returns is obtained by drawing T random samples of joint returns from a multivariate standard normal distribution with these parameters. This provides a new vector of means and a new covariance matrix. The optimisation is performed as before

and the value of the F statistic recorded. This process is undertaken a total of 1,000 times in order to estimate the empirical distribution of the F statistic.

The second test is that proposed by Jobson & Korkie (1981) which tests for the equality of the Sharpe ratios of any two portfolios. The test statistic can be formulated as:

$$Z = \frac{\sigma_a(\mu_b - R_f) - \sigma_b(\mu_a - R_f)}{\sqrt{\Theta}} \quad (3)$$

where: μ_a , μ_b are the mean returns of portfolios under investigation, R_f is the risk-free rate of return (assumed to be zero) and where Θ is calculated as follows:

$$\Theta = \frac{1}{T} \left[2\sigma_a^2\sigma_b^2 - 2\sigma_a\sigma_b\sigma_{ab} + \frac{1}{2}\mu_a^2\sigma_b^2 + \frac{1}{2}\mu_b^2\sigma_a^2 - \frac{\mu_a\mu_b}{2\sigma_a\sigma_b}(\sigma_{ab}^2 + \sigma_a^2\sigma_b^2) \right] \quad (4)$$

where T is the number of observations and σ_a , σ_b and σ_{ab} are estimates of the standard deviation and covariance's of the excess returns of the two portfolios over the evaluation period. Jobson and Korkie (1981) showing that the test statistic Z is approximately normally distributed with a zero mean and a unit standard deviation for large samples. A significant Z statistic would reject the null hypothesis of equal risk-adjusted performance and would suggest that one of the investment strategies outperforms the other. Jobson and Korkie (1981) and Jorian (1985) note that the statistical power of the test is low, especially for small sample sizes. Observing a statistically significant Z score between two portfolios can be seen as a strong evidence of a difference in risk-adjusted performance¹.

IV Data and Initial Results

Data

The data consist of the IPDMI, broken down both by region and sector, the data covering the period 1987:1 to 1998:12, a total of 144 observations. We separate the UK commercial market into the three primary commercial sectors, office, retail and industrial, and also into three 'super' regions. The reasons for separating geographically on this basis are four fold. First, the composition of the majority of institutional portfolios and therefore by definition the IPD index, is heavily concentrated in London and the south east. Second, if the remainder of the UK outside the southeast was further broken down, it may lead to problems in the number of properties within sector regional markets and could bring into question how representative the measure was of actual fund performance. Third, UK institutional investors hold on average less than 60 properties, thus limiting the number of property diversification categories a fund manger can consider and yet be represented in all market segments. Fourth, the work of Eichholtz et

¹ As the results of the two tests produced similar results in the case of the expanded portfolios and since the Gibbons et al test cannot be used to test the equivalence of the portfolios against a benchmark only those results from the Jobson & Korkie test are reported.

al. (1995), Hoesli, et al (1997) and Hamelink, et al (2000) clearly shows that for all but the largest investors, a diversification approach based on a 9-category classification scheme in the UK is a reasonable investment strategy. Finally using the same classification scheme as previous authors means that we can see whether their conclusions are maintained over a different period and with a different data source. Therefore, for purposes of comparison the following analysis also employs the three sector and three regional classification scheme.

Table 1 presents the summary statistics of the 3x3 real estate market segments in the UK over the period 1987:1 to 1998:12. The results shows that the highest returns were achieved the further the region is from London and especially so in the industrial sector, while the London office market showed the least return. However, high return was not necessarily associated with high risk. The segment with the lowest returns, the London office market, had the highest risk, while the retail sector, showing returns well above those of offices in London and the south east, had the least risk. Consequently in terms of risk-adjusted performance the retail sector in general showed the best Sharpe performance along with industrial properties in the rest of the UK.

Table 1: Summary Statistics of the Nine Property Market Segments and Significance Tests of Performance: Full Period

Property Portfolios	Mean	SD	Sharpe	Benchmarks		
				Optimal	Naïve	IPDMI
London Retail	0.84	1.00	0.84	-4.56	-2.31	-1.31
South East Retail	0.71	0.85	0.84	-7.26	-2.31	-1.76
Rest of UK Retail	0.83	0.80	1.04	-2.69	2.87	3.58
London Office	0.68	1.33	0.51	-9.91	-9.45	-10.31
South East Office	0.72	1.15	0.63	-8.90	-7.01	-7.68
Rest of UK Office	0.93	1.17	0.79	-4.54	-1.86	-2.00
London Industrial	1.05	1.19	0.88	-3.48	-0.32	-0.48
South East Industrial	1.01	1.08	0.94	-2.83	-0.00	0.91
Rest of UK Industrial	1.23	1.16	1.06	-0.53	2.56	3.46
Optimal	1.00	0.90	1.12	-	3.36	5.70
Naïve	0.89	0.95	0.94	-	-	3.30
IPDMI	0.83	0.92	0.90	-	-	-

Note: A positive value indicates the property portfolio out performed the benchmark portfolio, a negative value indicates underperformance.

The nine property market segments were also compared against the optimal benchmark and the performance of the equally weighted naïve portfolio and the IPDMI, using the Jobson & Korkie statistic. As is to be expected no market segment out-performed the optimal portfolio benchmark, as indicated by the negative value for each Z statistics in column 5, although the risk-adjusted performance of industrials in the rest of the UK is statistical insignificantly different from that of the most efficient portfolio that can be constructed over this time period. In contrast a number of market segments out performed the other two benchmarks, as indicated by the positive values in columns 6 and 7. Once again this proved to be industrial and retail properties in the rest of the UK and industrial properties in the south east. In particular industrial and retail properties in the rest of the UK showed significantly greater performance than the IPDMI and the

naïve portfolio. Retail in London and the south east and industrials in London showed levels of risk-adjusted performance insignificantly different from that of the IPDMI and the naïve benchmark. In contrast the office market showed negative performance in all regions, significantly so in London and the south east. These results confirm the findings of Eichholtz et al (1995) and Lee and Byrne (1998) and Byrne and Lee (2000) who all show that performance, over the eighties and nineties, was greatest outside the office sector and the further away the property is from London.

In addition the results in Table 1 show that by concentrating a real estate portfolio within the London region, in whatever sector, leads to a level of performance which is statistically inferior to that of any benchmark. A prudent fund manager should therefore be encouraged to diversify. However, how should this be done to reap the greatest benefit? Given that it seems reasonable to assume that fund managers will try to capitalise on their expertise, staying within a sector and diversifying regionally would be considered the best alternative. However, this needs to be balanced against the advantages of staying within the most researched, most liquid and largest region by value, London, even if it means moving into property types with which the fund manager is unfamiliar. In addition previous studies would suggest that it is better to expand across different sectors within a region rather than staying with the same sector and expanding across the regions. The following sections will therefore test the benefits of each form of diversification to see which offers the greatest improvement in performance and which produces levels of performance comparable with the three portfolio benchmarks.

Regional Expansion within a Sector

In order to test the improvement in performance from regional diversification the London based portfolios were expanded across the regions of the UK, on a naïve and optimal portfolio basis, the results presented in Table 2. The Jobson & Korkie statistics presented in Columns 5-7 show that compared with the one-sector, one-region portfolios expansion would have led to a significantly increased performance in almost all situations, equally weighted or not, especially compared with the London office sector. The exceptions being the office portfolios expanded regionally when compared with London retail and industrial sectors, due to the inferior performance of the office sector in general. Nonetheless diversification even on a naïve basis leads to a significant increase in performance!

Columns 8-10 of Table 2 present the results of the performance of the expanded portfolios against the benchmarks. In the case of the retail portfolio expanded on an equal-weighted basis outside London, when compared against the optimal benchmark, there is initially a worsening in performance, due to the extremely poor performance of South East retail properties, see Table 1, but the inclusion of retail in the rest of the UK redresses the balance. In comparison the performance of retail against the IPDMI continues to improve with the inclusion of further regions. Indeed the equal-weighted retail portfolio significantly out-performs the market benchmark at the 3% level. Similar results are shown when the expansion is done on an optimal basis. In contrast industrial property shows that expansion out of London leads to out performance against the IPDMI

as soon as the south east region enters the equation, irrespective of whether the expansion is equally weighted or optimal. Further, when industrial expansion is compared against the optimal benchmark as soon as the South East region enters the portfolio industrial property shows a level of performance insignificantly different from that of optimal benchmark at the usual levels of statistical significance. In contrast office property never outperforms the optimal benchmark or the IPDMI, on an equal or optimal allocation basis.

Table 2: The Performance of Regional Diversification out of London; Equal and Optimal Allocation and Jobson & Korkie Statistics: Full Period

Property Segment	Mean	SD	Sharpe	London			Benchmarks		
				Retail	Office	Indust	Optimal	Naïve	IPDMI
Equal-Weighted									
L+SE Retail	0.78	0.86	0.90	2.05	6.80	0.32	-5.24	-1.61	-0.15
L+SE+REST	0.79	0.82	0.96	3.50	6.14	1.37	-4.66	0.06	1.92
L+SE Office	0.70	1.18	0.59	-4.31	1.01	-3.68	-10.67	-11.51	-11.30
L+SE+REST	0.78	1.11	0.70	-2.29	2.26	-7.26	-7.95	-10.42	-7.77
L+SE Industrial	1.03	1.07	0.96	1.89	7.58	1.67	-3.51	0.07	1.50
L+SE+REST	1.10	1.06	1.03	2.86	8.63	3.70	-1.19	2.78	3.50
Optimal									
L+SE Retail	0.77	0.86	0.90	1.90	5.36	0.34	-5.38	-1.63	-0.03
L+SE+REST	0.83	0.80	1.04	3.85	7.20	2.37	-2.45	1.80	3.66
L+SE Office	0.72	1.15	0.63	-3.59	1.41	-3.03	-9.13	-9.70	-8.38
L+SE+REST	0.93	1.16	0.80	-0.49	3.08	-1.32	-5.37	-3.69	-2.81
L+SE Industrial	1.02	1.06	0.97	2.04	7.51	1.65	-2.37	0.29	1.78
L+SE+REST	1.18	1.11	1.07	3.19	9.24	3.60	-0.75	3.43	4.31

Note: A positive value indicates the expanded portfolios out performed the London based sectors or the benchmark portfolios, a negative value indicates under performance.

The other important feature in Table 2 is that the Jobson & Korkie statistics for the optimal allocation are always better than the comparable equal-weighted strategy. Indeed tests, presented in Table 1 of the Appendix, show that the optimal allocation outperforms equal allocation in all cases, although the difference is marginal, supporting the conclusions of Cheng and Liang (2000).

Sector Expansion within London

Table 3 shows the performance of equal and optimal expansion across the London region starting from the retail, office and industrial property types and adding the other sectors in turn. Columns 5-7 of Table 3 show the Jobson & Korkie statistics of the performance of the equally weighted and optimally allocated London portfolios compared against each sector in turn. Once again diversification leads to a significant increase in performance in the majority of cases. The exceptions are the portfolios that include the office sector on an equal-weighted basis.

Column 8 of Table 3 shows that in the case of the retail sector when adding offices, in an equal-weighted fashion leads to a reduction in return and an increase in risk and consequently a worsening in performance, against the optimal portfolio and other

benchmarks. In contrast adding industrial properties to the retail sector leads to an increase in return and a reduction in risk and so an increase in Sharpe performance. Indeed this portfolio shows a level of performance almost as good as that of the naïve portfolio and significantly outperforms the IPDMI. However, when industrials are added to offices the resultant portfolio shows little improvement. While the equal-weighted portfolio spread across all property types fails to outperform all benchmarks, due to the dismal performance of the office sector.

Table 3: The Performance of Property Type Diversification within London; Equal and Optimal Allocation and Jobson & Korkie Statistics: Full Period

Property Segment	Mean	SD	Sharpe	London			Benchmark		
				Retail	Office	Indust	Optimal	Naïve	IPDMI
Equal-Weighted									
R+O	0.76	1.08	0.70	-3.65	6.61	-2.55	-8.80	-6.70	-6.38
R+I	0.94	0.98	0.96	2.91	8.84	1.61	-3.49	0.11	1.59
O+I	0.86	1.14	0.75	-1.33	3.94	-2.06	-7.86	-6.97	-4.58
R+O+I	0.86	1.03	0.83	-0.16	4.96	-1.46	-5.95	-4.99	-2.76
Optimal									
R+O	0.84	1.00	0.84	0.00	4.48	-0.65	-4.74	-2.30	-1.13
R+I	0.94	0.98	0.96	2.91	7.55	1.43	-3.49	0.11	1.59
O+I	1.05	1.19	0.88	0.51	4.12	0.00	-4.39	-1.80	-0.48
R+O+I	0.94	0.98	0.96	2.91	7.42	2.39	-3.49	0.11	1.59

Note: A positive value indicates the property portfolios outperformed the London based sectors or the benchmark portfolios, a negative value indicates under performance.

The optimal portfolios show similar levels of performance as the retail and industrial portfolios, since the optimal allocation always assigns a zero weight to the office sector. Consequently although the London portfolios fail to out-perform the optimal portfolio, those constructed from the retail and industrial sectors show Sharpe performance statistically above that of the IPDMI and comparable with the naïve portfolio. In addition the optimal portfolios show significantly better Sharpe performance than the equal-weighted strategy, although the difference is marginal, see Table 2 in the appendix.

Sector or Regional Diversification?

The results so far suggest that efficient diversification is superior to an equal-weighted strategy, irrespective of whether the expansion is across the regions of the UK or within the London area. Indeed in some cases simply expanding the portfolio naively into another region or sector can lead to the creation of portfolios that produce levels of risk/return performance statistically insignificant from the most diversified portfolio that can be constructed! Indeed some of these portfolios show performance levels that are significantly greater than the naïve portfolio or market benchmark. However, one question yet to be answered is whether diversification across the three property types within the London region offers comparable or even superior to diversification within a sector across the regions of the UK. In order to investigate this issue Table 4 presents the Jobson & Korkie statistics testing the equivalence of the equal-weighted and optimal

London portfolios against the three property types diversified across the regions of the UK.

Table 4: Jobson & Korkie Statistics for the London Portfolio against the Regional Portfolios; Equal and Optimal Allocation: Full Period

Regional Portfolios	London Portfolio	
	EW	Optimal
Equal-Weighted		
L+SE Retail	-1.77	1.70
L+SE+REST	-3.35	0.03
L+SE Office	8.00	7.76
L+SE+REST	3.99	5.62
L+SE Industrial	-3.82	0.05
L+SE+REST	-5.38	-1.82
Optimal		
L+SE Retail	-1.77	1.67
L+SE+REST	-4.22	-1.47
L+SE Office	4.98	7.12
L+SE+REST	0.53	2.64
L+SE Industrial	-4.01	-0.12
L+SE+REST	-4.91	-2.04

Note: A positive value indicates that the London portfolio out performed the regional portfolios, a negative value indicates under performance.

The results in Table 4 show that diversification within London across the three property types, on a naïve basis, was generally inferior in terms of Sharpe performance to a real estate portfolio expanded across the regions. The only exception is that of the office sector. In contrast optimal diversification within London generally offered superior performance to diversification across the regions! The only exceptions here are the equal and optimally weighted industrial portfolios. This suggests that sector diversification, within only one region, may offer comparable performance to regional diversification, but only with the benefit of sophisticated portfolio analysis techniques, which may not be feasible or practicable for most managers.

V. Stability of Results

In order to test the stability of these findings the analysis was repeated for the two six-year sub-periods, 1987:1 to 1992:12 and 1993:1 to 1998:12. Summary statistics for these two periods shown in Table 5. The first sub-period covering the severe market rise and fall, and the second period was one of greater stability. As a consequence the picture in the two sub-periods shows significant shifts in performance. The best performing segment in the first period is the industrial sector in the rest of the UK. Indeed the risk-adjusted performance of this market segment dominated all other segments and as a consequence the optimal portfolio was made up of a 100% allocation in this area. However, during the second sub-period performance was more evenly spread with all the property types in London showing the good performance, especially the retail and industrial property types.

Table 5: Summary Statistics of the Nine Property Market Segments and Significance Tests of Performance 87-92 and 93-98

Property Portfolios	Mean	SD	Sharpe	Benchmarks		
				Naïve	IPDMI	Optimal
87-92						
London Retail	0.65	1.20	0.54	-2.29	-1.45	-4.65
South East Retail	0.59	0.93	0.63	-1.42	-0.33	-5.37
Rest of UK Retail	0.63	0.80	0.79	1.04	2.32	-3.58
London Office	0.41	1.71	0.24	-8.34	-8.46	-9.74
South East Office	0.55	1.41	0.39	-6.65	-4.80	-8.75
Rest of UK Office	1.09	1.48	0.74	0.25	1.19	-4.08
London Industrial	1.06	1.54	0.69	-0.42	0.50	-4.49
South East Industrial	1.09	1.38	0.79	1.54	3.03	-3.55
Rest of UK Industrial	1.43	1.38	1.04	6.22	6.70	0.00
Naï ve	0.84	1.16	0.72	-	-2.66	-6.22
IPDMI	0.72	1.10	0.65	-	-	-6.70
Optimal	1.43	1.38	1.04	-	-	-
93-98						
London Retail	1.03	0.71	1.45	0.86	1.20	-2.01
South East Retail	0.83	0.74	1.12	-6.95	-7.35	-7.66
Rest of UK Retail	1.02	0.74	1.38	-0.55	-0.04	-3.27
London Office	0.95	0.68	1.40	-0.14	0.19	-3.08
South East Office	0.89	0.80	1.11	-5.95	-5.28	-8.16
Rest of UK Office	0.77	0.71	1.08	-6.05	-5.41	-7.14
London Industrial	1.03	0.71	1.45	0.77	0.98	-2.05
South East Industrial	0.93	0.65	1.43	0.58	0.89	-2.81
Rest of UK Industrial	1.02	0.83	1.23	-3.73	-3.19	-5.13
Naï ve	0.94	0.67	1.40	-	0.86	-4.83
IPDMI	0.94	0.68	1.38	-	-	-4.56
Optimal	0.98	0.63	1.56	-	-	-

Note: A positive value indicates the market segment out performed the benchmark portfolio, negative value indicates under performance.

Table 6 presents the results of expanding the London portfolios across the regions of the UK. The first feature of interest is that during the first sub-period there is an immediate gain in performance from diversification, both on a naï ve and optimum basis, across the regions, in comparison with the London based sectors. In contrast during the second sub-period the gains from diversification are much less and in most cases show little improvement compared with the superior performance of the three London property types. In addition the results show that during the first sub-period all of the portfolios, with the exception of the optimal industrial portfolio (with a 100% allocation in the rest of the UK) showed highly significant under-performance against the optimal portfolio benchmark. However, the under performance in the second sub-period was much less. The number of expanded portfolios which out-performed the other two benchmarks was also greater and at higher levels of significance during the second sub-period.

**Table 6: The Performance of Regional Diversification out of London;
Equal and Optimal Allocation and Jobson & Korkie Statistics 87-92 and 93-98**

Property Segment	Mean	SD	Sharpe	London			Benchmarks		
				Retail	Office	Indust	Optimal	Naïve	IPDMI
EW 87-92									
L+SE Retail	0.62	0.98	0.64	2.03	4.90	-0.67	-4.75	-0.33	-1.55
L+SE+REST	0.63	0.90	0.70	2.86	5.78	0.06	-4.41	0.93	-0.46
L+SE Office	0.48	1.49	0.33	-2.54	2.21	-4.14	-10.05	-8.91	-9.69
L+SE+REST	0.69	1.42	0.48	-0.63	4.65	-2.44	-8.66	-4.46	-6.49
L+SE Industrial	1.08	1.37	0.79	2.56	7.15	2.18	-3.99	2.54	1.53
L+SE+REST	1.20	1.33	0.90	3.69	8.86	4.09	-3.14	5.16	4.81
Optimal 87-92									
L+SE Retail	0.60	0.93	0.65	1.47	4.98	-0.45	-5.11	-0.02	-1.30
L+SE+REST	0.63	0.80	0.79	2.87	6.24	0.99	-3.58	2.32	1.04
L+SE Office	0.55	1.41	0.39	-1.67	2.04	-3.47	-8.75	-4.80	-6.65
L+SE+REST	1.09	1.48	0.74	1.76	5.67	0.48	-4.08	1.19	0.25
L+SE Industrial	1.09	1.35	0.80	2.91	8.21	1.79	-3.75	3.33	2.12
L+SE+REST	1.43	1.38	1.04	4.65	9.74	4.49	0.00	6.70	6.22
EW 93-98									
L+SE Retail	0.93	0.71	1.32	-3.57	-0.97	-1.57	-4.88	-2.79	-1.55
L+SE+REST	0.96	0.71	1.35	-2.29	-0.55	-1.20	-4.22	-1.32	-0.84
L+SE Office	0.92	0.71	1.29	-2.12	-2.36	-2.04	-6.06	-3.68	-1.87
L+SE+REST	0.87	0.68	1.27	-2.47	-2.35	-2.52	-6.88	-3.76	-2.90
L+SE Industrial	0.98	0.65	1.51	0.83	1.52	1.59	-1.38	1.76	2.40
L+SE+REST	0.99	0.69	1.45	0.03	0.78	0.09	-2.80	1.46	1.70
Optimal 92-98									
L+SE Retail	1.03	0.71	1.45	-0.02	0.71	0.02	-2.02	0.86	1.20
L+SE+REST	1.03	0.71	1.46	0.44	0.92	0.18	-2.00	1.33	1.80
L+SE Office	0.95	0.68	1.39	-0.71	0.00	-0.62	-3.08	-0.14	-0.19
L+SE+REST	0.95	0.66	1.44	-0.94	-0.46	-0.86	-2.92	0.45	0.02
L+SE Industrial	0.97	0.65	1.50	0.78	1.49	1.26	-1.53	2.31	2.38
L+SE+REST	0.97	0.65	1.50	0.78	1.49	1.26	-1.53	2.31	2.38

Note: A positive value indicates the property portfolios out performed the London based sector or the benchmark portfolios, a negative value indicates under performance.

A similar picture is also observable in the London portfolios, see Table 7. That is during the first sub-period there is an immediate gain from diversification compared with the London property sectors, while the gain in the second period is much less, due to the superior performance of the three property types in the London region. In contrast the performance of the portfolios was still poor during the first sub-period even after the expansion across the various property types in comparison with the benchmarks. However, in the second period diversification resulted in far better performance with a number of the London based portfolios significantly out performing the IPDMI and the naïve benchmark. This implies that in periods of market stability it is easier to achieve greater risk-adjusted performance by diversification than during periods of market turbulence.

Table 7: The Performance of Property Type Diversification within London; Equal and Optimal Allocation and Jobson & Korkie Statistics, 87-92 and 93-98

Property Segment	Mean	SD	Sharpe	London			Benchmark		
				Retail	Office	Indust	Optimal	Naïve	IPDMI
Equal-Weighted 87-92									
R+O	0.53	1.35	0.39	-2.55	3.62	-3.04	-7.63	-6.70	-5.86
R+I	0.86	1.21	0.71	2.50	5.64	0.29	-4.33	-0.89	0.94
O+I	0.74	1.48	0.50	-0.43	5.07	-3.40	-8.34	-6.21	-3.45
R+O+I	0.71	1.30	0.55	0.09	5.89	-2.19	-7.09	-5.57	-2.79
Optimal 87-92									
R+O	0.65	1.20	0.54	0.00	3.26	-1.36	-4.65	-2.64	-1.45
R+I	0.93	1.29	0.72	2.10	5.46	0.68	-4.37	-0.67	1.01
O+I	1.06	1.54	0.69	1.36	4.52	0.00	-4.49	-0.88	0.50
R+O+I	0.93	1.29	0.72	2.10	5.46	0.68	-4.37	-0.67	1.01
Equal-Weighted 92-98									
R+O	0.99	0.66	1.50	1.44	2.50	0.73	-1.62	2.71	2.23
R+I	1.03	0.67	1.54	1.78	2.03	2.12	-0.80	3.45	3.62
O+I	0.99	0.65	1.52	1.02	2.70	1.65	-1.27	2.92	2.91
R+O+I	1.00	0.64	1.56	2.70	3.20	2.10	-0.03	4.43	4.46
Optimal 92-98									
R+O	1.00	0.66	1.50	1.65	2.28	0.76	-1.59	2.75	2.27
R+I	1.03	0.67	1.54	1.81	2.03	2.11	-0.80	3.46	3.62
O+I	0.99	0.65	1.52	1.04	2.51	1.84	-1.24	2.92	2.95
R+O+I	1.01	0.65	1.57	2.16	2.92	2.35	0.00	4.43	4.59

Note: A positive value indicates the property portfolios out performed the London based sectors or the benchmark portfolios, a negative value indicates under performance.

Table 8 presents an even more interesting picture. During the first sub-period the equally weighted London portfolios only outperformed the office sector regionally diversified portfolios, as a result of the poor office market performance during this period. The optimally diversified portfolios outperformed all but industrial properties spread regionally. In contrast during the second sub-period all the London based portfolios significantly outperformed the regionally diversified portfolios because of the excellent performance of all the market sectors in London, see Table 5. This implies that a fund manager who wished to stay in the London region can achieve good risk-adjusted performance and even out-perform a number of benchmark portfolios, but that this is dependent on the time period and the weighting scheme employed.

Table 8: Jobson & Korkie Statistics for the London Portfolios against the Regional Portfolios; Equal and Optimal Allocation, 87-92 and 93-98

Regional Portfolios	London Portfolio 87-92		London Portfolio 93-98	
	EW	Optimal	EW	Optimal
Equal-Weighted				
L+SE Retail	-1.57	-1.23	5.22	4.95
L+SE+REST	-1.64	0.03	4.48	4.29
L+SE Office	5.02	5.34	5.94	6.04
L+SE+REST	2.50	3.04	6.65	6.76
L+SE Industrial	-5.22	-1.77	1.10	1.12
L+SE+REST	-5.63	-3.62	2.27	2.20
Optimal				
L+SE Retail	-1.64	0.73	2.26	2.12
L+SE+REST	-3.22	1.17	2.24	2.10
L+SE Office	-2.50	4.64	3.20	3.24
L+SE+REST	-2.32	0.41	4.30	4.10
L+SE Industrial	-5.63	-1.88	1.16	1.16
L+SE+REST	-7.09	-4.33	1.16	1.16

Note: A positive value indicates the London portfolio out performed the regional portfolios a negative value indicates under performance.

VI. Concluding Comments

In investigating the potential gains from sector and regional diversification within the UK property market, this paper has used mean-variance analysis to examine in turn the role of a number of different investment strategies. Furthermore, to make the portfolio analysis more applicable to actual portfolio experiences all the scenarios start with a realistic portfolio base, i.e. a large holding in London. The mean-variance approach requires foresight of future mean returns, variances and correlations between investment opportunities to show whether the potential gains are likely to be realised. However, by examining realistic scenarios in which perfect foresight is not required the findings here provide the real estate manager with insights into the sector/regional decision choice.

First, the results in Table 1 show that staying within only one sector and one region (London) is undesirable in terms of risk and return compared with all three benchmark portfolios considered here. Secondly diversification on a naïve basis or in an optimal fashion leads to significant improvements in performance, irrespective of whether it is across different property types within London or within the same sector across the regions. Finally the results in Tables 4 and 8 show that staying within London and diversifying across the various property types may offer performance comparable with regional diversification, although this conclusion largely depends on the time period and the fund manager's ability to diversify efficiently.

However, it should be noted that risk factors other than simple mean-variance considerations play a role in the decision making process in the real estate market. Thus although spreading the portfolio across various sectors or regions may enhanced returns

and make a valuable contribution to the reduction in portfolio risk, such diversification is achieved at a cost. In deciding to move out of say the London office market the fund manager may face additional 'risks' not accounted for in portfolio returns. For example, the fund manager may face potentially higher transaction costs, lower liquidity and less information. In practice real estate fund managers may need to moderate the conclusions of a mean-variance analysis, refining their investment strategy in the light of these other considerations. Thus the extent of inclusion of a particular sector or region of the market depends largely on the individual's tolerance to specific risks not accounted for in the mean-variance-based procedure. Nonetheless the results suggest that diversification almost always offers increased performance. Indeed a little diversification can quickly lead to levels of performance that is superior to number of benchmarks as well as performance insignificantly different from that of the most diversified portfolio that could be constructed! Consequently fund managers should be encouraged to diversify, be it across the regions or across the sectors of the UK.

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Appendix 1

Table 1: The Performance of Equal and Optimal Allocation of Sector/Regional Diversification Out of the London Region: Full Period

Property Market	Jobson & Korkie
	Equal v Optimal
Retail	
L+SE	-0.05
L+SE+REST	-3.07
Office	
L+SE	-1.14
L+SE+REST	-2.55
Industrial	
L+SE	-0.44
L+SE+REST	-1.50

Note: A positive value indicates the equal-weighted portfolio out performed the optimal allocation a negative value indicates the optimal allocation out performed the naï ve strategy

Table 2: The Performance of Equal and Optimal Sector Allocation Within the London Region: Full Period

Property Segment	Jobson & Korkie
	Equal v Optimal
R+O	-3.65
R+I	0.00
O+I	-3.12
R+O+I	-5.34

Note: A positive value indicates the equal-weighted portfolio out performed the optimal allocation a negative value indicates the optimal allocation out performed the naï ve strategy